QFT Beyond Fixed Order

Introduction to Bremsstrahlung and Jets

1. Radiation from Accelerated Charges

Soft Bremsstrahlung in Classical E&M, and in QED. The soft eikonal & coherence.

2. Infrared Singularities and Infrared Safety

IR Poles & Probabilities > 1, Sudakov logs, ***IRC Safety***

3. QCD as a Weakly Coupled Conformal Field Theory

The ***emission*** *probability; Double-Logarithmic Approximation*

The ***no-emission*** *probability; Sudakov Factor; exponentiation; example:* ***jet mass****.*

4. Parton Showers

DLA as differential evolution kernels; unitarity and detailed balance

Sampling the Sudakov; perturbation theory as a Markov Chain; Monte Carlo

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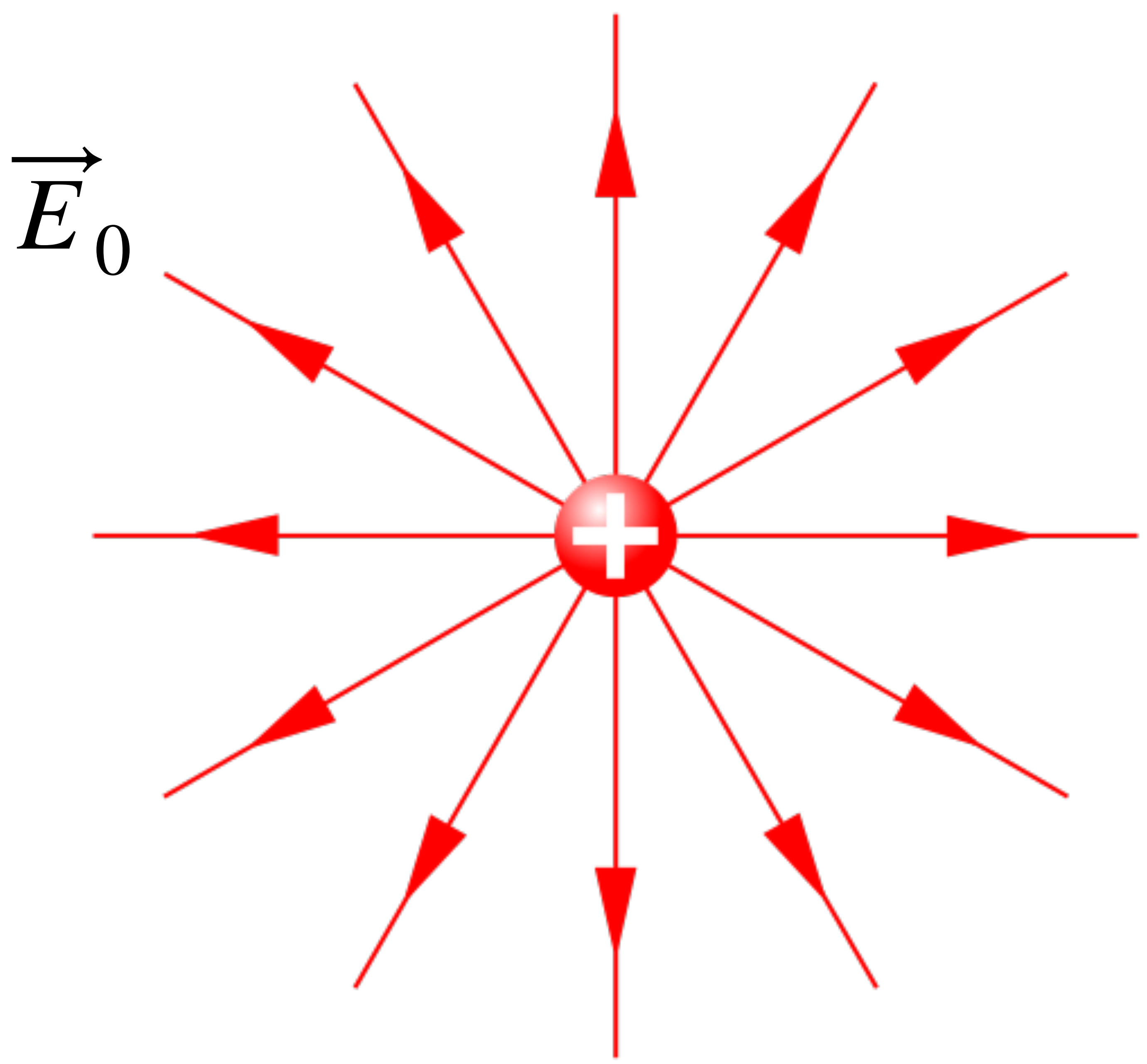
To start with, consider a classical charged particle

