Something Strange? A glance into Parton Fragmentation

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Hadronisation in Particle Collisions is modelled by some Monte-Carlo generators by a model called the Lund Model. With a recent increase in strange enhancement seen in high multiplicity collisions at ALICE, a review of the Lund Model and it's formulation is undertaken. Further, investigation into modifying the Schwinger Model in QCD is conducted in order to realise the new phenomena, by using a lower order exponential dependence, similar to a thermal distribution like Boltsmann's Distribution. We find that this sort of parametrisation suits desired suppressive behaviour, however have yet been able to test this modification in event generators such as PYTHIA due to timing constraints of this project.

This article is aimed for undergraduate use, seeming the project was completed by an Undergraduate Student. The language should be accessible for anyone who has studied undergraduate physics.

I. INTRODUCTION

In recent results, seen at the ALICE collaboration[2] at CERN, a higher density of strange particles has been observed in charged particle high multiplicity collisions. 'High multiplicity' refers to the presence of other particles. In this case, we talk about the presence of other charged quarks.

Current Monte-Carlo event generators such as PYTHIA[8], DIPSY or EPOS, which simulate the production rates of different particles by running Monte-Carlo algorithms on theoretical models, currently do not reflect the behaviour seen in these results. This motivates investigation into the underlying models that dictate the production probability spectrum.

This paper specifically looks into the Lund Model, which is used in PYTHIA. The Lund Model is a theoretical framework for modelling the production of Quarks in particle collisions. Largely developed by Bo Andersson[1], this theory has successfully provided insight to processes observed at the LHC, supporting modelling and providing predictions for experimental data. This report focuses specifically on String Fragmentation, which describes the decay of strings (strong fields) between quarks. This model uses the "Massless Relativistic String" to derive it's probabilities. This provides necessary foundations for the Lund Model.

To explain the Lund Model with enough detail for an undergraduate audience to understand, this paper will cover background theory to the Massless Relativistic String, such as light cones in Minkowski space and quark basics. We will then investigate the Schwinger Model in Quantum Chromodynamics (QCD), and begin analysing the distribution effects of these models and modifications we make.

A. Just 3 Dimensions? 'No way', Minkowski says...

Although we perceive the world in three dimensions, for particle physicists it's much more natural to consider a fourth dimension; time. We call this set of dimensions *space-time*. By considering a distance between two coordinates, given by the difference in spacial and time co-ordinates, physicists can categorise various events.

$$\begin{aligned} X &= (ct, x, y, z) \\ &= (t, \vec{\mathbf{x}}) \end{aligned} \tag{1}$$

Here 'c' is the speed of light. The distance from the origin can be calculated as such:

$$\begin{aligned} |X| &= \sqrt{X^2} \\ &= \sqrt{c^2t^2 - \vec{\mathbf{x}}^2} \end{aligned} \tag{2}$$

You may be surprised by the negative sign, but that is essential to describing how time and space can interact. This metric is called *Minkowski Space*, and it calculates a magnitude by the evaluating the difference between the light and space components (squared). We also apply the same metric for other similar vectors, such as Total Energy and Momentum (*energy-momentum* or *four-momentum*), and *four-velocity*.

Given Minkowski space, the notion of *lightcones* can be described. Imagine you travel at the speed of light from one point in space to another. Then we can describe your coordinate transform as:

$$\begin{aligned} X &\to X + \Delta X \\ &\to (c\,(t + \Delta t)\,, \vec{\mathbf{x}} + \Delta\vec{x}) \end{aligned} \tag{3}$$

By using the distance light travels in time,

$$\Delta\vec{\mathbf{x}} = c\Delta t \tag{4}$$

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we can now consider your displacement from the original point.

$$
\begin{aligned}
|\Delta X| &= \sqrt{(X_2 - X_1)^2} \\
&= \sqrt{\left[\left(c\left(t+\Delta t\right), \vec{\mathbf{x}} + \Delta \vec{x}\right) - \left(ct, \vec{\mathbf{x}}\right)\right]^2} \\
&= \sqrt{c^2 (\Delta t)^2 - (\Delta \vec{x})^2} \\
&= 0
\end{aligned}
\tag{5}
$$

We call this behaviour *lightlike*, when there is a zero magnitude difference between the coordinates. We can think about this like having travelled the same speed as light between to spacial coordinates.

Analogously, if the distance between two coordinates is negative, we have a *spacelike* event. How might you think of this? It is the case where light itself cannot travel fast enough to cover the space gap, or something has travelled faster than the speed of light (impossible). It suggests the events are independent of each other.

Lastly, *timelike* events are the usual ordinary events, for anything with matter and mass. Our physical world is bounded by *causality*, the idea that events have a dependence on past events in time. We also consider that we have finite velocity smaller than the speed of light.

Interpreting these light cones as geometric shapes, we can add meaning to a cone. Consider a 1D space that we can move in. The 'lightcone' will contain all possible velocities that anything physical can travel at, limited by the speed of light. From here on this paper uses natural units, which assume $c = 1$.

B. Motivations To Go Deeper

Scattering and observing the collisions of particles has been done ever since Rutherford scattered alpha nuclei off nuclear targets. By doing so, physicists have been able to gain appreciable understanding of atomic physics, decays and surface charges. This was progressed by using more significant, energetic beams, probing into the structure of the nucleus. Later, single nucleons were used allowing the study of protons and neutrons. We began to discover that a proton wasn't a single point charge, but rather a smear of charges. Then began the discovery of quarks, and their resonances seen in distributions. As we keep probing deeper into the foundations of our Universe, it's exciting to reflect on the progress made within the last 100 years. We can expect to understand more exciting physics, and if not we're much better off for trying.

C. Brief introduction to Quarks

Quarks are one of the achievements seen in the late 20th century by particle accelerators. Quarks, Leptons and Bosons make up the core of the 'Standard Model', the current used model for fundamental particles. Quarks (and Gluons) are also referred to as Partons, which were proposed by Richard Feynman in the late 1960's. There are 6 different known Quarks, as seen in Figure 1.

