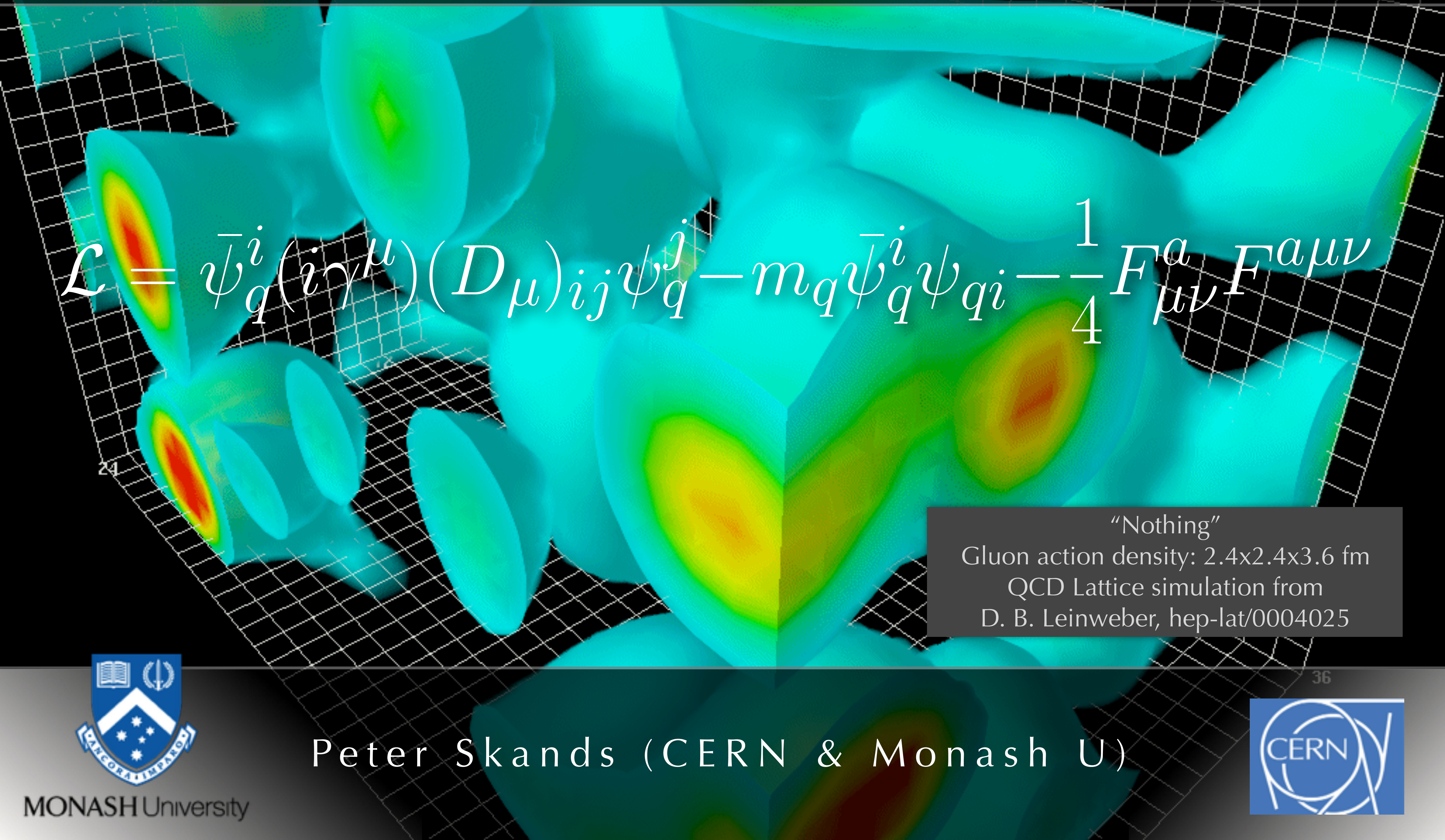
Asia-Europe-Pacific School of High Energy Physics
Puri, India, 2014

Introduction to QCD

Lecture 1

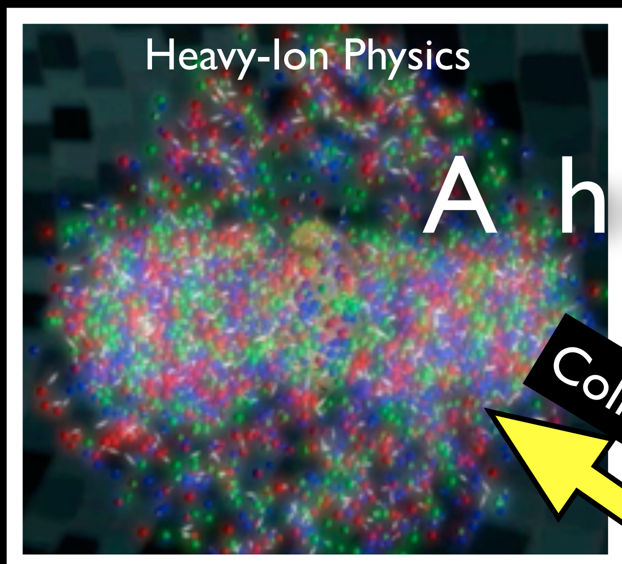
$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij}\psi_q^j - m_q \bar{\psi}_q^i \psi_{qi} - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

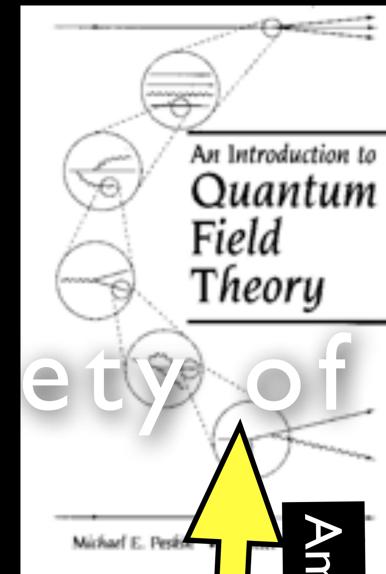


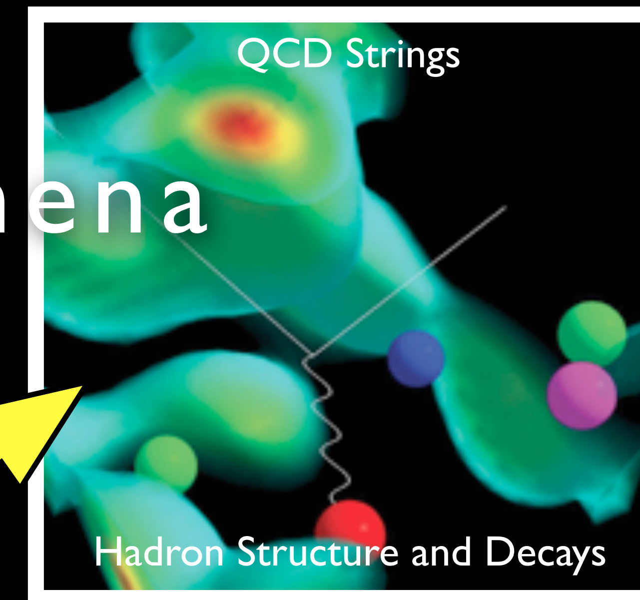
"Nothing"
Gluon action density: 2.4x2.4x3.6 fm
QCD Lattice simulation from
D. B. Leinweber, hep-lat/0004025

Peter Skands (CERN & Monash U)

A huge variety of phenomena







Collective Effects

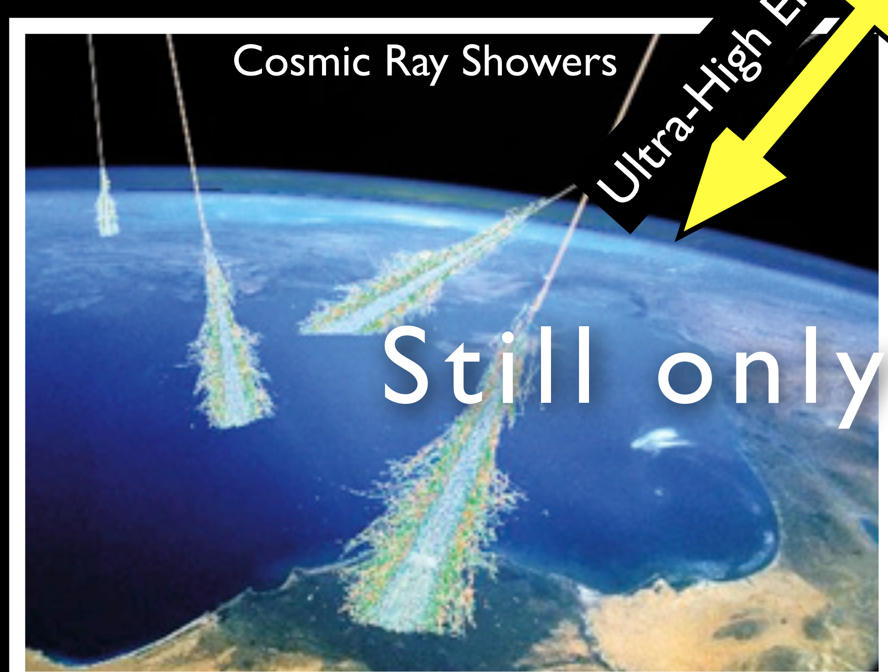
Amplitudes

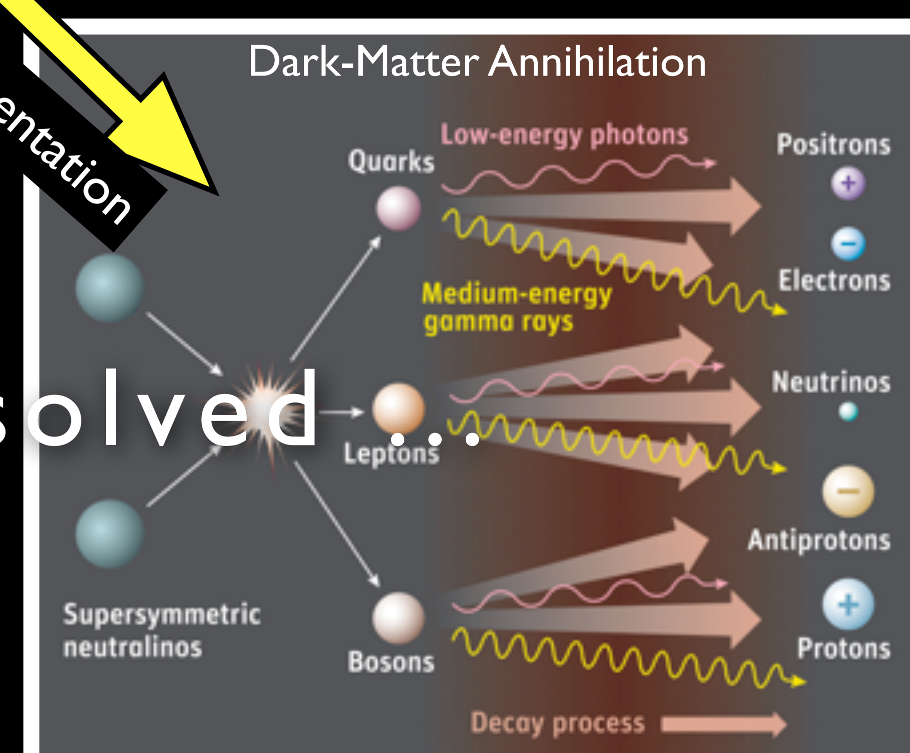
Confinement

$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij} \psi_q^j - m_q \bar{\psi}_q^i \psi_{qi} - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

Ultra-High Energies

Fragmentation





Still only partially solved ...

There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy

W. Shakespeare, Hamlet.

$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij}\psi_q^j - m_q\bar{\psi}_q^i\psi_{qi} - \frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu}$$

$$+ \ldots\ \ldots\ \ldots ?$$

LHC: still no explicit signs of new physics

→ we're still looking for *deviations* from SM

Disclaimer

Based on TASI lectures (2012)
P. Skands, arXiv:1207.2389

Focus on QCD for **collider physics**

- Quantum Chromodynamics
- The Ultraviolet (hard processes and jets)
- The Infrared (hadronization and underlying event)
- Monte Carlo Event Generators (shower Markov chains)

Still, some topics not touched, or only briefly

- Physics of hadrons (Lattice QCD, Heavy flavor physics, diffraction, …)
- Heavy ion physics
- New Physics
- \+ Many specialized topics (DIS, prompt γ, polarized beams, low-x, …)

Introduction to QCD

1. Fundamentals of QCD
2. PDFs, Fixed-Order QCD, and Jet Algorithms
3. Parton Showers and Event Generators
4. QCD in the Infrared

Slides posted at:
www.cern.ch/skands/slides

Lecture Notes (updated for this school):
P. Skands, arXiv:1207.2389

Before QCD

1951: the first hint of colour?

Discovery of the Δ^{++} baryon

Meson-Nucleon Scattering and Nucleon Isobars*

KEITH A. BRUECKNER
Department of Physics, Indiana University, Bloomington, Indiana
(Received December 17, 1951)

K. A. Brueckner
Phys.Rev.86(1952)106

satisfactory agreement with experiment is obtained. It is concluded that the apparently anomalous features of the scattering can be interpreted to be an indication of a resonant meson-nucleon interaction corresponding to a nucleon isobar with spin $\frac{3}{2}$, isotopic spin $\frac{3}{2}$, and with an excitation of 277 Mev.