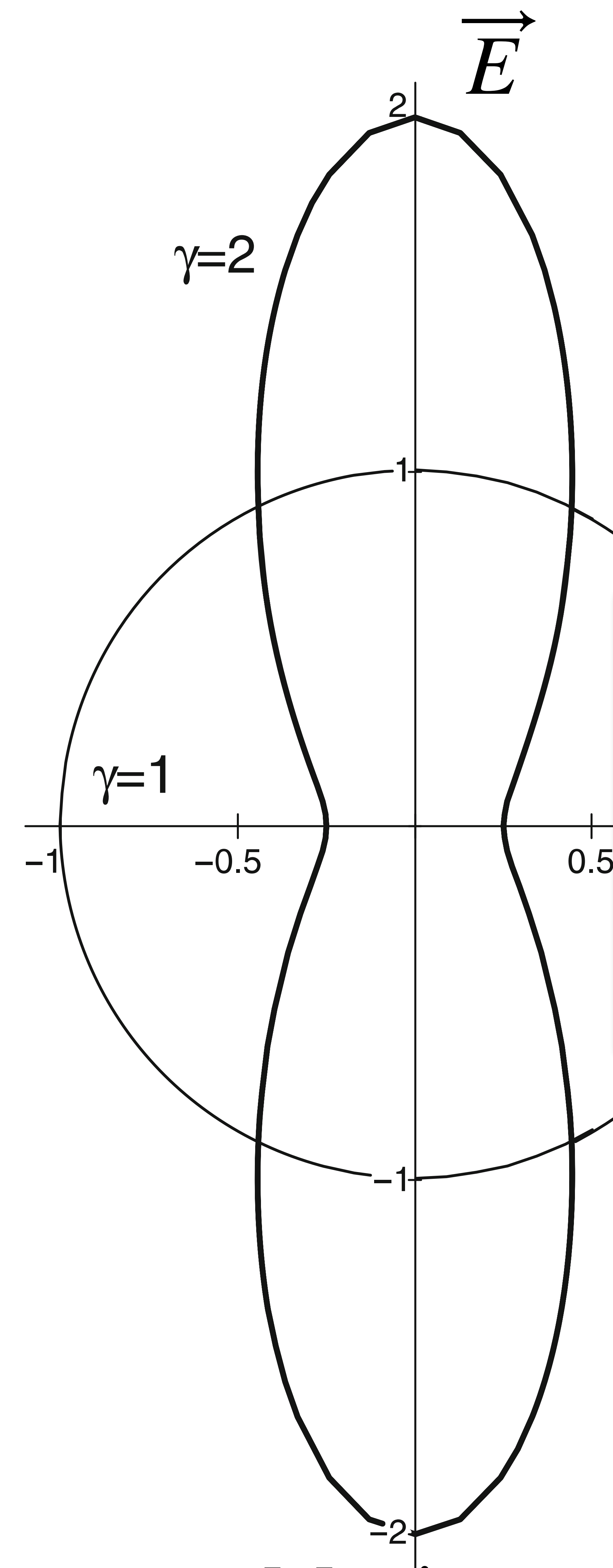
If it is **charged**, it has a **Coulomb field**



Stationary

Lorentz boost

Cf, Eg., Scheck 2.4 (PHS3201)