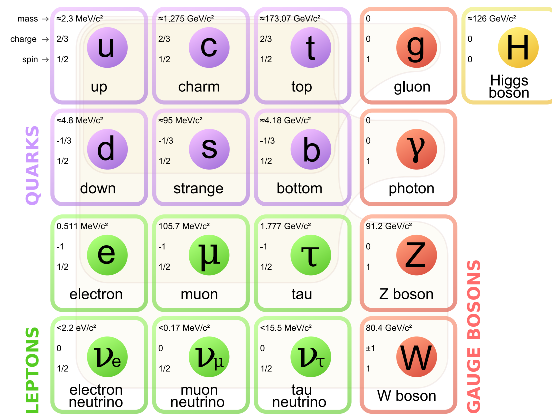

FIG. 1. The Standard Model Of Particle Physics[5]

Each quark is distinguished experimentally by it's mass. The Top quark was only discovered in 1995 due requiring high enough beam energies to appropriately detect it's large mass, seen by the Tevatron, at Fermilab, in Illinois USA. Up and Down quarks, the two lightest quarks, are what constitute the protons and neutrons that we commonly talk about in nuclear science. A Strange quark is the next heaviest quark, however it is not seen in what we consider stable particles like the Neutron and Proton. For example, a Kaon (K^+,K^-,K^0), which is constituted by a strange quark and a up/down quark, has a lifetime of order 10ns, where as a proton hasn't been observed to naturally decay, but some 'theories' give it a lifetime of order $10^2 9$ years. Because of it's heaviness, the energy needed to create it, we expect to see strange quarks produced less in energetic particle collisions.

A very nice concept of quarks is **confinement**. Quarks interact through a force called the strong force, it's study called Quantum Chromodynamics (QCD). Quarks are never seen individually, because they have a colour charge in addition to their electric charge. This colour can never have a net colour - it needs to maintain neutrality. There are three colours (six include anti-colour of antiparticles), and they are often represented as Red, Green & Blue (RGB), conforming to the primary colours of human vision. Confinement requires there to be a neutrality of colour locally for quarks to exist. For example, having $\{1 \times R,\ 1 \times G,\ 1 \times B\}$ (a hadron) gives neutrality, like white, and so does $\{1 \times R,\ 1 \times \bar{R}\}$ (a meson).

II. THE LUND MODEL

The Lund Model is used to formulate a distribution function for generating a shower of partons - the process of fragmentation. In this section, it will be shown how to derive the two key probability distribution functions that contribute to the fragmentation of 'strings' (to be explained).

The first of these is the probability of having a string break at a particular coordinate. This coordinate is given as the hyperbolic coordinate Γ, but it's conceptual meaning is a bit harder to understand. This will be elaborated on below.

$$H(\Gamma) = C\Gamma^a \exp(-b\Gamma) \tag{6}$$

The second of these is the probability of using a percentage z of a pool of energy momentum.

$$f(z) = N\frac{1}{z}(1-z)^{-a}\exp\left(\frac{-bm^2}{z}\right) \tag{7}$$

The origin of these equations was investigated in order to see if one could change the order of mass term in the function $f(z)$.

A. The Massless Relativistic String

This finally brings us the beginning of the massless relativistic string (MRS) model!

From studying the behaviour of QCD (Quantum Chromodynamics) it is known that the potential of quarks has a linear component which becomes dominant at still relatively short distances. This implies we can approximate a constant force between quarks, and a linear 'string' potential.

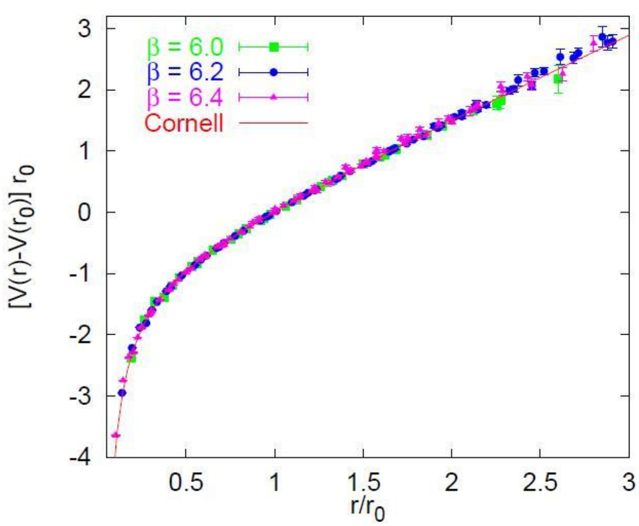

FIG. 2. Lattice QCD Potential[7]

The 'string' actually represents the field lines between quarks, which are approximately linear at a small distance scale. You can think of these field lines similar to electric charge field lines.

When the quarks are really close, it's not so linear, but it's a useful approximation for our purposes. This is used to explain the confinement behaviour of quarks; that as spatially separate, they rapidly loose momentum to this linearly growing potential.

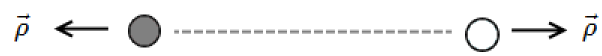

FIG. 3. A string between two quarks with momenta p.

Jets, which are collections of Partons (Quarks and Gluons), can be observed in particle detectors. The general idea in quark production is that initially, a pair(s) of quarks (a quark and it's anti-quark) will be created in the high energy collision, with a large amount of momentum. We refer to this combined quantity as the Energy-momentum. These quarks travel short distances before producing more particles, eventually seen as 'showers' of hadrons, in a process called 'hadronization'. This is seen in Figure 4.