~ 1960:

Isospin: Wigner, Heisenberg
Strangeness ('53): Gell-Mann, Nishijima
Eightfold Way ('61): Gell-Mann, Ne'eman
Quarks ('63): Gell-Mann, Zweig, (Sakata)

$|\Delta^{++}\rangle = |u_\uparrow u_\uparrow u_\uparrow\rangle$?!?!?

Fermion (spin-3/2).
Symmetric in space, spin & flavour
Antisymmetric in what?

1965: Additional SU(3) *Han, Nambu, Greenberg*

$|\Delta^{++}\rangle = \varepsilon_{ijk}|u_{i\uparrow} u_{j\uparrow} u_{k\uparrow}\rangle$

degree = 3; dimension = 8
Or larger?

The Width of the π^0

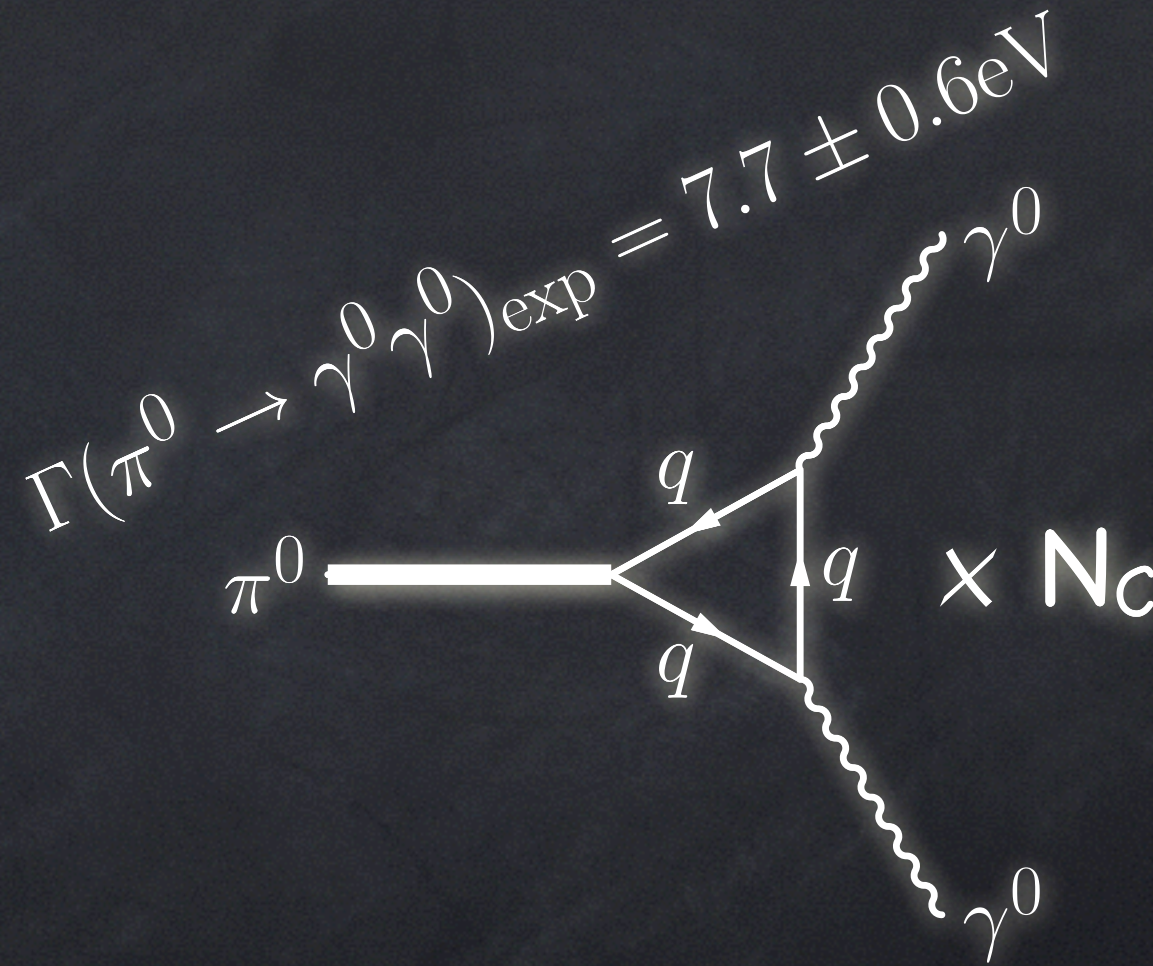
Δ^{++}, Δ^-, and Ω^-

Strictly speaking, we only know $N \geq 3$

$\pi \to \gamma\gamma$ decays

Get pion decay constant f_π from

$\pi^- \to \mu^- \nu_\mu$

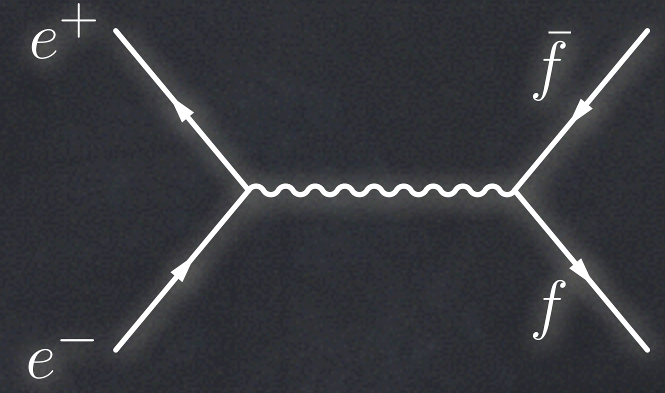


$$\Rightarrow \quad \Gamma(\pi^0 \to \gamma^0\gamma^0)_{\rm th} = \frac{N_C^2}{9}\frac{\alpha_{\rm em}^2}{\pi^2}\frac{1}{64\pi}\frac{m_\pi^3}{f_\pi^2} = 7.6\left(\frac{N_C}{3}\right)^2 \text{eV}$$

See, e.g., Ellis, Stirling, & Webber, "QCD and Collider Physics", Cambridge Monographs

"R"

$$R = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



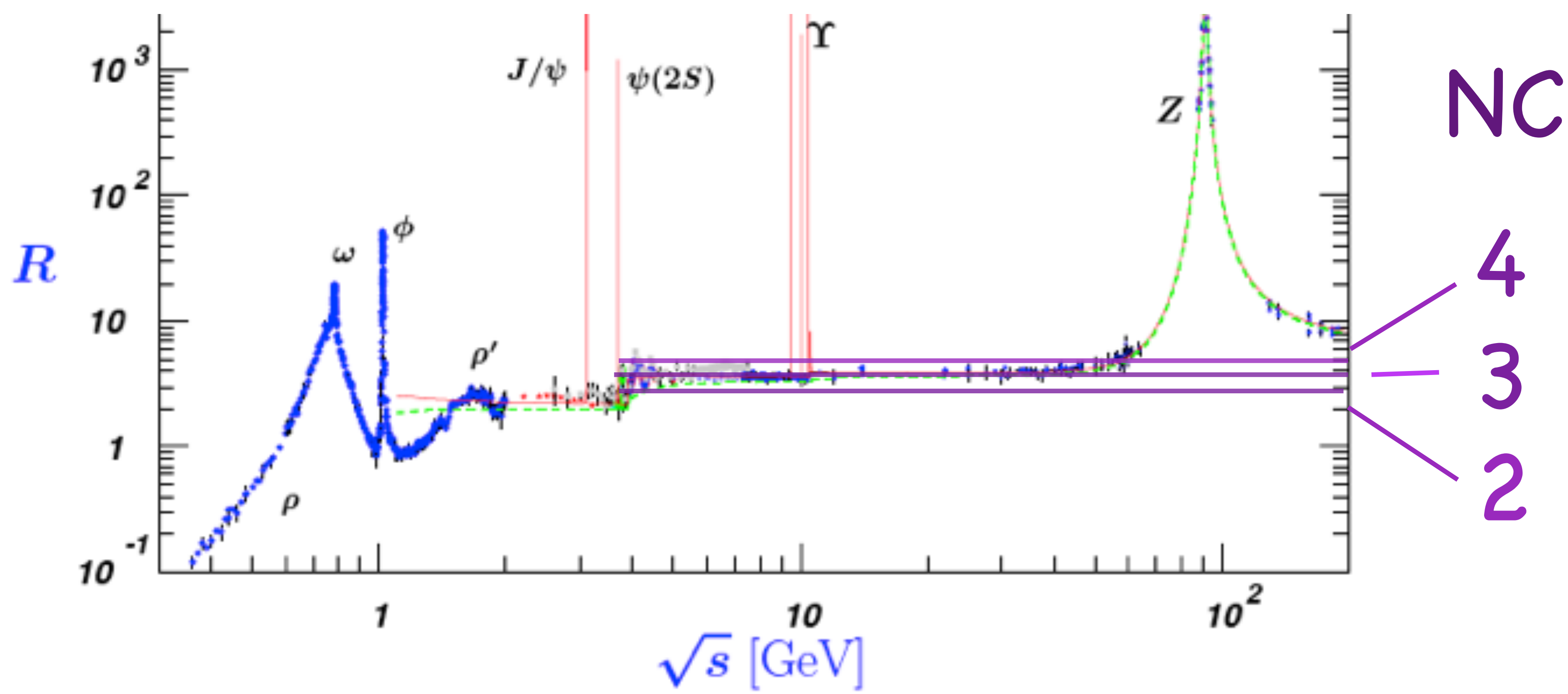
$$= n_u \left(\frac{2}{3}\right)^2 + n_d \left(-\frac{1}{3}\right)^2$$

Question: why does $\pi^0 \to \gamma^0\gamma^0$ go with N_C^2 and R only with N_C?

$$= \begin{cases} 2\,(N_C/3) & q = u, d, s \\ 3.67\,(N_C/3) & q = u, d, s, c, b \end{cases}$$

"R"

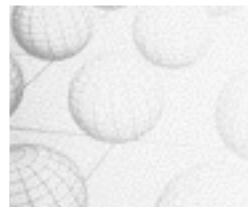
$$R = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = n_u \left(\frac{2}{3}\right)^2 + n_d \left(-\frac{1}{3}\right)^2$$



Quantum Chromodynamics

$$\mathcal{L} = \bar{\psi}^i_q(i\gamma^\mu)(D_\mu)_{ij}\psi^j_q - m_q\bar{\psi}^i_q\psi_{qi} - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

Quark fields



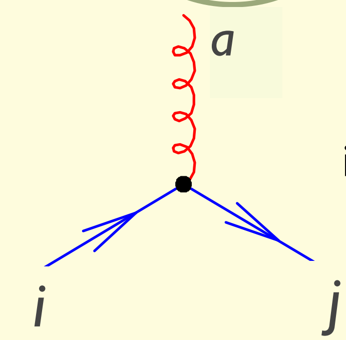
$$\psi^j_q = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

SU(3)
Local Gauge
Symmetry

Covariant Derivative

$$D_{\mu ij} = \delta_{ij}\partial_\mu - ig_s T^a_{ij} A^a_\mu$$

⇒ Feynman rules



a∈[1,8]

i,j∈[1,3]

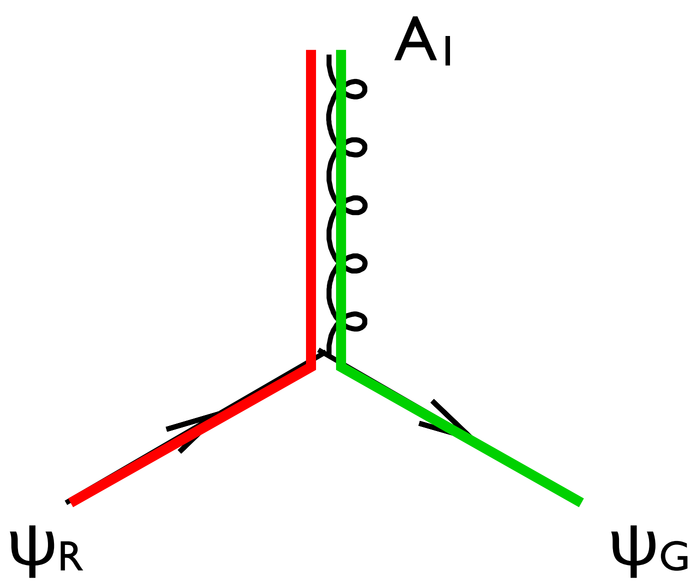
Gell-Mann Matrices *(T^a = ½λ^a)*

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix}$$

Interactions in Colour Space

Quark-Gluon interactions



$$\underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{A_I} \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{\psi_R} = \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}_{\psi_G}$$

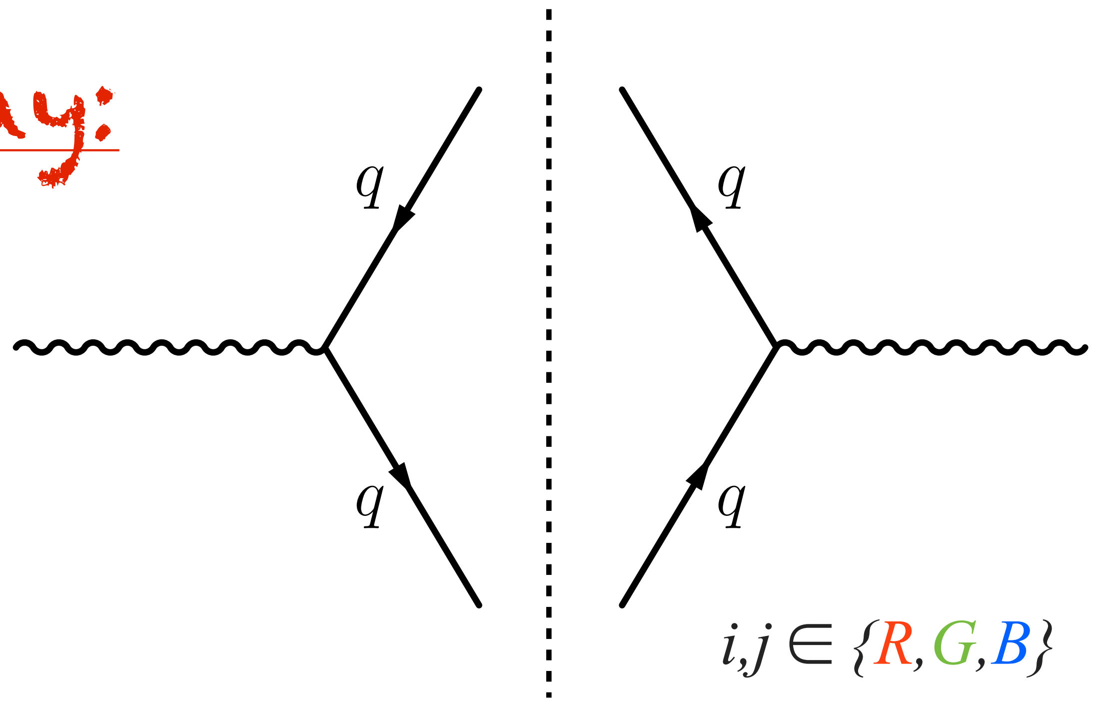
Colour Factors

We already saw pion decay and the "R" ratio depended on how many "colour paths" we could take

All QCD processes have a "colour factor". It counts the enhancement from the sum over colours.