Transverse components get larger $\to \gamma\,|E_0|$

Longitudinal component gets smaller $\to \dfrac{1}{\gamma^2}\,|\vec{E}_0|$

Scheck, fig.2.5

Moving

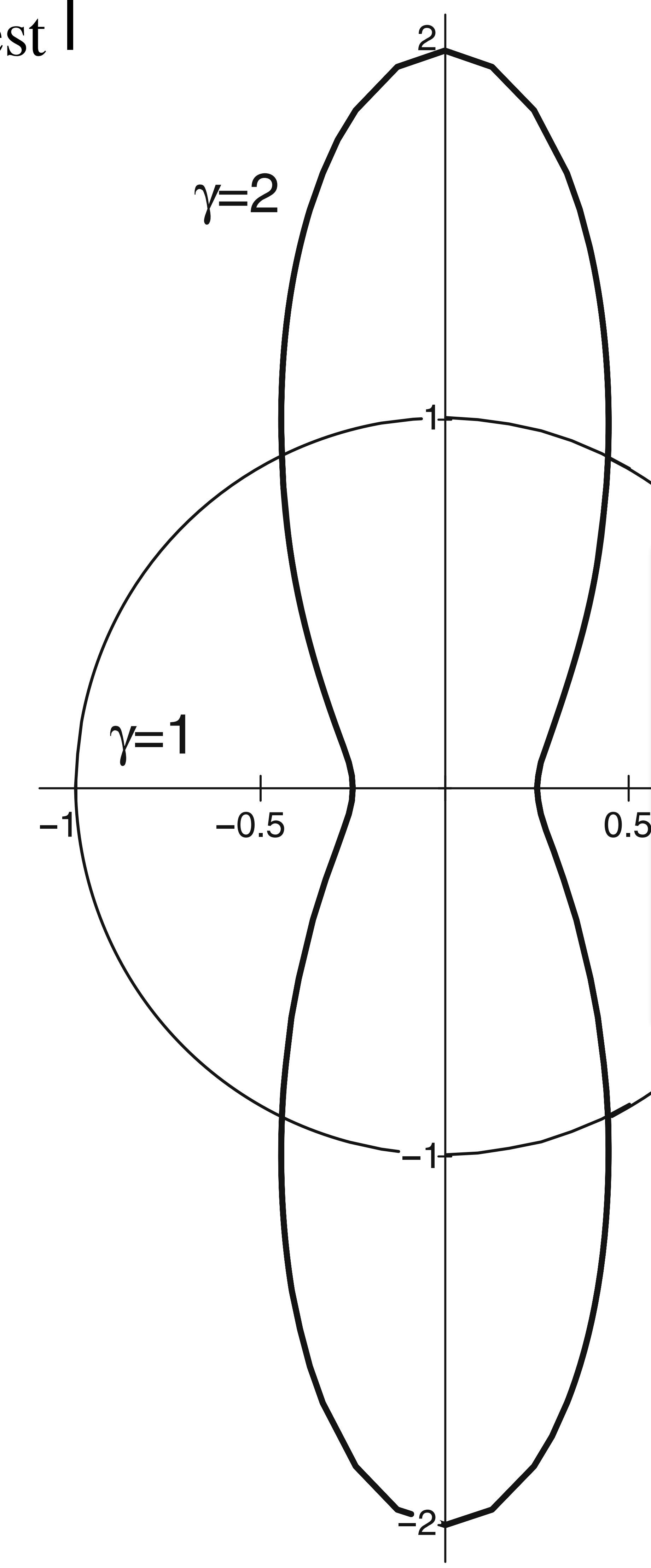
Warmup: Classical Fields of a Charge in Uniform Motion

The EM fields of an electron **in uniform relativistic motion** are predominantly **transverse**, with $|E| \approx |B| \approx \gamma |E_{\rm rest}|$

Weiszäcker (1934) & Williams (1935) noted that, in the limit of a large boost, that starts to look a lot like (a superposition of) plane waves!