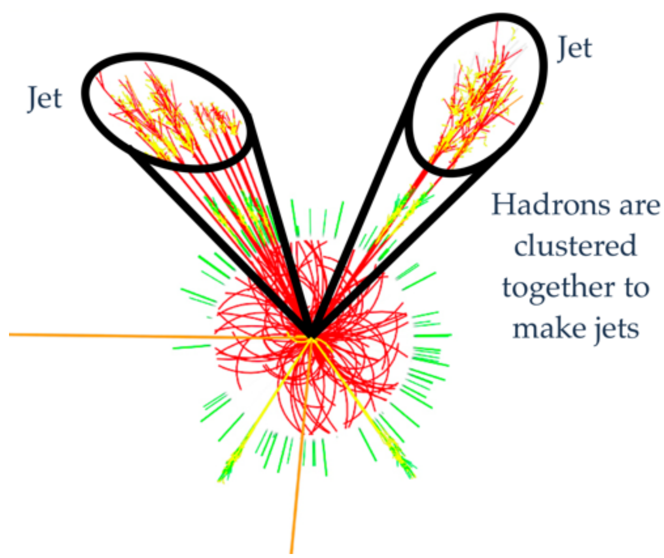

FIG. 4. Cross Section of a Detector, in a Collider Event[10]

The role of The Lund Model and the MRS is to generate this distribution of hadronic constituents, the quarks. This is modelled by the breakup of the 'string' as the length of the string increases as the quarks separate from each other.

In this article we will primarily investigate the 1D MRS model, which assumes the strings have momentum and fragment in the same axis. Of course there is momentum transverse to this, which can be included in the mass terms as a transverse mass (ie. $m_\perp^2 = m^2 + p_\perp^2$).

We also approximate particles to be ‘massless’, meaning that they will move exactly along the lightcone axis. Realistically, their paths would be hyperbolic in nature, curving as they slow down to a stationary point.

B. Coordinates

A discussion of the coordinates used in these derivations is required to conceptually understand the Lund Model.

Firstly, space-time coordinates are used to locate points of interest, such as where a string breaks (a vertex) or when a quark becomes stationary. A good example to discuss is a produced pair of quarks that have a small amount of energy momentum; the string won't break (Figure 5).

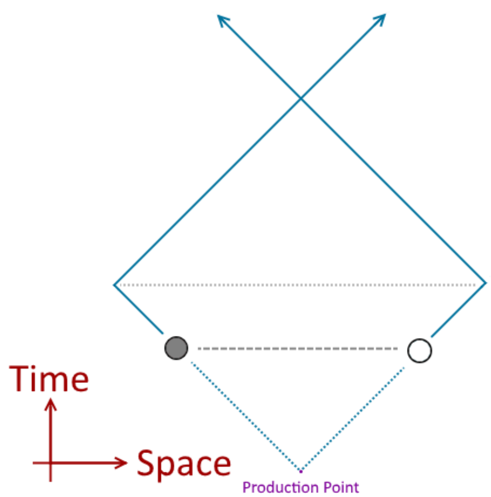

FIG. 5. Yoyo-mode - Pair of Quarks

Analysing the diagram, we can observe that at a certain time, the quarks become stationary. At this point all the kinetic energy is stored as potential in the string, or strong field. After this time the quarks proceed to come back towards each other. For this reason, the state is considered a ‘yoyo’ mode, and alludes to a stable meson (hadron consisting of two quarks).

However, space-time coordinates aren't that useful in Particle Physics. We're more concerned about the amount of energy the particles have, and consequently it's more useful to describe evolution of the string in terms of the lightcone energy-momentum components. So first we'll transform the space-time to lightcone coordinates, by the following transformation:

$$x_{\pm} = t \pm x \tag{8}$$

Now that we're in lightcone space-time, we can *scale* from space-time into energy-momentum space. The field strength is defined a constant between quarks.

$$\frac{\partial E}{\partial x} = \frac{\partial p}{\partial t} = -\kappa \tag{9}$$

Because the Lund String Model begins with the production of a pair of quarks with some energy-momentum quantity, it's actually more intuitive to work in this space. The space-time model depends on the energy-momentum space; The initial energy-momentum determines the amount of space a pair of quarks will traverse.

We now need to consider a hyperbolic conversion of the coordinates. We are doing this because it will prove useful in deriving our function $H(\Gamma)$. Traditionally, hyperbolic coordinates map from {x,y}→{u,v}. We do the same, with $\{x_-,x_+\}\rightarrow\{\sqrt{\Gamma},y\}$, but treat Γ as the coordinate.

$$\Gamma = \kappa^2 x_- x_+ \tag{10}$$

$$y = \frac{1}{2}\ln\left(\frac{x_+}{x_-}\right) \tag{11}$$

The iterations of these transformations is shown in Figure 6.

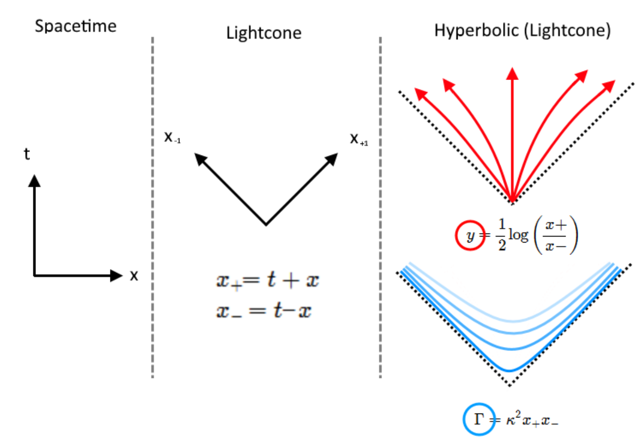

FIG. 6. Spacetime, Lightcone and Hyperbolic Coordinates

In often trying to find some intuition as to how to understand Γ and y in relation to a vertex, it's clear that y is the same quantity known as the Rapidity.

A rapidity in this 1D case will provide a ratio of momentum to energy of the particle, with high magnitude rapidities producing large velocity particles. However, this is only useful in considering the distance between two vertexes and the resultant produced particle, rather y in this case is just a coordinate for a vertex.

Γ, is known as the ‘squared proper time of the vertex’. This seems to suggest a magnitude of energy-momentum.

Interestingly enough, in (energy-)momentum space, the encapsulated by a yoyo-mode determines it's effective mass. An example of this is the orange and blue areas seen in Figure 7.

Intuiting this, the larger that the Γ of a vertex becomes the less kinetic energy will be seen in a resulting particle. This is because the potential area of yoyo-modes will be *diminished* the *earlier* a vertex is created (or when the string breaks). An example is shown in Figure 7.