Z Decay:

$$\sum_{\text{colours}} |M|^2 =$$



Interactions in Colour Space

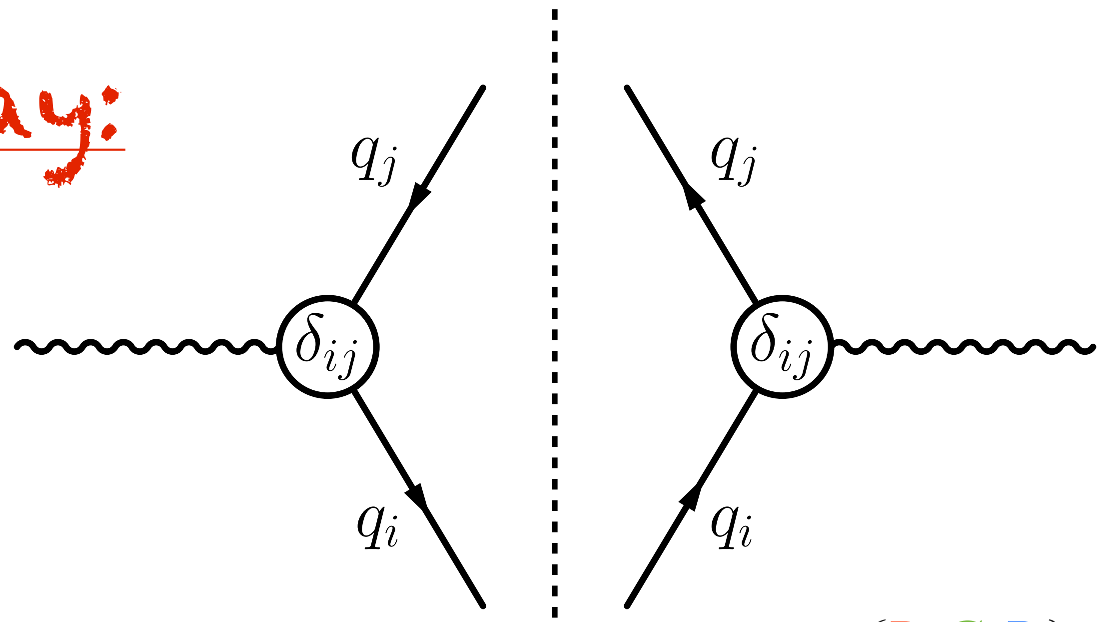
Colour Factors

We already saw pion decay and the "R" ratio depended on how many "colour paths" we could take

All QCD processes have a "colour factor". It counts the enhancement from the sum over colours.

Z Decay:

$$\sum_{\text{colours}} |M|^2 =$$



$i,j \in \{R,G,B\}$

$$\propto \delta_{ij}\delta^*_{ji} = \mathrm{Tr}[\delta_{ij}] = N_C$$

Interactions in Colour Space

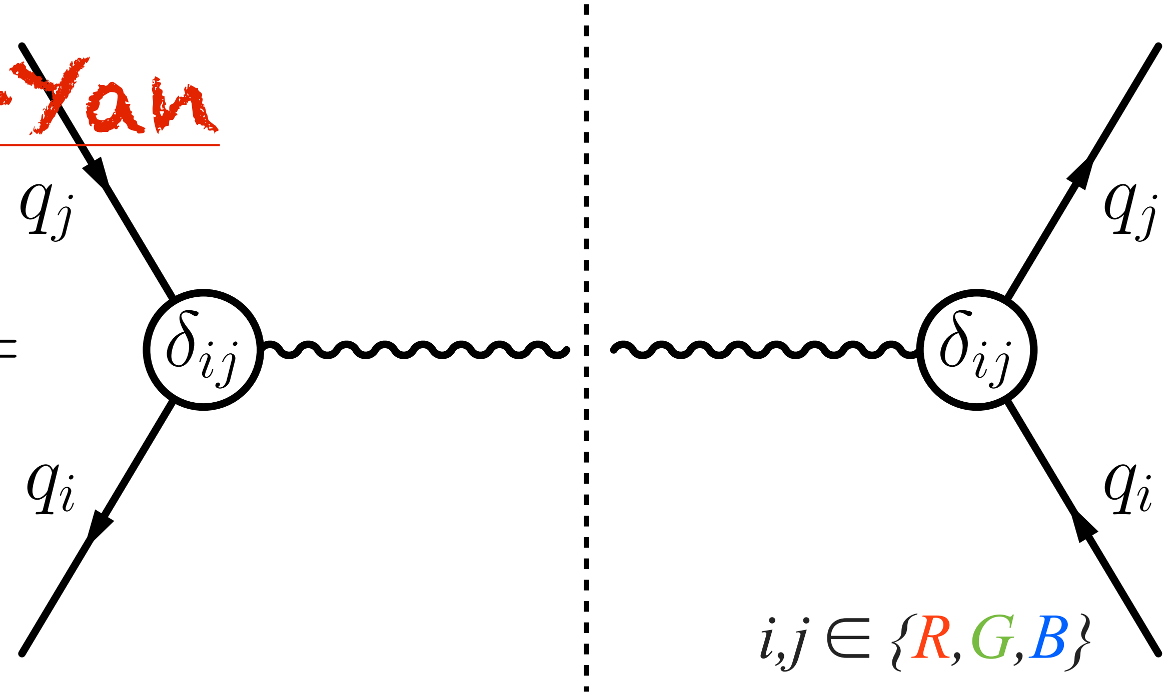
Colour Factors

We already saw pion decay and the "R" ratio depended on how many "colour paths" we could take

All QCD processes have a "colour factor". It counts the enhancement from the sum over colours.

Drell-Yan

$$\sum_{\text{colours}} |M|^2 =$$



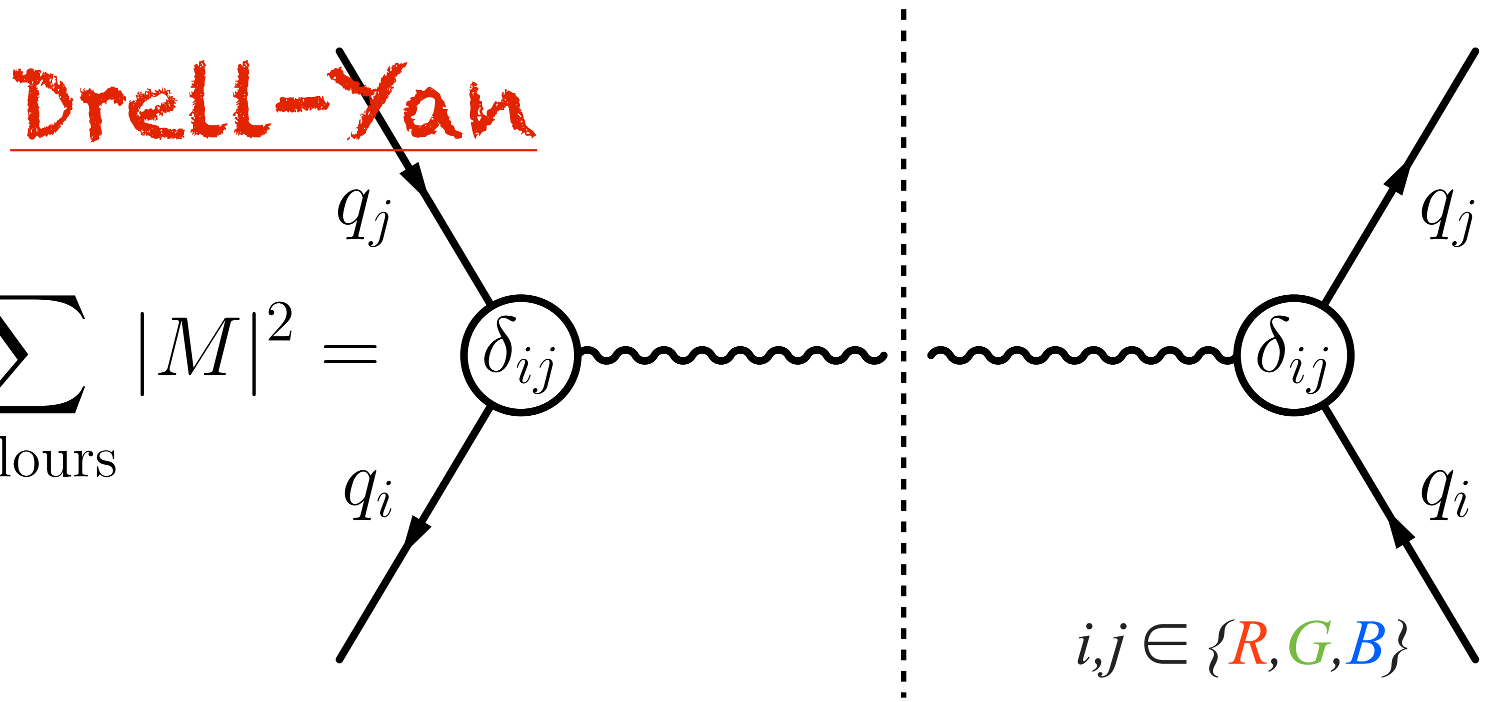
$$\propto \delta_{ij}\delta^*_{ji} = \mathrm{Tr}[\delta_{ij}] = N_C$$

Colour Factors

We already saw pion decay and the "R" ratio depended on how many "colour paths" we could take

All QCD processes have a "colour factor". It counts the enhancement from the sum over colours.

$$\frac{1}{9}\sum_{\text{colours}} |M|^2 =$$



$$\propto \delta_{ij}\delta^*_{ji}\,\frac{1}{N_C^2}$$
$$= \mathrm{Tr}[\delta_{ij}]\,\frac{1}{N_C^2}$$
$$= 1/N_C$$

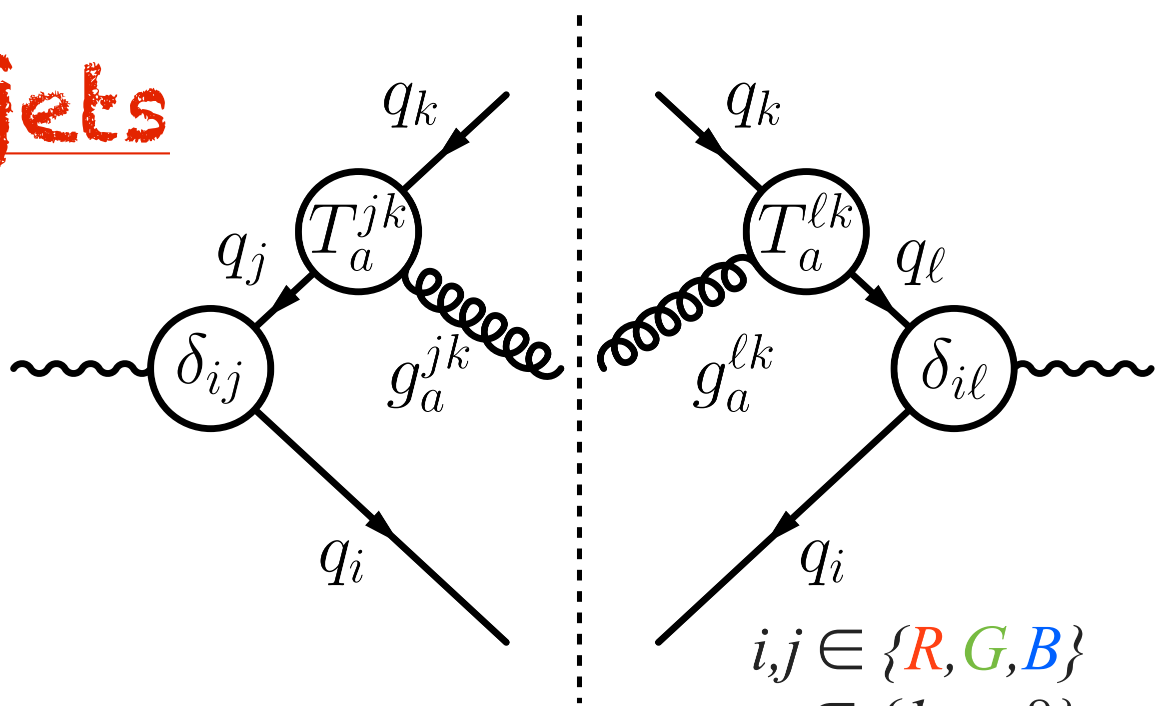
Colour Factors

We already saw pion decay and the "R" ratio depended on how many "colour paths" we could take

All QCD processes have a "colour factor". It counts the enhancement from the sum over colours.