They interpreted this to mean that **fast electrically charged particles** can be regarded as carrying with them **clouds of virtual photons**

That was for a charge in uniform motion.
What happens if we **give it a kick?**



Transverse components get larger $\to \gamma |E_0|$

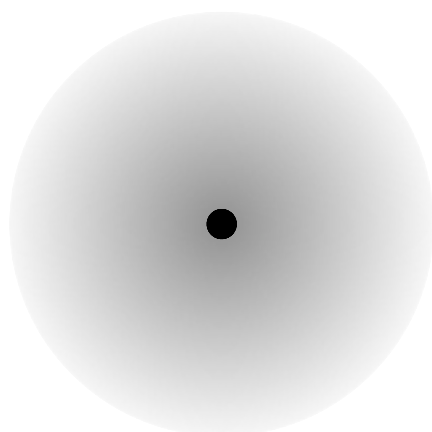
Longitudinal component gets smaller $\to \frac{1}{\gamma^2} |\vec{E}_0|$

Scheck, fig.2.5

a.k.a. "the method of virtual quanta" (e.g., Jackson, *Classical Electrodynamics*) or "the equivalent photon approximation" (EPA)

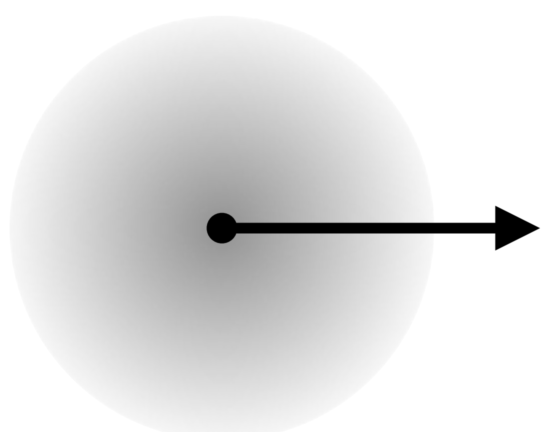
Consider a (QED or QCD) charge that receives a kick at $t = 0$

(Analogous to the elastic-scattering form-factor situation we discussed earlier in the course)



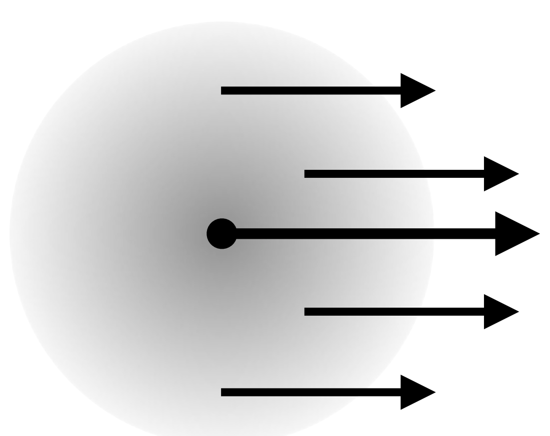
Before t_0: Coulomb field at rest with respect to particle.

An observer far away sees the fields of a uniformly moving charge. No plane waves; no radiation.



At t_0: instantaneously replace stationary charge by one moving at velocity v'

If $v \neq v'$, the Coulomb field will need to be rearranged (sped up).



Much later than t_0: moving charge, with boosted Coulomb field.

No plane waves; no radiation.

*The far-away observer experiences a **disturbance** in the EM field (generated around $t \sim t_0$), which upon Fourier transformation ➤ a spectrum of plane-wave radiation*