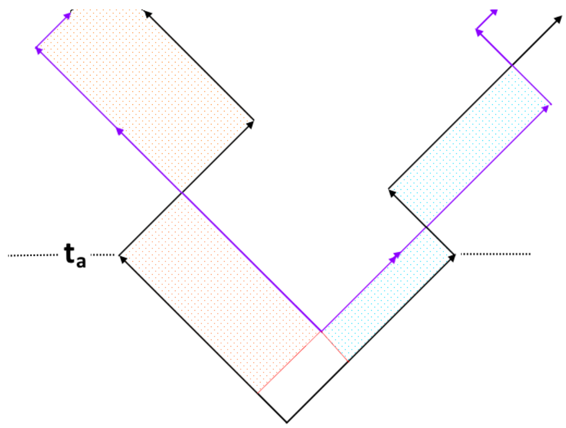

FIG. 7. String Fragmentation. Here splitting occurs at an early time. Consider if splitting occurred at t_a.

C. Breakup Distribution

Having introduced most of the terminology and background to the MRS in the Lund Model, the distribution for the production of a hadron (in this case meson) will be derived.

There is a scenario where we consider that the string has already broken at an arbitrary number of places counting down a light cone. This process is considered independent of past events; each vertex is spacelike from each other. This is shown in Figure 8. In particular we consider two arbitrary vertexes (where the string breaks) that will produce a hadron between them. They have coordinates $\{\Gamma_1,y_1\}$ and $\{\Gamma_2,y_2\}$.

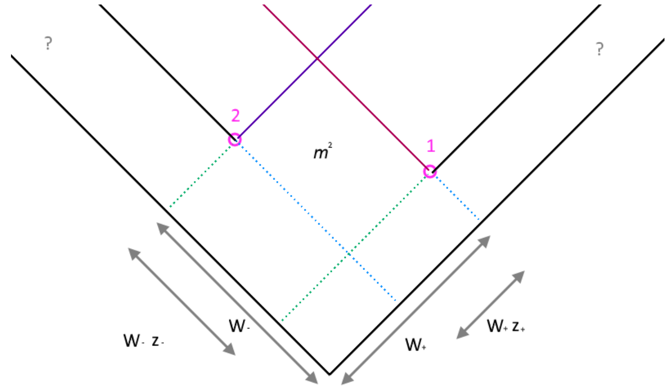

FIG. 8. Hadron Production between Vertexes 1 & 2

Because there are an arbitrary number of vertexes before considered vertexes 1 & 2, it is noted that there is initially $W_\pm$ amount of energy-momentum left in the $\pm$ lightcone to produce the hadron from. However, the hadron doesn't necessarily use all this energy-momenta, and consequently two random variables $z\pm$ determine the percentage of the remaining lightcone to be used.

Using Figure 8, the equations for the mass of the system and the coordinates can be noted.

$$\Gamma_1 = (1 - z_-)W_-W_+ \tag{12}$$
$$\Gamma_2 = W_-(1 - z_+)W_+ \tag{13}$$
$$m^2 = (z_-W_-)(z_+W_+) \tag{14}$$

We can now write the probability of producing a hadron.

First, we have a distribution $H(\text{Vertex}_1)$, which determines the probability of arriving at vertex 1 with W_+ remaining energy-momentum in the positive lightcone. Therefore, H is actually a function of the coordinates Γ and y. However, the Lund Model argues that H is independent of y, for reasons that will be explored in subsection E.

Secondly we have a distribution $f_{12}(z_+)$ which determines the probability of taking another 'step' to vertex 2, using z percentage of the remaining W_+ lightcone.

The probability of producing the hadron can then be written as a joint product of these two probabilities:

$$H(\Gamma_1)\,\partial\Gamma_1\partial y_1 f_{12}(z_+)\partial z_+ \tag{15}$$

By the same method, we can also write a second probability for considering the energy-momenta of the negative Lightcone.

$$H(\Gamma_2)\,\partial\Gamma_2\partial y_2 f_{21}(z_-)\partial z_- \tag{16}$$

We will proceed to equate these two probabilities together, in an effort to generate independent solutions for

H and f. Note that the integrals over y can be cancelled, seeming all functions are independent of y.

Using equations (12-14) to remove $W_\pm$ dependence, and then differentiating Γ_1 & Γ_2 WRT $z_\mp$, we find that:

$$\frac{\partial \Gamma_1}{\partial z_-} = \frac{-m^2}{z_-^2 z_+} \tag{17}$$
$$\frac{\partial \Gamma_2}{\partial z_+} = \frac{-m^2}{z_- z_+^2} \tag{18}$$

This results in:

$$H(\Gamma_1) z_+ f_{12}(z_+) = H(\Gamma_2) z_- f_{21}(z_-) \tag{19}$$

Taking the logarithm of this, the probability can be written in terms of a sum. Here $g(z) = \exp(z_- f(z_-))$ and $h(z) = \exp(H(\Gamma))$.

$$h(\Gamma_1) + g_{12}(z_+) = h(\Gamma_2) + g_{21}(z_-) \tag{20}$$
$$h_1 + g_+ = h_2 + g_- \tag{21}$$

Differentiate WRT z_+

$$\frac{\partial h_1}{\partial z_+} + \frac{\partial g_+}{\partial z_+} = \frac{\partial h_2}{\partial z_+} + \frac{\partial g_-}{\partial z_+} \tag{22}$$

Note that neither $g_\pm$ has a dependence on $g_\mp$.

$$\frac{\partial h_1}{\partial z_+} + \frac{\partial g_+}{\partial z_+} = \frac{\partial h_2}{\partial z_+} \tag{23}$$

and now Differentiate WRT z_-.

$$\frac{\partial}{\partial z_-}\left(\frac{\partial h_1}{\partial z_+} + \frac{\partial g_+}{\partial z_+}\right) = \frac{\partial}{\partial z_-}\left(\frac{\partial h_2}{\partial z_+}\right) \tag{24}$$
$$\frac{\partial}{\partial z_-}\left(\frac{\partial h_1}{\partial z_+}\right) = \frac{\partial}{\partial z_-}\left(\frac{\partial h_2}{\partial z_+}\right) \tag{25}$$