Z→3 jets

$$\sum_{\text{colours}} |M|^2 =$$



$i,j \in \{R,G,B\}$
$a \in \{1,\ldots,8\}$

$$\propto \delta_{ij} T_a^{jk} (T_a^{\ell k} \delta_{i\ell})^*$$
$$= \mathrm{Tr}[T_a T_a]$$
$$= \frac{1}{2} \mathrm{Tr}\delta_{ab}$$
$$= 4$$

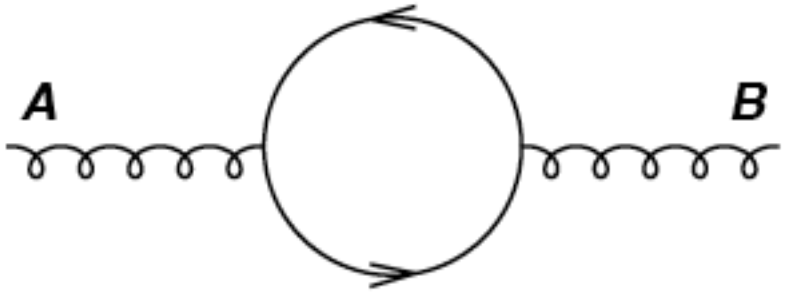
Quick Guide to Colour Algebra

Colour factors squared produce traces

Trace Relation

$$\mathrm{Tr}(t^A t^B) = T_R \delta^{AB}\,, \quad T_R = \frac{1}{2}$$

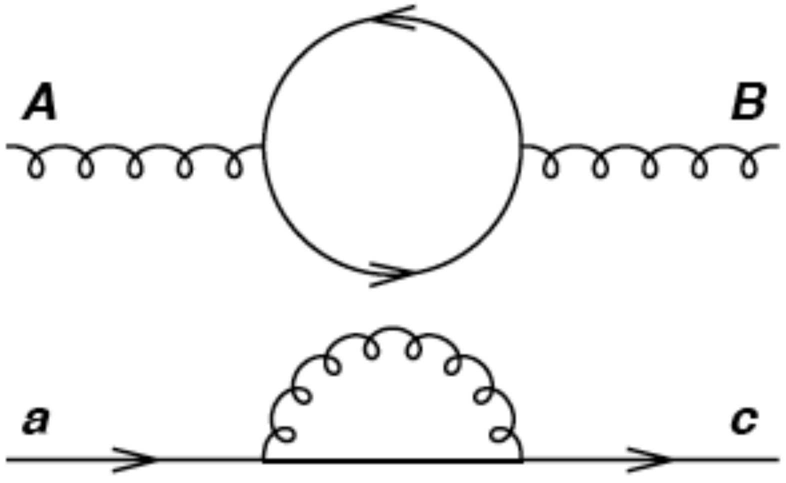
Example Diagram



(from ESHEP lectures by G. Salam)

Quick Guide to Colour Algebra

Colour factors squared produce traces

Trace Relation	Example Diagram
$\mathrm{Tr}(t^A t^B) = T_R \delta^{AB}\,, \quad T_R = \frac{1}{2}$	
$\sum_A t^A_{ab} t^A_{bc} = C_F \delta_{ac}\,, \quad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$	

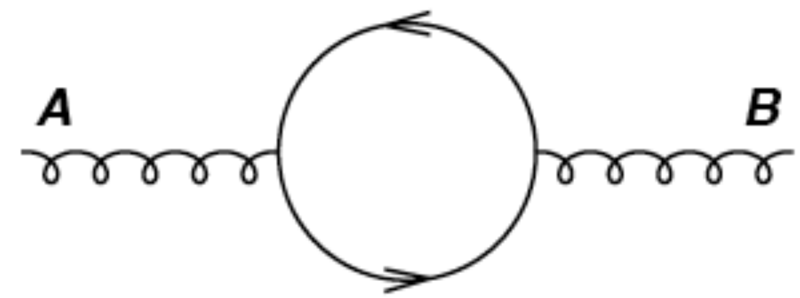
(from ESHEP lectures by G. Salam)

Quick Guide to Colour Algebra

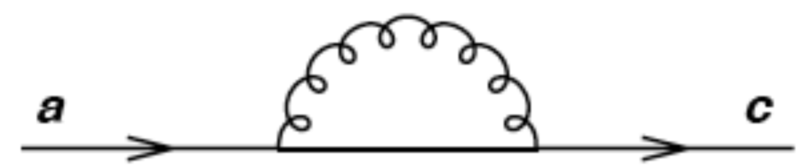
Colour factors squared produce traces

Trace Relation | **Example Diagram**

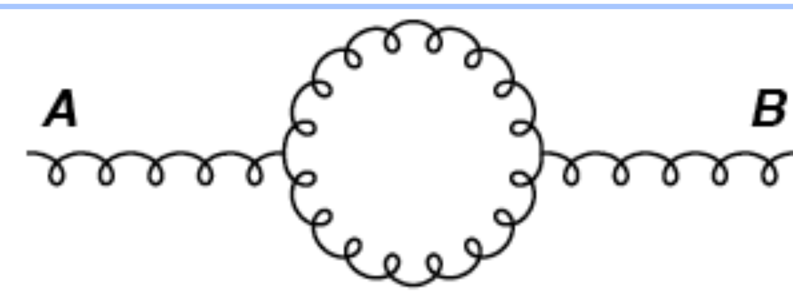
$$\mathrm{Tr}(t^A t^B) = T_R \delta^{AB}\,, \quad T_R = \tfrac{1}{2}$$



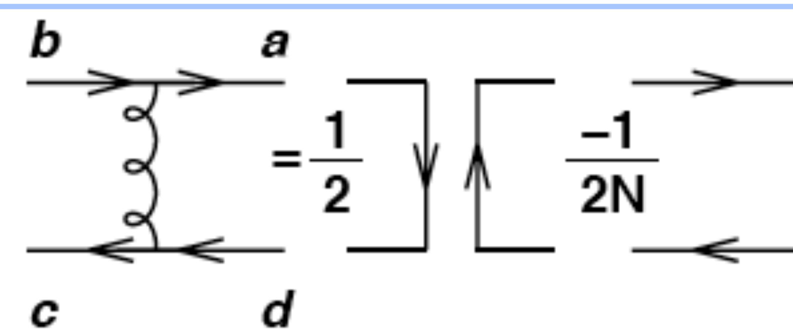
$$\sum_A t^A_{ab} t^A_{bc} = C_F \delta_{ac}\,, \quad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$



$$\sum_{C,D} f^{ACD} f^{BCD} = C_A \delta^{AB}\,, \quad C_A = N_c = 3$$



$$t^A_{ab} t^A_{cd} = \frac{1}{2}\delta_{bc}\delta_{ad} - \frac{1}{2N_c}\delta_{ab}\delta_{cd} \quad \text{(Fierz)}$$



(from ESHEP lectures by G. Salam)

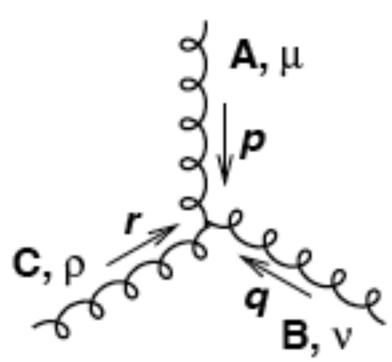
The Gluon

Gluon-Gluon Interactions

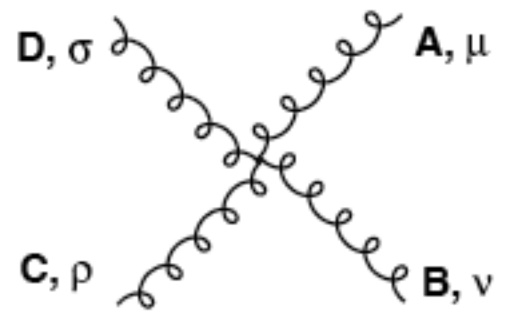
$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij}\psi_q^j - m_q \bar{\psi}_q^i \psi_{qi} - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

Gluon field strength tensor:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_\mu^b A_\nu^c$$



$$-g_s f^{ABC}[(p-q)^\rho g^{\mu\nu} + (q-r)^\mu g^{\nu\rho} + (r-p)^\nu g^{\rho\mu}]$$



$$-ig_s^2 f^{XAC} f^{XBD}[g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\gamma}] + (C,\gamma) \leftrightarrow (D,\rho) + (B,\nu) \leftrightarrow (C,\gamma)$$

Structure constants of SU(3):

$$f_{123} = 1$$

$$f_{147} = f_{246} = f_{257} = f_{345} = \frac{1}{2}$$

$$f_{156} = f_{367} = -\frac{1}{2}$$

$$f_{458} = f_{678} = \frac{\sqrt{3}}{2}$$

Antisymmetric in all indices

All other $f_{ijk} = 0$

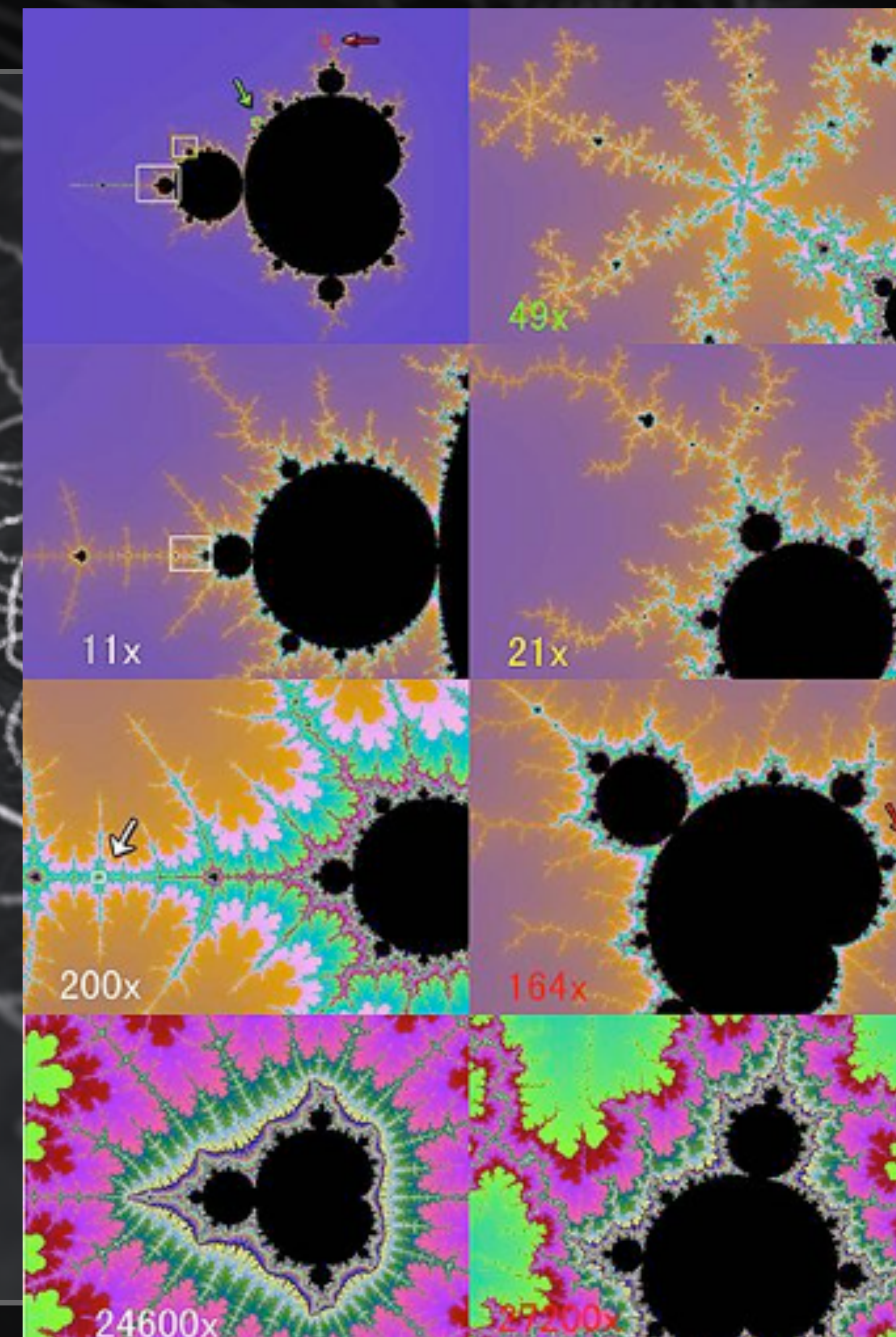
The Strong Coupling

Bjorken scaling
To first approximation, QCD is
SCALE INVARIANT
(a.k.a. conformal)

A jet inside a jet inside a jet inside a jet ...