Peskin & Schroeder shows the calculations for actual electrons

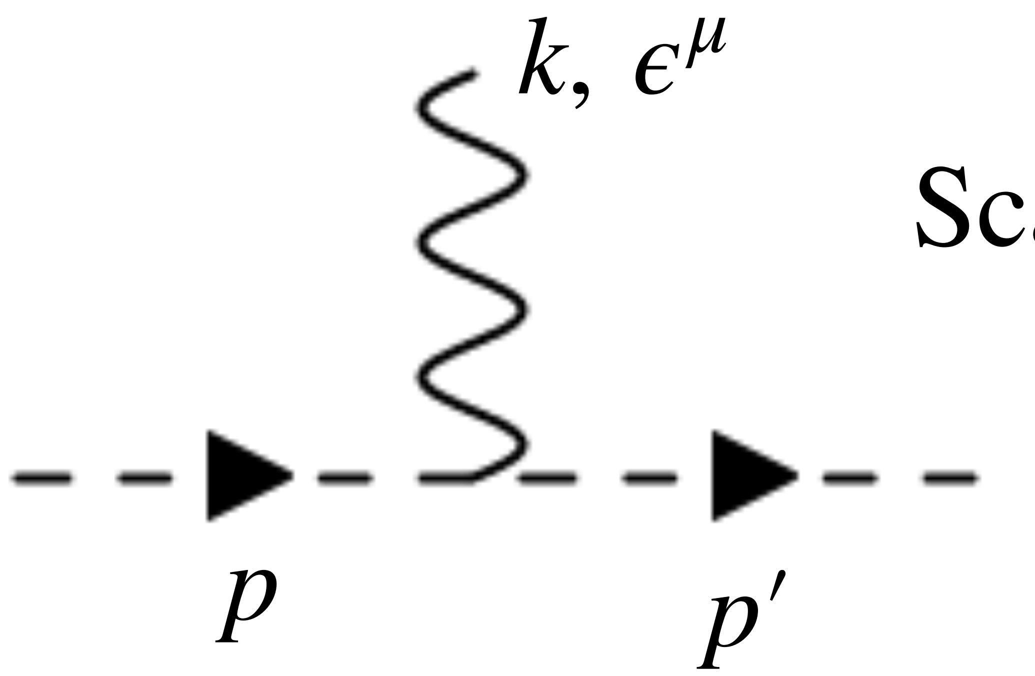
I will not care about spin; will use **scalar charged particles** instead (for simplicity)

We will see that we get the **same** result(s) in the end (in the **soft** limit, $k_\gamma \to 0$)