By the chain rule:

$$\frac{\partial}{\partial z_-}\left(\frac{\partial h_1}{\partial \Gamma_1}\frac{\partial \Gamma_1}{\partial z_+}\right) = \frac{\partial}{\partial z_-}\left(\frac{\partial h_2}{\partial \Gamma_2}\frac{\partial \Gamma_2}{\partial z_+}\right) \tag{26}$$

Expanding:

$$\begin{aligned}\frac{\partial \Gamma_1}{\partial z_+}\frac{\partial}{\partial z_-}\left(\frac{\partial h_1}{\partial \Gamma_1}\right) + \frac{\partial h_1}{\partial \Gamma_1}\frac{\partial}{\partial z_-}\left(\frac{\partial \Gamma_1}{\partial z_+}\right) = \\ \frac{\partial \Gamma_2}{\partial z_-}\frac{\partial}{\partial z_+}\left(\frac{\partial h_2}{\partial \Gamma_2}\right) + \frac{\partial h_2}{\partial \Gamma_2}\frac{\partial}{\partial z_+}\left(\frac{\partial \Gamma_2}{\partial z_-}\right)\end{aligned} \tag{27}$$

Simplifying:

$$\begin{aligned}\frac{\partial \Gamma_1}{\partial z_+}\frac{\partial \Gamma_1}{\partial z_-}\frac{\partial h_1^2}{\partial \Gamma_1^2} + \frac{\partial h_1}{\partial \Gamma_1}\frac{\partial^2 \Gamma_1}{\partial z_+ \partial z_-} = \\ \frac{\partial \Gamma_2}{\partial z_+}\frac{\partial \Gamma_2}{\partial z_-}\frac{\partial h_2^2}{\partial \Gamma_2^2} + \frac{\partial h_2}{\partial \Gamma_2}\frac{\partial^2 \Gamma_2}{\partial z_+ \partial z_-}\end{aligned} \tag{28}$$

Recognising that from (12-14) the following factors can be derived:

$$\frac{\partial^2 \Gamma_1}{\partial z_+ \partial z_-} = \frac{\partial^2 \Gamma_2}{\partial z_+ \partial z_-} = \frac{m^2}{z_-^2 z_+^2} \tag{29}$$
$$\frac{\partial \Gamma_1}{\partial z_+}\frac{\partial \Gamma_1}{\partial z_-} = \frac{m^4}{z_-^3 z_+^3}(1 - z_-) \tag{30}$$
$$\frac{\partial \Gamma_2}{\partial z_+}\frac{\partial \Gamma_2}{\partial z_-} = \frac{m^4}{z_-^3 z_+^3}(1 - z_+) \tag{31}$$

We observe that (30)/(29) = Γ_1 and (31/29) = Γ_2. Dividing (28) by (29), and using the observed property, gives the following form:

$$\frac{\partial h_1}{\partial \Gamma_1} + \Gamma_1 \frac{\partial^2 h_1}{\partial \Gamma_1^2} = \frac{\partial h_2}{\partial \Gamma_2} + \Gamma_2 \frac{\partial^2 h_2}{\partial \Gamma_2^2} \tag{32}$$

We can simplify this:

$$\frac{\partial}{\partial \Gamma_1}\left(\Gamma_1 \frac{\partial h_1}{\partial \Gamma_1}\right) = \frac{\partial}{\partial \Gamma_2}\left(\Gamma_2 \frac{\partial h_2}{\partial \Gamma_2}\right) \tag{33}$$
$$\tag{34}$$

Lastly, we recognise that each side of this equation is independent upon it's own Γ. This means that expression is constant.

$$\frac{\partial}{\partial \Gamma}\left(\Gamma \frac{\partial h}{\partial \Gamma}\right) = b \tag{35}$$

By integrating and re-arranging, the form of $h(\Gamma)$ is:

$$h(\Gamma) = -b\Gamma + a \ln(\Gamma) + \ln(C) \tag{36}$$

Here C and a are arbitrary integration constants. $H(\Gamma)$ is found by taking the exponential of this:

$$H(\Gamma) = C\Gamma^a \exp(-b\Gamma) \tag{37}$$

Substituting (36) back into (20) allows us to solve for $g(z)$, in a similar manner.

$$\begin{aligned}g_{12}(z_+) - b\Gamma_1 + a_1 \ln(\Gamma_1) + \ln(C_1) = \\ g_{21}(z_-) - b\Gamma_2 + a_2 \ln(\Gamma_2) + \ln(C_2)\end{aligned} \tag{38}$$

Substituting Γ from (12-14) in terms of mass and $z_\pm$, the common constant factor cancels out.

$$\begin{aligned}g_{12}(z_+) + \frac{bm^2}{z_+} + a_1 \ln\left(\frac{m^2}{z_+ z_-} - \frac{m^2}{z_+}\right) + \ln(C_1) = \\ g_{21}(z_-) + \frac{bm^2}{z_-} + a_2 \ln\left(\frac{m^2}{z_+ z_-} - \frac{m^2}{z_-}\right) + \ln(C_2)\end{aligned} \tag{39}$$

Rewrite the log terms '$a \ln\left(\frac{1-z_+}{z_+ z_-} m^2\right)$' as a sum:

$$g_{12}(z_+) + \frac{bm^2}{z_+} + a_1 \ln\left(\frac{m^2}{z_-}\right) + a_1 \ln\left(\frac{1-z+}{z_+}\right) + \ln(C_1) = g_{21}(z_-) + \frac{bm^2}{z_-} + a_2 \ln\left(\frac{m^2}{z_+}\right) + a_2 \ln\left(\frac{1-z-}{z_-}\right) + \ln(C_2) \tag{40}$$

Again, re-arrange to gain independence in $z_\pm$:

$$g_{12}(z_+) + \frac{bm^2}{z_+} - a_2 \ln\left(\frac{m^2}{z_+}\right) + a_1 \ln\left(\frac{1-z+}{z_+}\right) + \ln(C_1) = g_{21}(z_-) + \frac{bm^2}{z_-} - a_1 \ln\left(\frac{m^2}{z_-}\right) + a_2 \ln\left(\frac{1-z-}{z_-}\right) + \ln(C_2) \tag{41}$$