If the strong coupling didn't "run", this would be absolutely true (e.g., N=4 Supersymmetric Yang-Mills)

As it is, α_s only runs slowly (logarithmically) → can still gain insight from fractal analogy

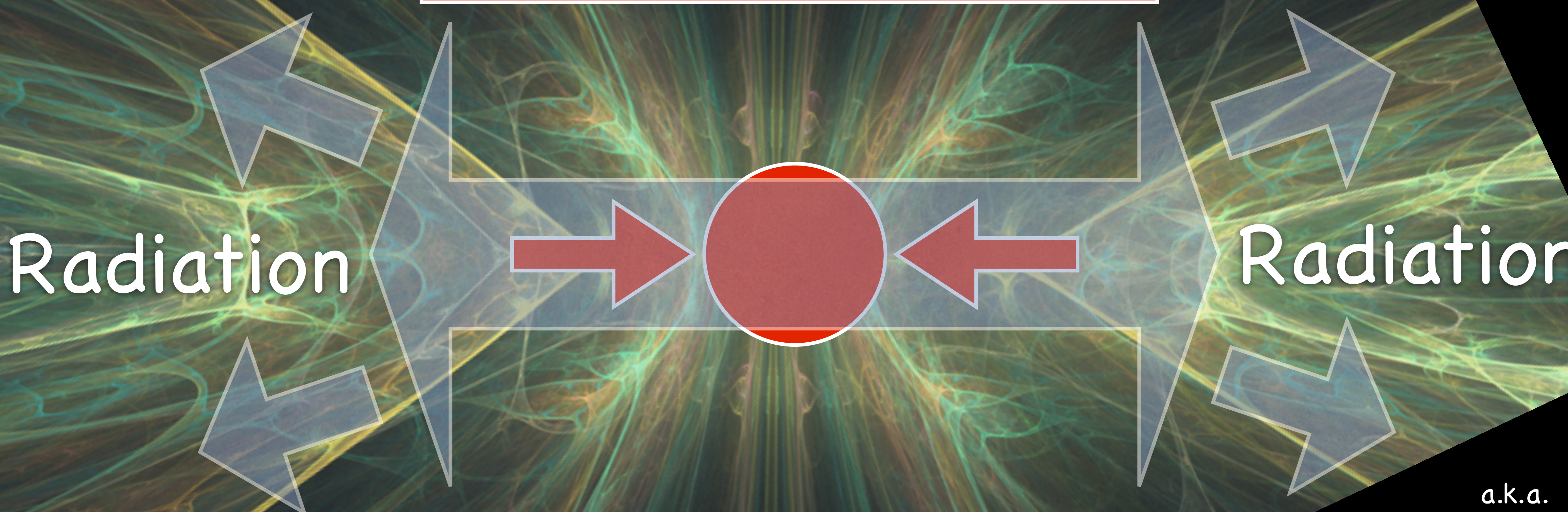


Note: I use the terms "conformal" and "scale invariant" interchangeably
Strictly speaking, conformal (angle-preserving) symmetry is more restrictive than just scale invariance
But examples of scale-invariant field theories that are not conformal are rare (eg 6D noncritical self-dual string theory)

(some) Physics

cf. equivalent-photon approximation
Weiszäcker, Williams
~ 1934

Charges Stopped or kicked

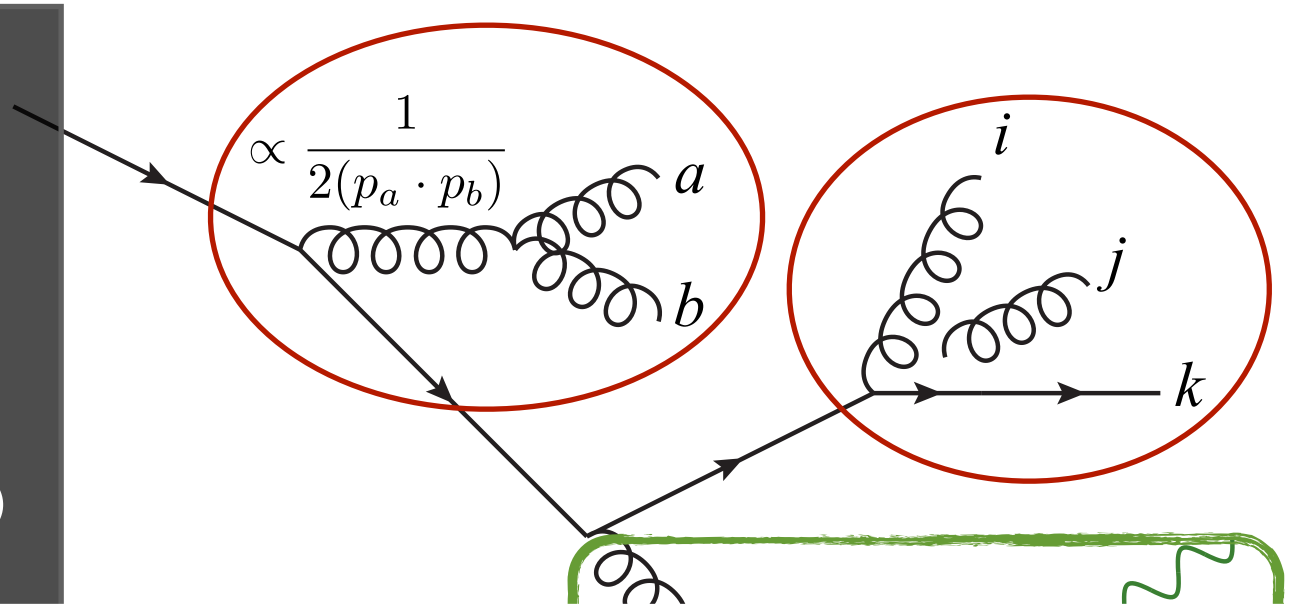


a.k.a.
Bremsstrahlung
Synchrotron Radiation

The harder they stop, the harder the fluctations that continue to become radiation

Jets ≈ Fractals

- **Most bremsstrahlung** is driven by divergent propagators → simple structure
- **Amplitudes factorize in singular limits** (→ universal "conformal" or "fractal" structure)



Partons ab → "collinear":

P(z) = DGLAP splitting kernels, with z = energy fraction = $E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \overset{a||b}{\to} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

Gluon j → "soft":

Coherence → Parton j really emitted by (i,k) "colour antenna"

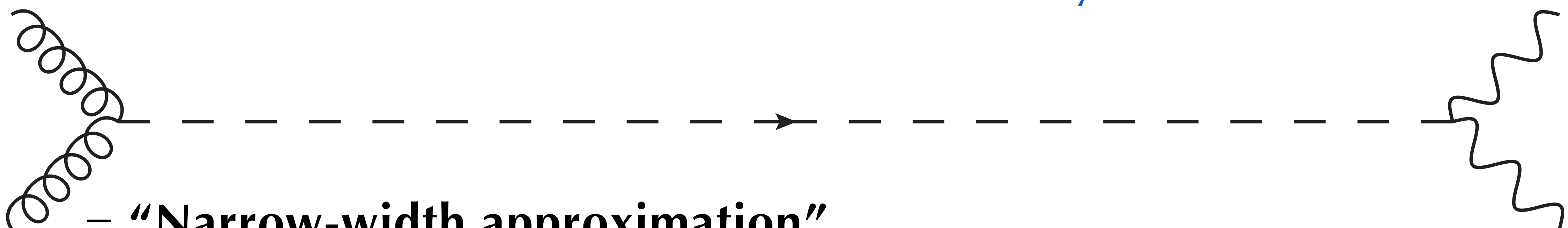
$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \overset{j_g\to 0}{\to} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

+ scaling violation: $g_s^2 \to 4\pi\alpha_s(Q^2)$

Can apply this many times
→ nested factorizations

Factorization: Separation of Scales

Factorization of Production and Decay:

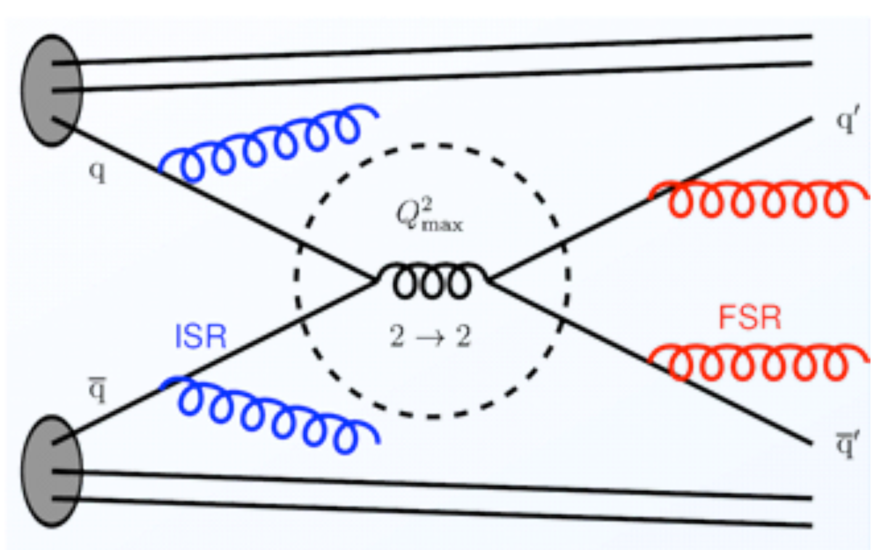


= **"Narrow-width approximation"**

Valid up to corrections Γ/m → breaks down for large Γ

More subtle when colour/charge flows *through* the diagram

Factorization of Long and Short Distances



Scale of fluctuations inside a hadron

~ Λ_{QCD} ~ 200 MeV

Scale of hard process » Λ_{QCD}

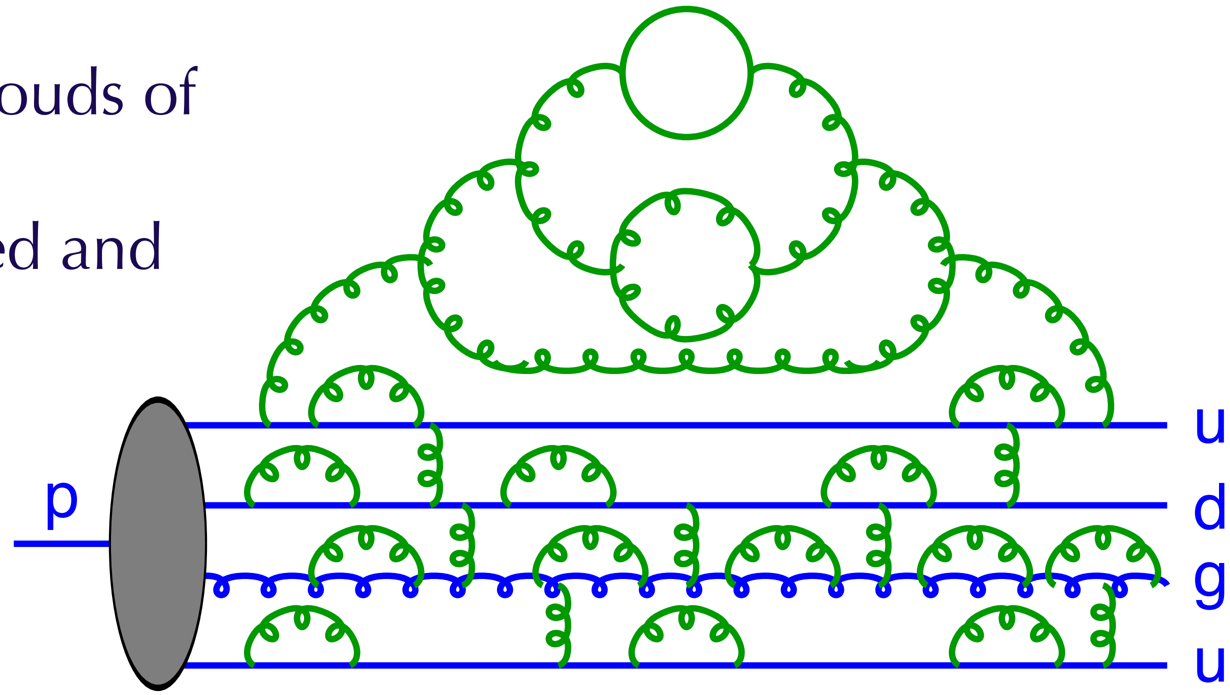
→ proton looks "frozen"

Instantaneous snapshot of long-wavelength structure, independent of nature of hard process

Factorization 2: PDFs

Hadrons are composite, with time-dependent structure:

Partons within clouds of further partons, constantly emitted and absorbed



For hadron to remain intact, virtualities $k^2 < M_h^2$
High-virtuality fluctuations suppresed by powers of

$$\frac{\alpha_s M_h^2}{k^2}$$

M_h : mass of hadron
k^2 : virtuality of fluctuation

→ Lifetime of fluctuations ~ $1/M_h$

Hard incoming probe interacts over much shorter time scale ~ 1/Q

On that timescale, partons ~ frozen

Hard scattering knows nothing of the target hadron apart from the fact that it contained the struck parton

Illustration from T. Sjöstrand

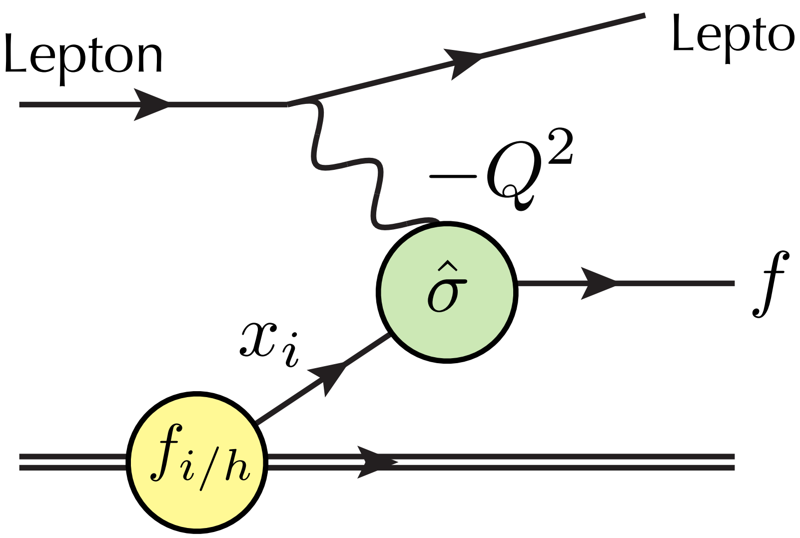
Factorization Theorem

In DIS, there is a formal proof of factorization

(Collins, Soper, 1987)

Deep Inelastic Scattering (DIS)

Surprise Question: What's the color factor for DIS?