Feynman rules for spin-0 particle coupled to a gauge field

1 Trivial wave function for incoming/outgoing scalars

$\frac{i}{p^2 - m^2}$ Scalar propagator



Scalar-Scalar-Vector vertex

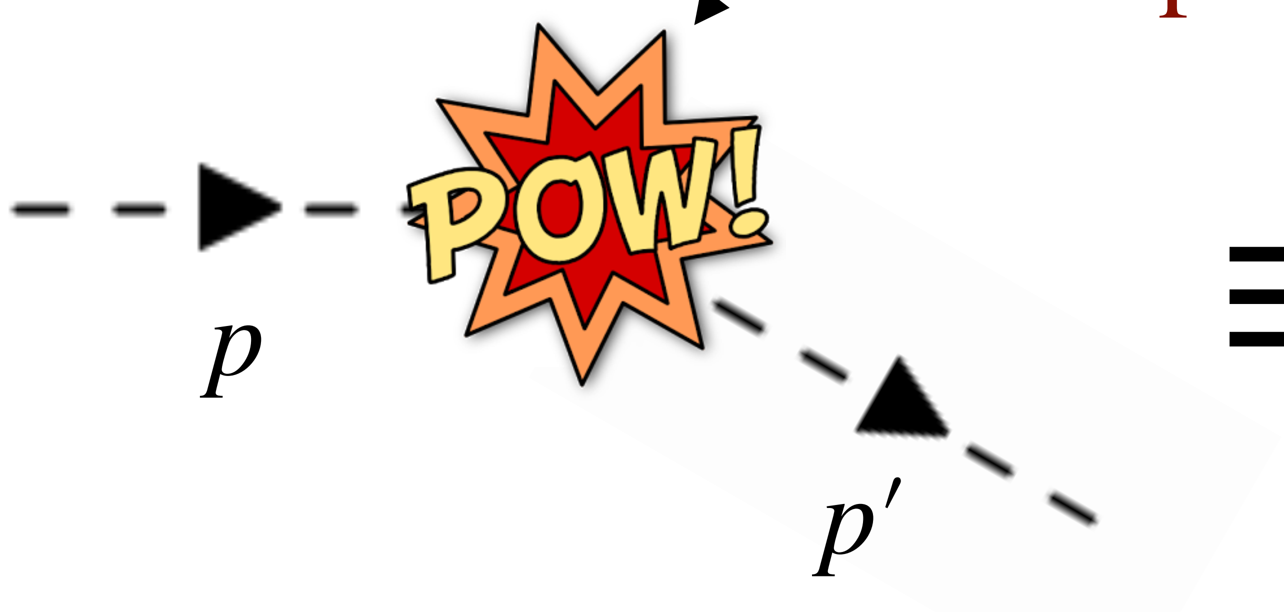
$-ie(p + p')^\mu$

$$\left(\begin{array}{l} D_\mu \phi D^\mu \phi^* = (\partial_\mu + ieA_\mu)\phi(\partial^\mu + ieA^\mu)\phi^* \\ \Longrightarrow \text{terms with } ieA_\mu \phi (\partial^\mu \phi)^* + \text{c.c.} \end{array} \right)$$

Note: in principle applies every time we disturb a charged particle!

Expect consequences to be universal, for every vertex that involves a charged particle

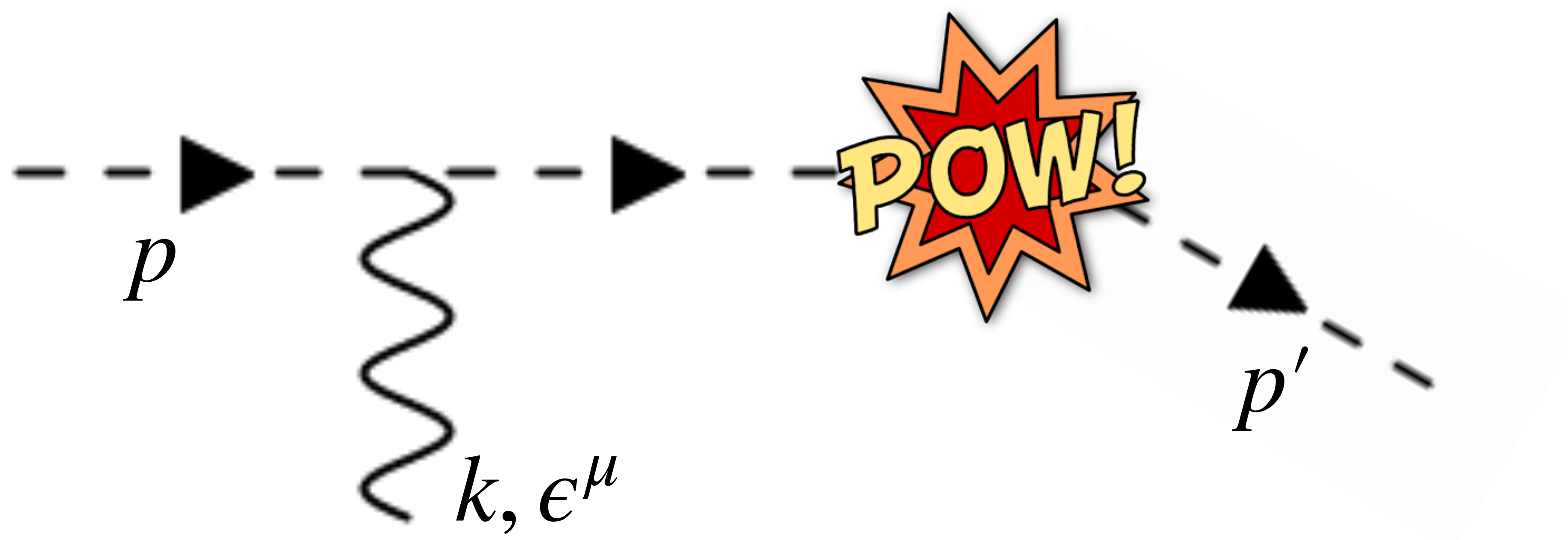
Some operator that annihilates the particle with p and creates the one with p'



$$\equiv i\mathcal{M}_0(p, p')$$

"Born-level" amplitude for the process at hand (precise form not important)

Radiative Correction (Bremsstrahlung) to first perturbative order in e:



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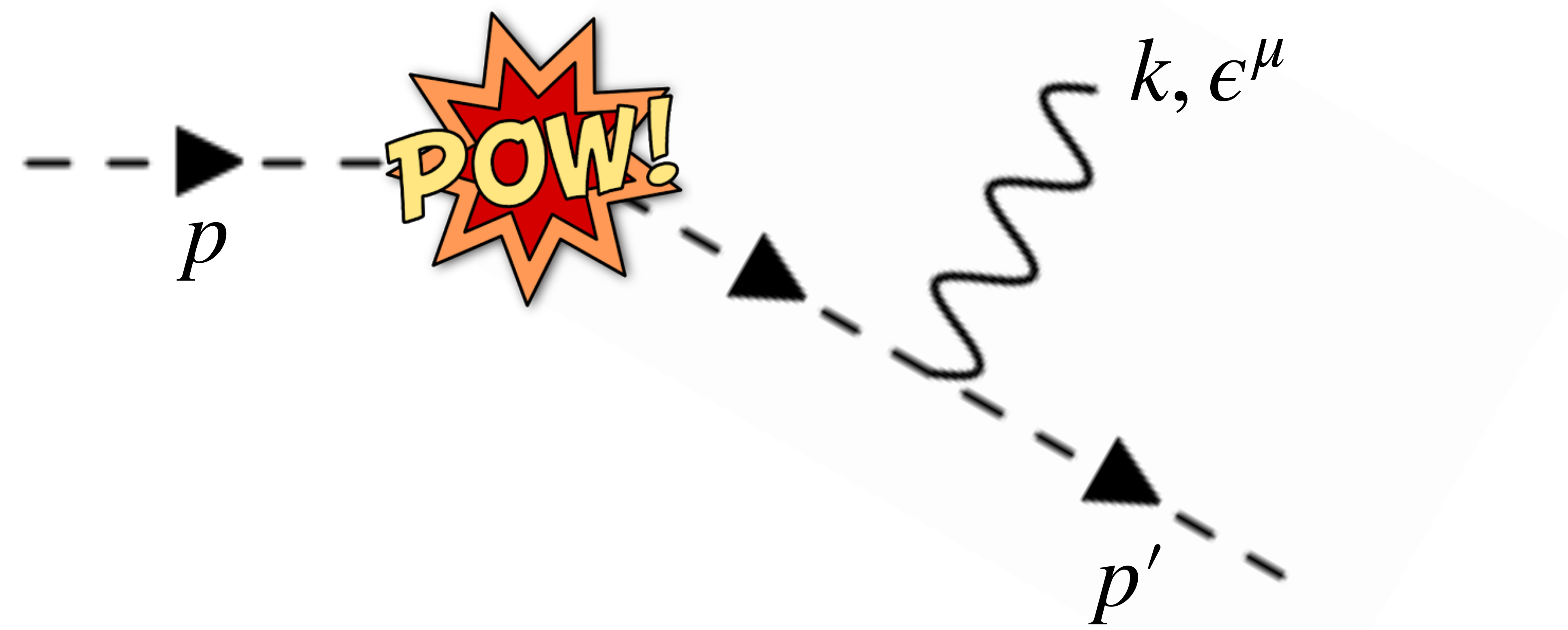


$$-ie\mathcal{M}_0(p-k, p')\frac{i}{(p-k)^2-m^2}(p+p-k)^\mu\epsilon^*_\mu(k)$$

Exercise: write down the amplitude for this diagram

A little algebra

On-shell photon has transverse polarisation; satisfies $k^\mu \epsilon_\mu = 0$

$$(2p - k)^\mu \epsilon^*_\mu \to 2(p \cdot \epsilon^*)$$

(a.k.a. "Lorenz gauge")

$$(2p' + k)^\mu \epsilon^*_\mu \to 2(p' \cdot \epsilon^*)$$

Propagators :