Recognising that both sides of this equation are independent, we use a constant n to describe their relationship. We can ignore the constants $\ln(C)$, and include them into n.

$$n = g_{12}(z_+) + \frac{bm^2}{z_+} - a_2 \ln\left(\frac{m^2}{z_+}\right) + a_1 \ln\left(\frac{1-z+}{z_+}\right) \tag{42}$$

Re-arrange for g_12:

$$g_{12}(z_+) = n - \frac{bm^2}{z_+} + a_2 \ln\left(\frac{m^2}{z_+}\right) - a_1 \ln\left(\frac{1-z+}{z_+}\right) \tag{43}$$

Taking the exponential of this equation:

$$z_+ f_{12}(z_+) = N\left(\frac{m^2}{z_+}\right)^{-a_1}\left(\frac{1-z_+}{z_+}\right)^{a_2} \exp\left(\frac{-bm^2}{z_+}\right) \tag{44}$$

$$f_{12}(z_+) = N(z_+)^{a_1 - a_2 - 1}(1-z_+)^{a_2} \exp\left(\frac{-bm^2}{z_+}\right) \tag{45}$$

where $N = \exp(n) \times m^{-2a_1}$.

If we consider the case where $a_1 == a_2$ then the expression simplifies to

$$f(z) = N\frac{1}{z}(1-z)^a \exp\left(\frac{-bm^2}{z}\right) \tag{46}$$

This is actually slightly different to the equations in The Lund Model[1], but only because of a few algebraic errors with signs, in the last few equations of the derivation. His normalisation constants in 8.18 for N are seemingly incorrect, as well as a sign for the general case of $a_1 \neq a_2$.

D. Outcomes?

Having re-derived this result, it's clear that there's no room to change the order of any of the terms, such as the order of the mass contribution. This was a desired change, because in current models a dependence on mass is a constant ratio for different quarks, roughly in the order observed[6] as:

$$1 \times \mathrm{U} : 1 \times \mathrm{D} : 0.3 \times \mathrm{S} : 10^{-11} \times \mathrm{C} \tag{47}$$

In the next section this article will touch further on this, and how else we might achieve our strange enhancement at high multiplicity.

E. H independent of Rapidity?

The assertion was made earlier that the rapidity elements $\partial y_1 == \partial y_2$. Although not rigorously addressed, some thought was put into why this might be. Consider skipping this subsection as it is not very rigorous and lacks conclusion.

In Hadron collisions, the production of particles across different rapidity angles is *observed* to be constant. Theoretically, it can be argued that only the magnitude of the coordinate matters with respect to H(Vertex).

If we Lorentz boost, the magnitude of the momentum vectors (the mass) stays the same, which Γ is proportionate to. However, the rapidity is additive in a Lorentz boost, suggesting if there was a dependency on y then our frame of reference would change the probability of a boosted vertex coordinate. But we require Lorentz invariance, as all frames should behave with the same physics.

We can see hints that this is possible. Consider that the mass is invariant.

$$m^2 = (z_+ W_+)(z_- W_-) \tag{48}$$
$$= \kappa^2 (z_{+1} x_{+1})(z_{-2} x_{-2}) \tag{49}$$

Using the energy-momentum components along the light-cones,

$$W_{-2}(1-z_-) = W_{-1} \tag{50}$$
$$W_{+1}(1-z_+) = W_{+2} \tag{51}$$

z can be expressed as

$$z_- = 1 - \frac{x_{-1}}{x_{-2}} \tag{52}$$

$$z_+ = 1 - \frac{x_{+2}}{x_{+1}} \tag{53}$$

Substituting these into (49) gives

$$m^2 = \kappa^2 (x_{+1} - x_{+2})(x_{-2} - x_{-1}) \tag{54}$$

Now, consider a very small change in the one of the coordinates. Because the mass is invariant, then to match the change, we see:

$$\begin{aligned}\partial(m^2) = 0 = \kappa^2 (\partial x_{+1} - \partial x_{+2})(x_{-2} - x_{-1}) \\ +\kappa^2 (x_{+1} - x_{+2})(\partial x_{-2} - \partial x_{-1})\end{aligned} \tag{55}$$

A possible solution to this is

$$\frac{\partial x_{+1}}{\partial x_{+2}} = 1 \tag{56}$$

$$\frac{\partial x_{-2}}{\partial x_{-1}} = 1 \tag{57}$$

If that is the case, consider the rapidity of each coordinate:

$$y_1 = \frac{1}{2}\ln\left(\frac{x_{+1}}{x_{-1}}\right) \tag{58}$$

$$y_2 = \frac{1}{2}\ln\left(\frac{x_{+2}}{x_{-2}}\right) \tag{59}$$

If we consider that ∂y_1 looks like $\frac{1}{2}\ln\left(\frac{\partial x_{+1}}{\partial x_{-1}}\right)$, then by (56 - 57) we know that $\frac{1}{2}\ln\left(\frac{\partial x_{+2}}{\partial x_{-2}}\right)$ looks the same:

$$\frac{\partial x_{+1}}{\partial x_{+2}} \times \frac{\partial x_{-2}}{\partial x_{-1}} = 1 \tag{60}$$

$$\frac{\partial x_{+1}}{\partial x_{-1}} = \frac{\partial x_{+2}}{\partial x_{-2}} \tag{61}$$

In discussion with Cody Duncan, he considered the possibility that the differential element ∂z covers any variation in y. The reason for this is because we can derive both rapidities to be:

$$y_1 = \frac{1}{2}\left(\frac{W_{+2}}{(1-z_+)}\frac{1}{W_{-1}}\right) \tag{62}$$

$$y_2 = \frac{1}{2}\left(\frac{W_{+2}}{W_{-1}}(1-z_-)\right) \tag{63}$$

Notice both forms have the same Momentum dependence, they only rely upon z_+ and z_-, and these are integrated by the $\partial z's$.