Note: Beyond LO, *f* can be more than one parton

→ We really can write the cross section in factorized form :

$$\sigma^{\ell h} = \sum_i \sum_f \int dx_i \int d\Phi_f \, f_{i/h}(x_i, Q_F^2) \, \frac{d\hat{\sigma}^{\ell i \to f}(x_i, \Phi_f, Q_F^2)}{dx_i \, d\Phi_f}$$

Sum over Initial (i) and final (f) parton flavors

Φ_f = Final-state phase space

$f_{i/h}$ = PDFs Assumption: $Q^2 = Q_F^2$

Differential partonic Hard-scattering Matrix Element(s)

A propos Factorization

Why do we need PDFs, parton showers / jets, etc.?
Why are Fixed-Order QCD matrix elements not enough?

F.O. QCD requires **Large scales** : to guarantee that α_s is small enough to be perturbative (not too bad, since we anyway *often* want to consider large-scale processes [insert your fav one here])

F.O. QCD requires **No hierarchies** : conformal structure implies that soft/collinear hierarchies are associated with on-shell singularities that ruin fixed-order expansion.

But!!! we collide - and observe - low-scale hadrons, with *non-perturbative structure*, that participate in hard processes, whose scales are *hierarchically greater* than $m_{had} \sim 1$ GeV.

→ A Priori, no perturbatively calculable observables in QCD

Conformal QCD in Action

Naively, QCD radiation suppressed by $\alpha_s \approx 0.1$

Truncate at fixed order = LO, NLO, …

But beware the jet-within-a-jet-within-a-jet …

→ More on this in lectures on Jets and Showers

Example: 100 GeV can be "soft" at the LHC

SUSY pair production at 14 TeV, with $M_{SUSY} \approx$ 600 GeV

LHC - sps1a - m~600 GeV

Plehn, Rainwater, PS PLB645(2007)217

FIXED ORDER pQCD	σ_{tot} [pb]	$\tilde{g}\tilde{g}$	$\tilde{u}_L\tilde{g}$	$\tilde{u}_L\tilde{u}_L^*$	$\tilde{u}_L\tilde{u}_L$	TT
$p_{T,j} >$ 100 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
inclusive X + 1 "jet" →	σ_{1j}	2.89	2.74	0.136	0.145	0.73
inclusive X + 2 "jets" →	σ_{2j}	1.09	0.85	0.049	0.039	0.26
$p_{T,j} >$ 50 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
	σ_{1j}	5.90	5.37	0.283	0.285	1.50
	σ_{2j}	4.17	3.18	0.179	0.117	1.21

(Computed with SUSY-MadGraph)

σ for X + jets much larger than naive estimate

σ_{50} ~ σ_{tot} tells us that there will "always" be a ~ 50-GeV jet "inside" a 600-GeV process

Factorization says we can still calculate!

Why is Fixed Order QCD not enough?
: It requires all resolved scales >> Λ_{QCD} **AND** no large hierarchies

PDFs: connect incoming hadrons with the high-scale process
Fragmentation Functions: connect high-scale process with final-state hadrons
(each is a non-perturbative function modulated by initial- and final-state radiation)

$$\frac{d\sigma}{dX} = \sum_{a,b} \sum_{f} \int_{\hat{X}_f} f_a(x_a, Q_i^2) f_b(x_b, Q_i^2) \frac{d\hat{\sigma}_{ab\to f}(x_a, x_b, f, Q_i^2, Q_f^2)}{d\hat{X}_f} D(\hat{X}_f \to X, Q_i^2, Q_f^2)$$

PDFs: needed to compute inclusive cross sections

FFs: needed to compute (semi-)exclusive cross sections

Resummed pQCD: All resolved scales >> Λ_{QCD} **AND** X Infrared Safe

*) pQCD = perturbative QCD

Will take a closer look at both PDFs and final-state aspects (jets and showers) in the next lectures

Scaling Violation

Real QCD isn't conformal

The coupling runs logarithmically with the energy scale

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = \beta(\alpha_s) \qquad \beta(\alpha_s) = -\alpha_s^2(b_0 + b_1\alpha_s + b_2\alpha_s^2 + \ldots) \ ,$$

$$b_0 = \frac{11C_A - 2n_f}{12\pi} \qquad b_1 = \frac{17C_A^2 - 5C_A n_f - 3C_F n_f}{24\pi^2} = \frac{153 - 19n_f}{24\pi^2}$$

1-Loop β function coefficient

2-Loop β function coefficient

$$b_2 = \frac{2857 - 5033n_f + 325n_f^2}{128\pi^3}$$

$$b_3 = \text{known}$$

Asymptotic freedom in the ultraviolet

Confinement (IR slavery?) in the infrared

Asymptotic Freedom

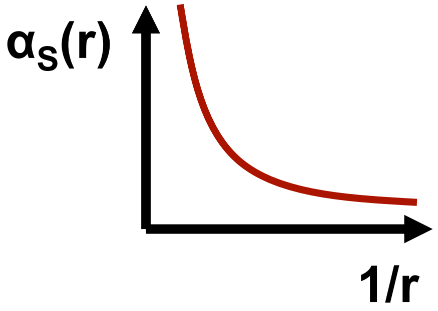
“What this year's Laureates discovered was something that, at first sight, seemed completely contradictory. The interpretation of their mathematical result was that the closer the quarks are to each other, the *weaker* is the 'colour charge'. When the quarks are really close to
*1 each other, the ~~force~~ charge is so weak that they behave almost as free particles. This phenomenon is called ‘asymptotic freedom’. The converse is true when the quarks move apart:
*2 the ~~force~~ potential becomes stronger when the distance increases.”

Nobelprize.org
The Official Web Site of the Nobel Prize

The Nobel Prize in Physics 2004
David J. Gross, H. David Politzer, Frank Wilczek

David J. Gross H. David Politzer Frank Wilczek

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek *"for the discovery of asymptotic freedom in the theory of the strong interaction"*.



*1 The force still goes to ∞ as $r \to 0$ (Coulomb potential), just less slowly

*2 The potential grows linearly as $r \to \infty$, so the force actually becomes constant (even this is only true in “quenched” QCD. In real QCD, the force eventually vanishes for r>>1fm)

Asymptotic Freedom

QED:

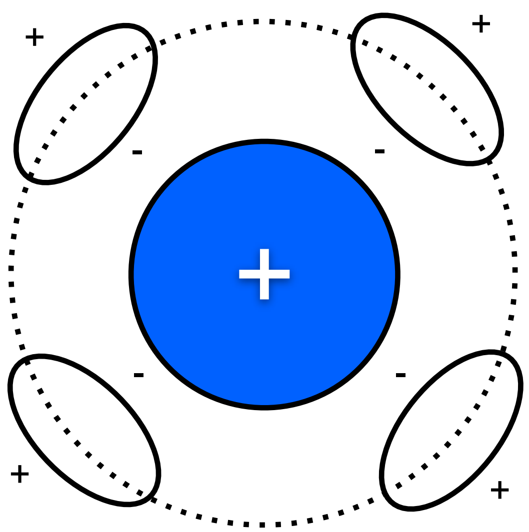
Vacuum polarization

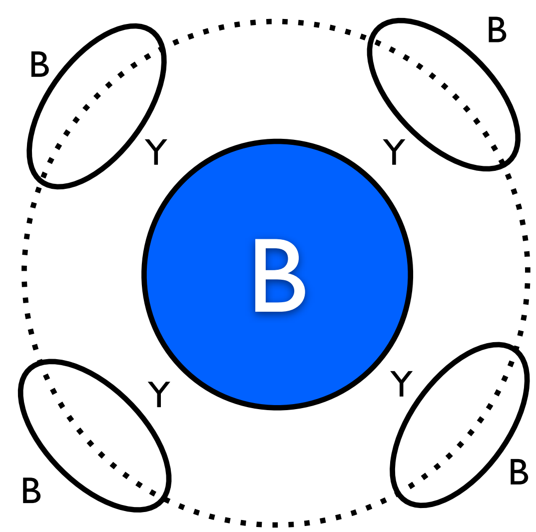
→ Charge screening

QCD:

Quark Loops

→ Also charge screening





But only dominant if > 16 flavors!

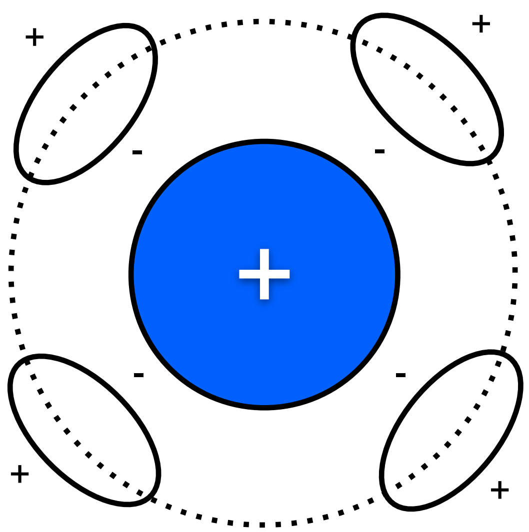
Asymptotic Freedom

$$b_0 = \frac{11C_A - 2n_f}{12\pi}$$

QED:

Vacuum polarization

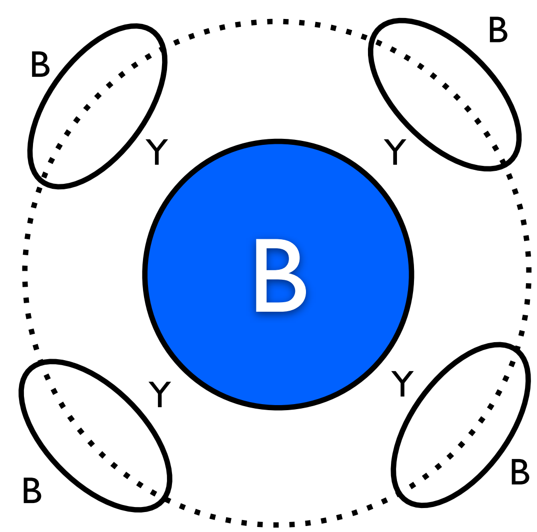
→ Charge screening

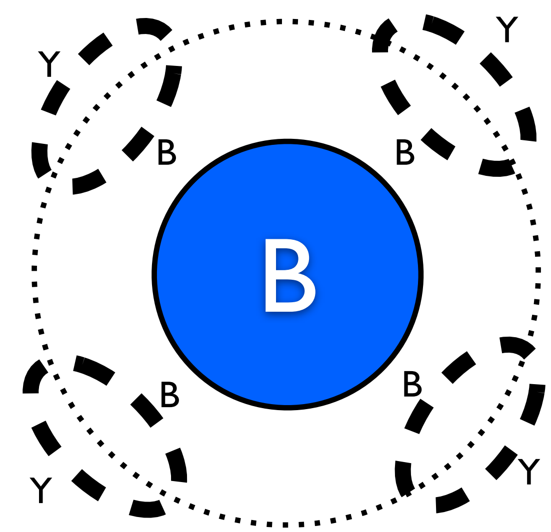


QCD:

Gluon Loops

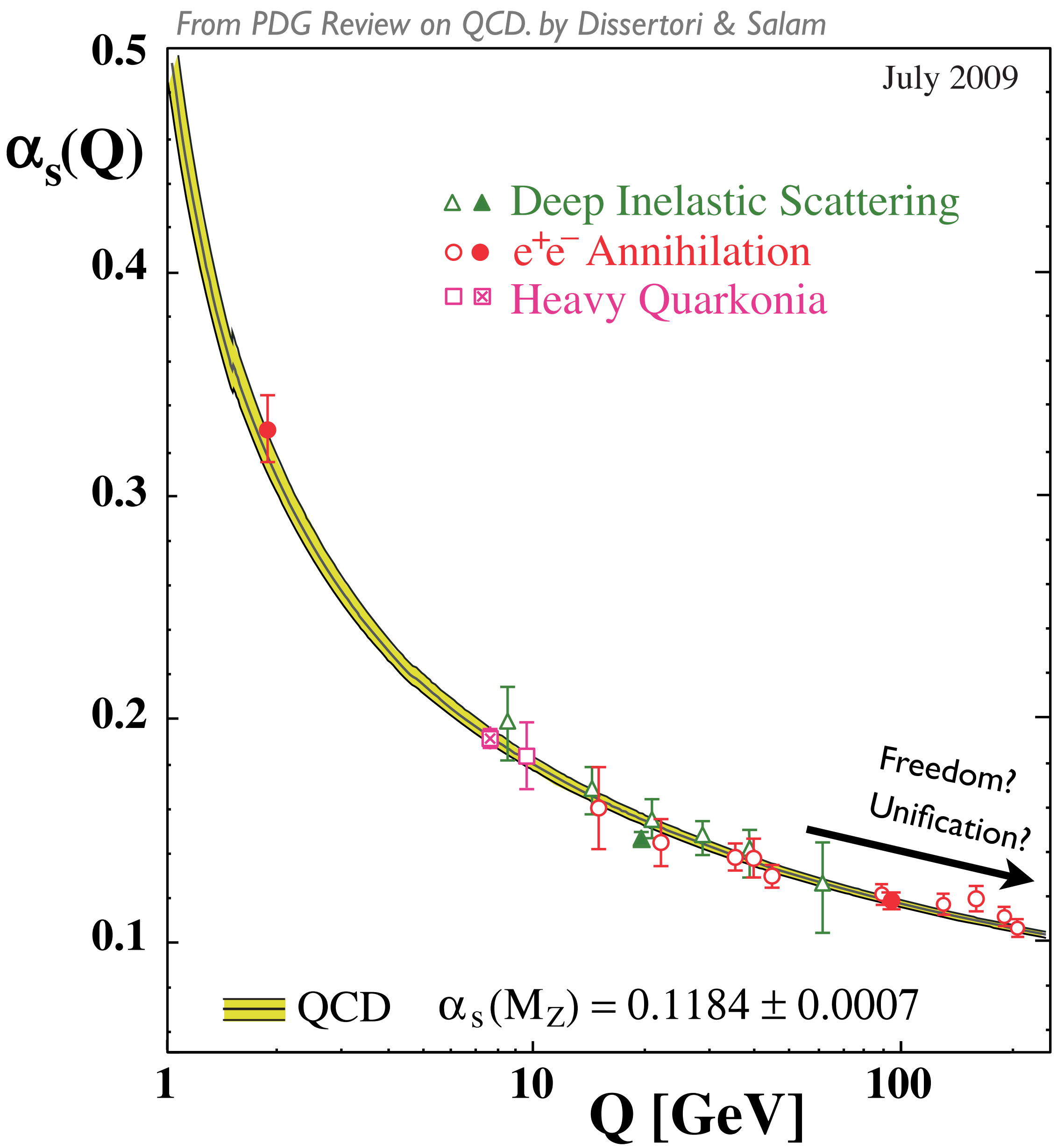
Dominate if ≤ 16 flavors





Spin-1 → Opposite Sign

UV and IR



At low scales

Coupling $\alpha_s(Q)$ actually runs rather fast with Q

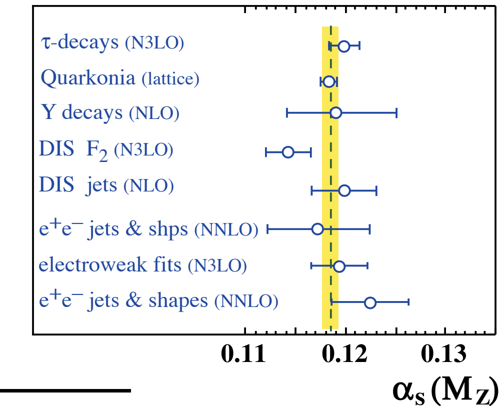
Perturbative solution diverges at a scale Λ_{QCD} somewhere below

$\approx$ 1 GeV

So, to specify the strength of the strong force, we usually give the value of α_s at a unique reference scale that everyone agrees on: M_Z

The Fundamental Parameter(s)

From PDG Review on QCD. by Dissertori & Salam



QCD has one fundamental parameter

$$\alpha_s(m_Z)^{\overline{\mathrm{MS}}}$$

$$\alpha_s(Q^2) = \alpha_s(m_Z^2)\frac{1}{1 + b_0\,\alpha_s(m_Z)\ln\frac{Q^2}{m_Z^2} + \mathcal{O}(\alpha_s^2)}$$

$$b_0 = \frac{11N_C - 2n_f}{12\pi}$$

… and its sibling

$$\Lambda_{\mathrm{QCD}}^{(n_f)\overline{\mathrm{MS}}}$$

$$\alpha_s(Q^2) = \frac{1}{b_0\ \ln\frac{Q^2}{\Lambda^2}}$$

(depends on n_f, scheme, and # of loops)

$$\Lambda \sim 200\ \mathrm{MeV}$$

… And all its cousins

$\Lambda^{(3)}$ $\Lambda^{(4)}$ $\Lambda^{(5)}$ Λ_{CMW} Λ_{FSR} Λ_{ISR} Λ_{MPI}, …

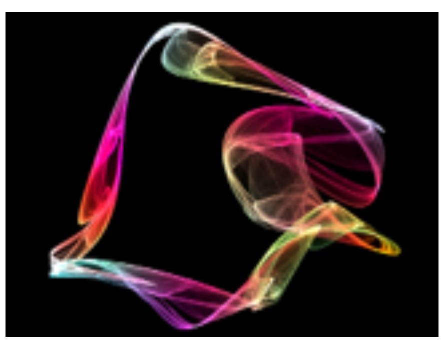
… + n_f and quark masses

Beyond α_s

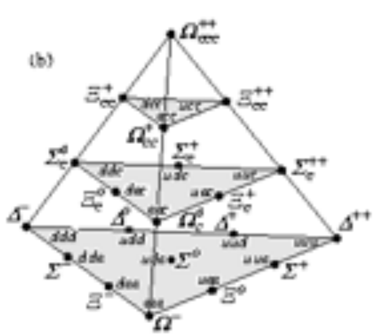
QCD is more than just a perturbative expansion in α_s

The relation between α_s, Feynman diagrams, and the full QCD dynamics is under active investigation. Emergent phenomena:

Jets (the QCD fractal) ↔ amplitude structures ↔ fundamental quantum field theory. Precision jet (structure) studies.



Strings (strong gluon fields) ↔ quantum-classical correspondence. String physics. Dynamics of hadronization phase transition.



Hadrons ↔ Spectroscopy (incl excited and exotic states), lattice QCD, (rare) decays, mixing, light nuclei. Hadron beams → MPI, diffraction, …

Other parameters

The emergent is unlike its components insofar as … it cannot be reduced to their sum or their difference."

G. Lewes (1875)

Emergent phenomena

Cannot guess non-perturbative phenomena from perturbative QCD → "Emerge" due to confinement

Hadron masses,

Decay constants,

Fragmentation functions

Parton distribution functions,…

Difficult/Impossible to compute given only knowledge of perturbative QCD

→

→

→

Image Credits: Yeimaya

The Way of the Chicken

- Who needs QCD? I'll use leptons
  - Sum inclusively over all QCD
    - Leptons almost IR safe by definition
    - WIMP-type DM, Z', EWSB → may get some leptons

The Way of the Chicken

- **Who needs QCD? I'll use leptons**
  - Sum inclusively over all QCD
    - Leptons almost IR safe by definition
    - WIMP-type DM, Z', EWSB → may get some leptons
  - Beams = hadrons for next decade (RHIC / Tevatron / LHC)
    - At least need well-understood PDFs
    - High precision = higher orders → enter QCD (and more QED)
  - Isolation → indirect sensitivity to QCD
  - Fakes → indirect sensitivity to QCD
  - Not everything gives leptons
    - Need to be a lucky chicken …

→ Next Lectures

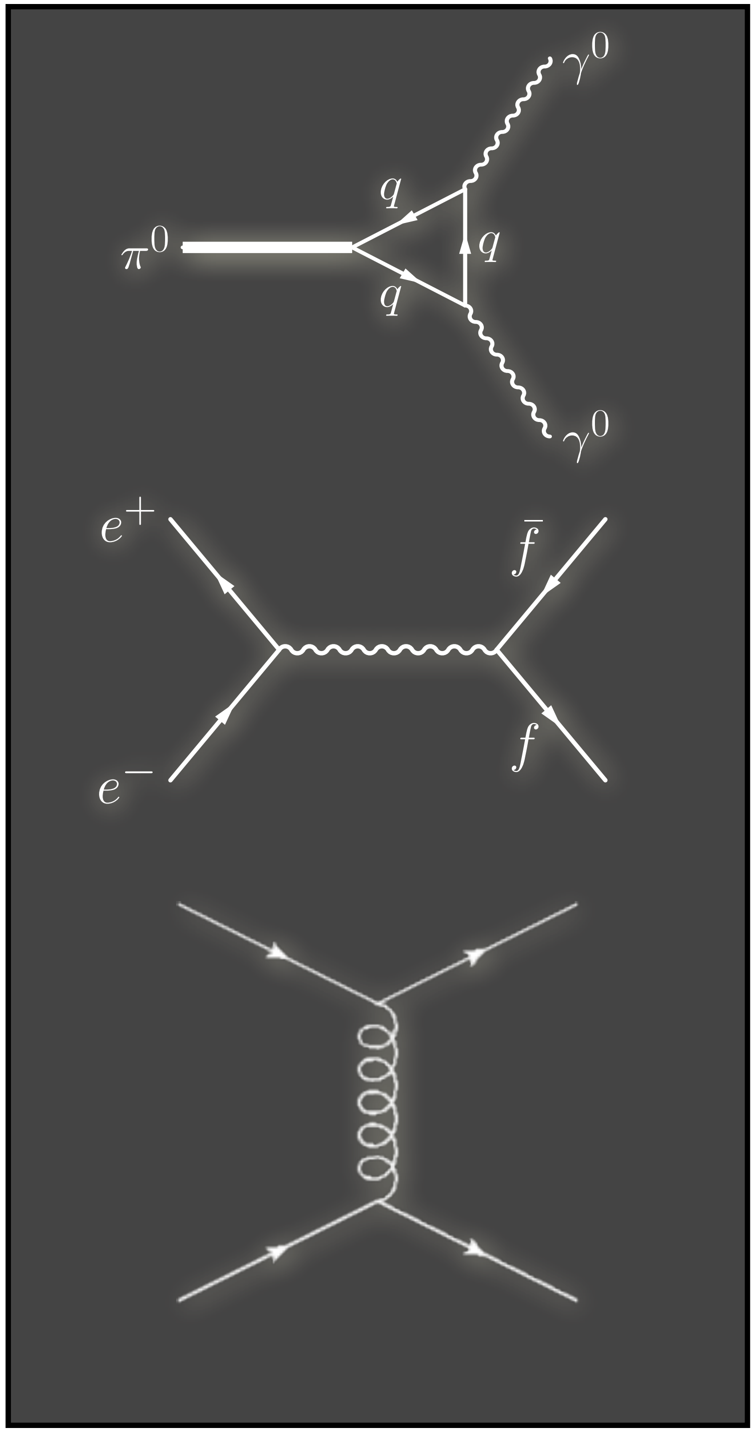
- **The unlucky chicken**
  - Put all its eggs in one basket and didn't solve QCD

Questions

1. Why is the color factor for $\pi^0 \to \gamma\gamma$ proportional to N_C^2 while the one for $e^+e^- \to$ quarks is proportional to N_C ?

(Note: treat the π^0 as a fundamental pseudoscalar)

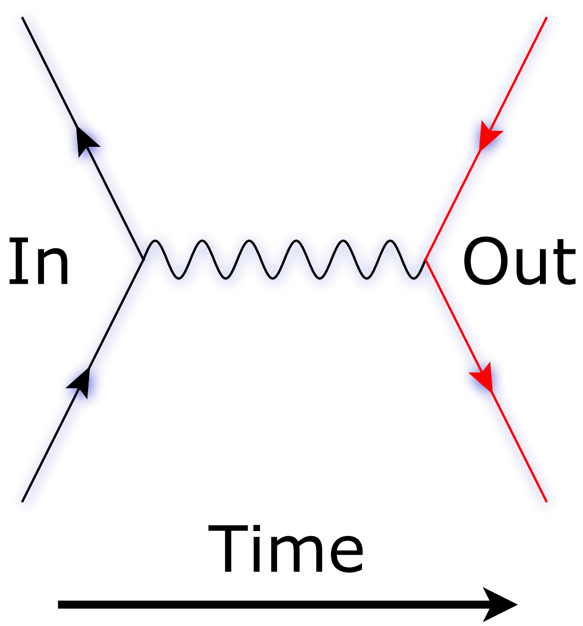
2. What is the colour factor for QCD Rutherford scattering, $qq \to qq$ via t-channel gluon exchange?



Crossings

$$e^+e^- \to \gamma^*/Z \to q\bar{q}$$

(Hadronic Z Decay)

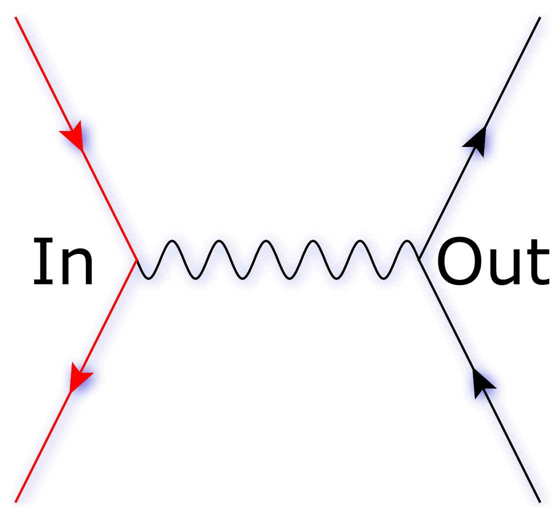


Color Factor:

$$\mathrm{Tr}[\delta_{ij}] = N_C$$

$$q\bar{q} \to \gamma^*/Z \to \ell^+\ell^-$$

(Drell & Yan, 1970)

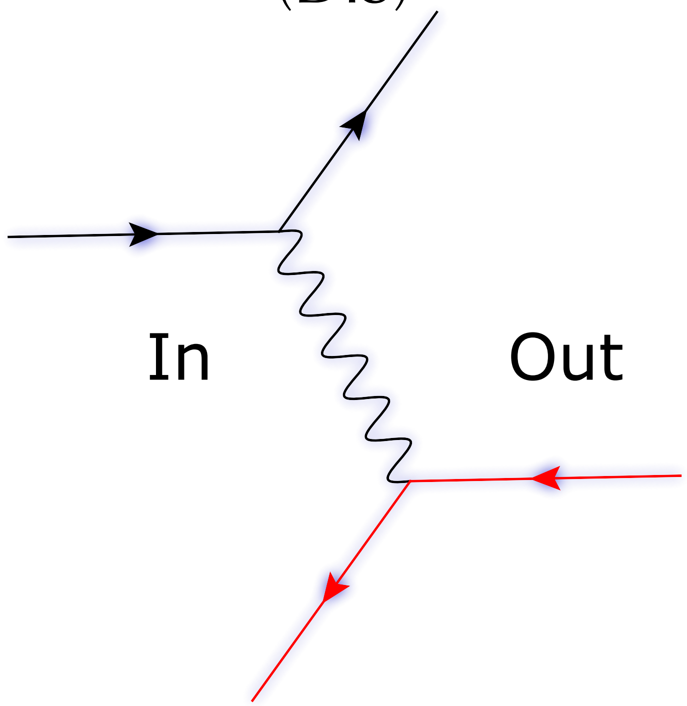


Color Factor:

$$\frac{1}{N_C^2}\mathrm{Tr}[\delta_{ij}] = \frac{1}{N_C}$$

$$\ell q \overset{\gamma^*/Z}{\to} \ell q$$

(DIS)



Color Factor:

$$\frac{1}{N_C}\mathrm{Tr}[\delta_{ij}] = 1$$

Uncalculated Orders

Naively O(α_s) - True in e^+e^- !

$$\sigma_{\mathrm{NLO}}(e^+e^- \to q\bar{q}) = \sigma_{\mathrm{LO}}(e^+e^- \to q\bar{q}) \left(1 + \frac{\alpha_s(E_{\mathrm{CM}})}{\pi} + \mathcal{O}(\alpha_s^2)\right)$$

Generally larger in hadron collisions

Typical "K" factor in pp (= $\sigma_{\mathrm{NLO}}/\sigma_{\mathrm{LO}}$) ≈ 1.5 ± 0.5

Why is this? Many pseudoscientific explanations

Explosion of # of diagrams ($n_{\mathrm{Diagrams}} \approx n!$)
New initial states contributing at higher orders (E.g., gq → Zq)
Inclusion of low-x (non-DGLAP) enhancements
Bad (high) scale choices at Lower Orders, …

Theirs not to reason why // Theirs but to do and die

Tennyson, The Charge of the Light Brigade

Changing the scale(s)

Why scale variation ~ uncertainty?

Scale dependence of calculated orders must be canceled by contribution from uncalculated ones (+ non-pert)

$$\alpha_s(Q^2) = \alpha_s(m_Z^2)\frac{1}{1 + b_0\ \alpha_s(m_Z)\ln\frac{Q^2}{m_Z^2} + \mathcal{O}(\alpha_s^2)}$$

$$b_0 = \frac{11N_C - 2n_f}{12\pi}$$

→ $$\left(\alpha_s(Q'^2) - \alpha_s(Q^2)\right)|M|^2 = \alpha_s^2(Q^2)|M|^2 + \ldots$$

→ Generates terms of higher order, but proportional to what you already have ($|M|^2$)→ a first naive* way to estimate uncertainty

*warning: some theorists believe it is the only way … but be agnostic! There are other things than scale dependence …

Dangers

$p_{\perp 1}$= 50 GeV
$p_{\perp 2}$= 50 GeV
$p_{\perp 3}$= 50 GeV

Complicated final states

Intrinsically Multi-Scale problems

with Many powers of α_s

E.g., W + 3 jets in pp

6 7 8

$$\alpha_s^3(m_W^2) < \alpha_s^3\left(m_W^2 + \langle p_\perp^2 \rangle\right) < \alpha_s^3\left(m_W^2 + \sum_i p_{\perp i}^2\right)$$

Global Scaling: jets don't care about m_W

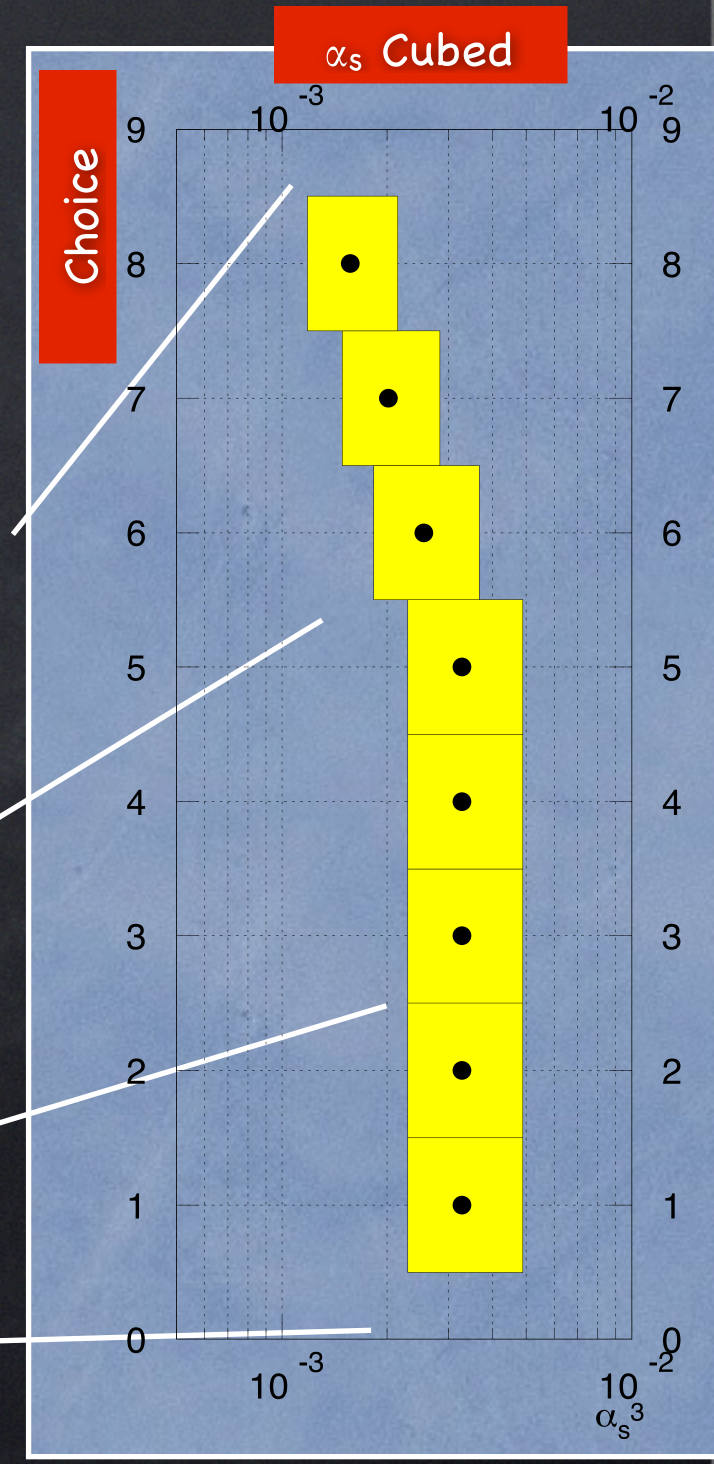
3 4 5

$$\alpha_s^3(\min[p_\perp^2]) < \alpha_s^3(\langle p_\perp^2 \rangle) < \alpha_s^3(\max[p_\perp^2])$$

MC parton showers: "Local scaling"

2 1

$$\alpha_s(p_{\perp 1})\alpha_s(p_{\perp 2})\alpha_s(p_{\perp 3}) \sim \alpha_s^3\left(\langle p_\perp^2 \rangle_{\mathrm{geom}}\right)$$



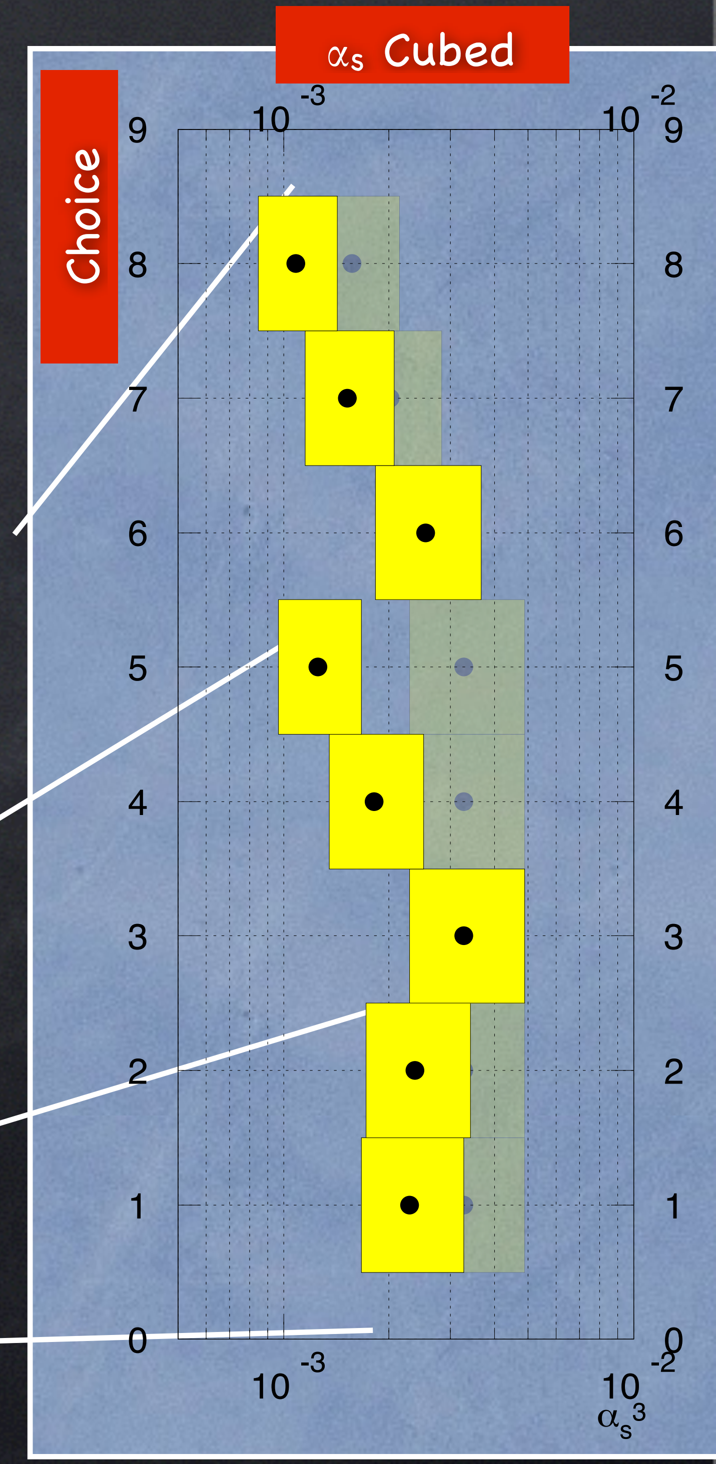
Dangers

$p_{\perp 1}$= 500 GeV
$p_{\perp 2}$= 100 GeV
$p_{\perp 3}$= 30 GeV

Complicated final states

Intrinsically Multi-Scale problems

with Many powers of α_s



If you have multiple QCD scales

→ variation of μ_R by factor 2 in each direction not good enough! (nor is × 3, nor × 4)

Need to vary also functional dependence on each scale!

(Factorization: Caveats)

1. The proof only includes the first term in an operator product expansion in "twist" = mass dimension - spin

→ Strictly speaking, only valid for $Q^2 \to \infty$. Neglects corrections of order

Higher Twist : $$\frac{\left[\ln\left(Q^2/\Lambda^2\right)\right]^{m<2n}}{Q^{2n}}$$ (n=2 for DIS)

2. The proof only applies to inclusive cross sections

In e^+e^- , in DIS, and in Drell-Yan. For everything else: factorization *ansatz*

3. Scheme dependence

In practice limited to MSbar + variations of Q_F

4. Interpretation of PDFs as parton number densities

Is only valid at Leading Order