III. LOOKING ELSEWHERE - THE SCHWINGER MODEL

As a result of not being able to find a modification that we could make to String Fragmentation at the breakup level, Peter decided to look into an extension of the MRS called the Schwinger Model.

The Schwinger Model was originally created for Quantum Electrodynamics (QED), however it has since been assumed that it's features also apply to the field model of Quantum Chromodynamics (QCD).

Because the Schwinger Model also contributes an exponential factor to the probability distribution, it is also a candidate of interest that we might be able to modify.

A. Intro to Schwinger Model

The Schwinger Model deals with a vacuum in which there are no particles. It deals with the idea that, due to an external field, quantum oscillations will occur in this vacuum drivings a breakdown of the no-particle state to a new state. In this case we are considering a colour field, opposed to QED.

In this project, the details of the Schwinger Model were not rigorously investigated, due to the time scope. Because of this, we assumed that the Schwinger Model had been ported from QED directly into QCD, without direct reasoning as to why it should behave the exact same.

The contribution of the Schinger Model is currently used given in a Gaussian format, meaning that the transverse momentum and mass terms are separable as a result.

$$P(p_\perp, m) = \exp\left(\frac{-\pi}{\kappa}m_\perp^2\right) \tag{64}$$

$$= \exp\left(\frac{-\pi}{\kappa}(p_\perp^2 + m^2)\right) \tag{65}$$

$$= \exp\left(\frac{-\pi}{\kappa}p_\perp^2\right)\exp\left(\frac{-\pi}{\kappa}m^2\right) \tag{66}$$

Our desired behaviour however, conforms more akin to (67).

$$P(p_\perp, m) = \exp(-am_\perp) \tag{67}$$

This is because, as the transverse momentum ($p_\perp$) increases around mass scale, the ratio of different probability distributions for different quark masses will no longer be a constant. Specifically, as a strange quark has a larger mass, it will be suppressed more significantly at smaller $p_\perp$ values compared to it's other flavours (quarks are distinguished by flavour).

B. Using a Thermal Distribution

As you can see in Figure , other reasonable exponential orders of $m_\perp$ will actually give the opposite effect to our desired numerical behaviour.

Thermal behaviour generally occurs with a constant scaling behaviour in the exponential (for example, a boltzmann distrubtion). Thermal models appeal to us, because we can think about the high multiplicity of charged particles as a 'soup' that is quite thermally 'hot'.

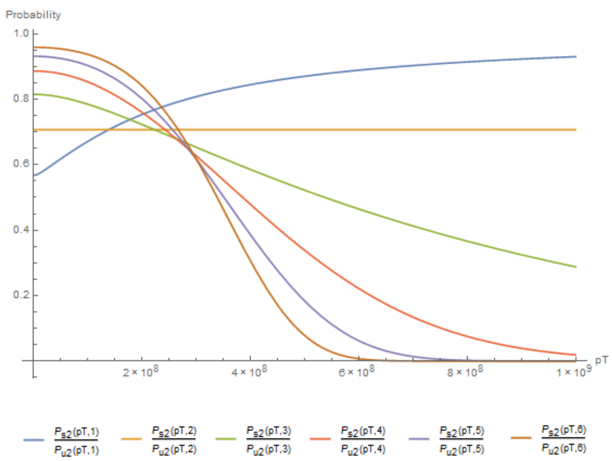

FIG. 9. Strange:Up Quark Supression - Different $m_\perp$ Order Behaviours

Note that the distributions for Up and Strange suppression were not normalised in this graph.

Rather interestingly, this graph was produced for natural units, where most measurements are in terms of eV. In Physics, different length scales are used for different processes. It's worth mentioning here that the field strength κ is measured experimentally as $0.18\text{GeV}^2 = 0.9\text{GeV / fm}$. Out of interest, we can actually calculate the length scale (what a meter is equivalent to) used for string fragmentation.

$$\ell = \frac{0.18\text{GeV}^2}{0.9\text{GeV / fm}} = \frac{0.18 \times 10^{18}}{0.9 \times 10^9 / 10^{-15}} = 2 \times 10^{-7} \quad (68)$$

Returning to using the distribution in (67), we call this a Thermal Distribution, because it is more akin to a thermal behaviour $\frac{1}{T}$, unlike the Gaussian.

C. Expectation of the Thermal Distribution

To investigate this formulation of the modified Schwinger contribution, we can consider an expectation value of the transverse momentum. This is useful because it provides a ballpark figure of what sorts of values a might take.

In the Gaussian case, the resulting value[9] for the expectation value is:

$$\langle p_\perp^2 \rangle = \sigma^2 = \frac{\kappa}{\pi} \approx (237 \text{ MeV})^2 \quad (69)$$

Here sigma is a 'width' of the $p_\perp$ spectrum. This is derived by the following steps (note that $\frac{\pi}{\kappa}$ out the front is the normalisation of the distribution itself.):

$$\begin{aligned}
\langle p_\perp^2 \rangle &= \langle p_x^2 + p_y^2 \rangle^2 && (70)\\
&= \frac{\pi}{\kappa} \int_{-\infty}^{\infty} \int_{\infty}^{\infty} dp_x dp_y p_\perp^2 \exp\left(-\frac{\pi}{\kappa} p_\perp^2\right) && (71)\\
&= \frac{\pi}{\kappa} \int dp_\perp^2 . p_\perp^2 \exp\left(-\frac{\pi}{\kappa} p_\perp^2\right) && (72)\\
&= \frac{\pi}{\kappa} \times \left(\frac{\kappa}{\pi}\right)^2 && (73)\\
&= \frac{\kappa}{\pi} && (74)
\end{aligned}$$

If we follow the same approach for our new model, then we find:

$$\begin{aligned}
\langle p_\perp^2 \rangle &= \langle p_x^2 + p_y^2 \rangle^2 && (75)\\
&= \frac{\int_{-\infty}^{\infty} \int_{\infty}^{\infty} dp_x dp_y p_\perp^2 \exp(-a m_\perp)}{\int_{-\infty}^{\infty} dp_x dp_y \exp(-a m_\perp)} && (76)\\
&= \frac{\pi \int_0^\infty dp_\perp^2 p_\perp^2 \exp\left(-a\sqrt{m^2 + p_\perp^2}\right)}{\pi \int_0^\infty dp_\perp^2 \exp\left(-a\sqrt{m^2 + p_\perp^2}\right)} && (77)\\
&= \frac{4\exp(-am)\left(3 + a^2m^2 + 3am\right)/a^4}{2\exp(-am)(1 + am)/a^2} && (78)\\
&= \frac{2}{a^2}\left(\frac{3(1 + am) + a^2m^2}{1 + am}\right) && (79)\\
\langle p_\perp^2 \rangle &= \frac{6}{a^2} + \frac{2m^2}{1 + am} && (80)
\end{aligned}$$

This is a very nice result to look at: there is now a mass dependence in the expectation $p_\perp$, that with an increase in mass the expectation will grow larger.

We note here that this is not a very rigorous way of trying to find the parameter a. a will vary for each mass body if we solve the last equation. In fact, solving this cubic through Mathematica produces complex solutions (extremely small magnitude though, assuming numerical precision errors). We won't display the algebra for the solutions here, it's quite messy and too large for these pages, and likely still unhelpful without further analysis.

D. Thoughts on an a_0 Value

One further approach Peter and I tried to follow was to assign a value for a when the mass is equal to 0.

$$a_{m=0} = \frac{\sqrt{6}}{\sigma} \quad (81)$$

By equating this to our known σ width we'd have a starting point for an a value.

This however, was not significantly useful when comparing various masses, although in Figure 9 a ballpark $p_\perp$ seems to give a value for the ratio of $S:U$ of about 0.3 as desired.

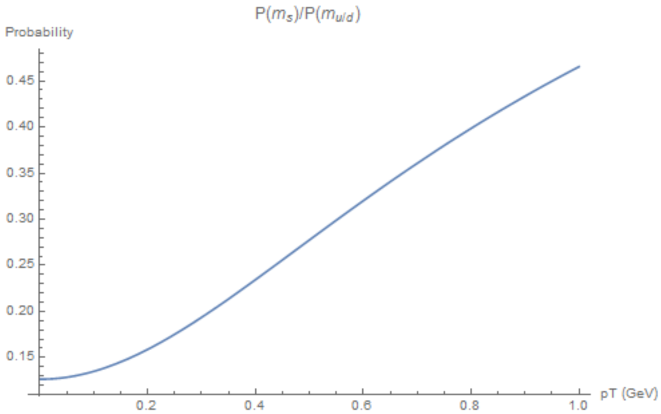

FIG. 10. Strange:Up Quark Supression - Thermal a_0

At this scale, the mass of a quark is not a rigorously known value. The values we commonly known for the masses of different particles come from a 'scheme'[4], which re-normalises and solves various equation complexities in a particular way. They are not defined in this same way at this scale. Ballpark masses that we use for different particles at this scale are as follows (in GeV):

$$m_\pi = 0.1 \tag{82}$$
$$m_U = 0.3 \tag{83}$$
$$m_D = 0.3 \tag{84}$$
$$m_S = 0.5 \tag{85}$$

Because we used expectation values to figure out a parameterisation of a, but couldn't find any simple solutions for a given mass like the Gaussian case, we decided to stop using the a_0 approach.

E. Convolution - dual dependence?

Another idea Peter and I wanted to explore was running a convolution of the thermal distribution with itself.

This was on the basis that in order to produce a hadron of some sort, multiple vertexes require creation, and consequently there is a Schwinger contribution to both quark constituents. This is also in the fact that if a strange quark is generated by string fragmentation, then an anti-strange quark is too, and both will combine to some final state hadron, before perhaps decaying at a later time.

To explore this idea, an attempt at deriving a convolution integral for two different masses was attempted. However, because of the square root in the exponential, the integral was found to be unsolvable.

$$P(p_\perp, m_1, m_1) = \int\int_{-\infty}^{\infty} dp_{x2}dp_{y2} \exp\left(-a_1 m_{\perp 1}\right) \times \exp\left(-a_2\sqrt{m_2^2 + (p_{x2}-p_{x1})^2 + (p_{y2}-p_{y1})^2}\right) \tag{86}$$

Low Order Taylor expansions were attempted, as well substitution methods used in deriving $\langle p_\perp^2 \rangle$. Also referring to expressions in Tables of Integrals[3],there was not an identified equivalent expression.

F. PYTHIA & VINCIA

A reasonable amount of time in the last few weeks of this project was investigating two C++ Libraries called Vincia and Pythia, which are Monte-Carlo generators for simulating particle colliders. In this time I was able to understand how the software applies factors into generating distributions for various parameters, and began starting to modify the code to direct to a thermal distribution for the p_perp. The software integrated with other various programs such as CERN-Root and Jetset3. Having gotten simulations running of various parameters, I unfortunately didn't find time to begin analysing the effects of our thermal model on the various parameters that Pythia doesn't quite perfectly match the experimental data.

IV. CONCLUDING REMARKS

In this project we have gained valuable insight to the fundamental theoretical processes of The Lund Model, and how it's used to computationally produce a hadronization spectrum. Unfortunately we couldn't find a way for the Lund Model to be modified to reflect the thermal behaviour of recent strange enhancement observations, but were able to instead investigate the Schwinger Model. The distributions and analytical results proved hard to analyse for the thermal modification, however we could still begin to simulate some of these effects in Monte Carlo generators, namely Pythia. This is still an exciting time to be rethinking particle theory and explaining hadronization distributions seen at the LHC!

V. THANKS AND ACKNOWLEDGEMENTS

Many thanks to my patient supervisor Peter Skands, for his effort and willingness in undertaking this project. I have learnt a great deal, and my passion for studying fundamental particles certainly hasn't diminished!

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Lastly to the CoEPP Node @ Monash University, all the PostDocs and Professors who included me in weekly discussions and lunches. That has been a fantastic experience, and a great opportunity to learn more in a collaborative environment.

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