$$\frac{1}{(p-k)^2 - m^2} = \frac{-1}{2(p \cdot k)}$$

$$\frac{1}{(p'+k)^2 - m^2} = \frac{+1}{2(p' \cdot k)}$$

+ Consider soft photon limit: $|k| \ll |p^0|$

$$\mathcal{M}_0(p-k, p') \sim \mathcal{M}_0(p, p') \sim \mathcal{M}_0(p, p'+k)$$

Probability for soft photon bremsstrahlung

Amplitude for diagram with radiation before scattering becomes:

$$-ie\mathcal{M}_0(p-k,p')\frac{i}{(p-k)^2-m^2}(p+p-k)^\mu\epsilon^*_\mu(k) \quad \rightarrow \quad -e\mathcal{M}_0(p,p')\frac{(p\cdot\epsilon^*)}{(p\cdot k)}$$

The other amplitude (radiation *after* scattering) becomes $\rightarrow +e\mathcal{M}_0(p,p')\frac{(p'\cdot\epsilon^*)}{(p'\cdot k)}$

Squaring and integrating to get the cross section, we get:

$$\mathrm{d}\sigma(p\rightarrow p'+k;\epsilon) = \mathrm{d}\sigma(p\rightarrow p')\int\frac{\mathrm{d}^3k}{(2\pi)^3 2k}e^2\left|\frac{(p'\cdot\epsilon)}{(p'\cdot k)}-\frac{(p\cdot\epsilon)}{(p\cdot k)}\right|^2$$

Same as classical result!

Denoting the one-particle phase space element by $\mathrm{d}\Phi_1(k)$ and summing over photon polarisations ➤ total **probability density per phase-space element**:

$$\frac{\mathrm{d}\sigma(p\rightarrow p'+k)}{\mathrm{d}\sigma(p\rightarrow p')\,\mathrm{d}\Phi_1} = e^2\left(\frac{2(p\cdot p')}{(p'\cdot k)(p\cdot k)}-\frac{m^2}{(p\cdot k)^2}-\frac{m^2}{(p'\cdot k)^2}\right)$$

a.k.a. the **"soft-eikonal"** or **"dipole"** factor

We considered a generic process: a charged particle got kicked

Expect to see this expression any time we look at the (soft limit of) a first-order QED correction to *any* scattering process involving a gauge-charged current:

$$d\sigma_{pp'+\gamma} = e^2 \left(\frac{2(p \cdot p')}{(p' \cdot k_\gamma)(p \cdot k_\gamma)} - \frac{m^2}{(p \cdot k_\gamma)^2} - \frac{m^2}{(p' \cdot k_\gamma)^2} \right) d\Phi_\gamma \, d\sigma_{pp'}$$

"Born process" with charge that got kicked from p to p' (Before adding photon)

Note: same expression for gluon emission in QCD, with $e^2 \to g_s^2 N_C$

(For completeness, note that for particles with spin there are further universal terms, called **collinear** which are also relevant to bremsstrahlung and which can be derived in a similar manner; here we focused only on the **soft** limit.)

Some immediate follow-up questions:

What is the **total probability** to emit a photon? How about two photons; or more?

What if there is **more than one charged** particle that gets kicked? (Or one gets several kicks?)

What about if the charged particle is **not pointlike** (e.g., a charged hadron)?

How does this relate to **gluons and jets in QCD**? How about weak $SU(2)_L$? Other (gauge) theories?

Towards the total probability

Want to integrate the dipole factor over $\mathrm{d}^3k = k^2\,\mathrm{d}k\,\mathrm{d}\cos\theta_k\,\mathrm{d}\varphi_k$

$$\int \frac{d^3k}{(2\pi)^3\, 2k}\,\frac{2(p\cdot p')}{(p\cdot k)(p'\cdot k)} = \frac{1}{4\pi^2}\int\frac{dk}{k}\int d\cos\theta_k\,\frac{d\varphi_k}{2\pi}\,\frac{1-\cos\theta_{pp'}}{(1-\cos\theta_{kp})(1-\cos\theta_{kp'})}$$

Looks **divergent** for small k (**soft** limit; our approximation *should* be OK there), **and/or** for small θ_{kp} , $\theta_{kp'}$ (**collinear**, with p or p' respectively). But let's forge ahead …

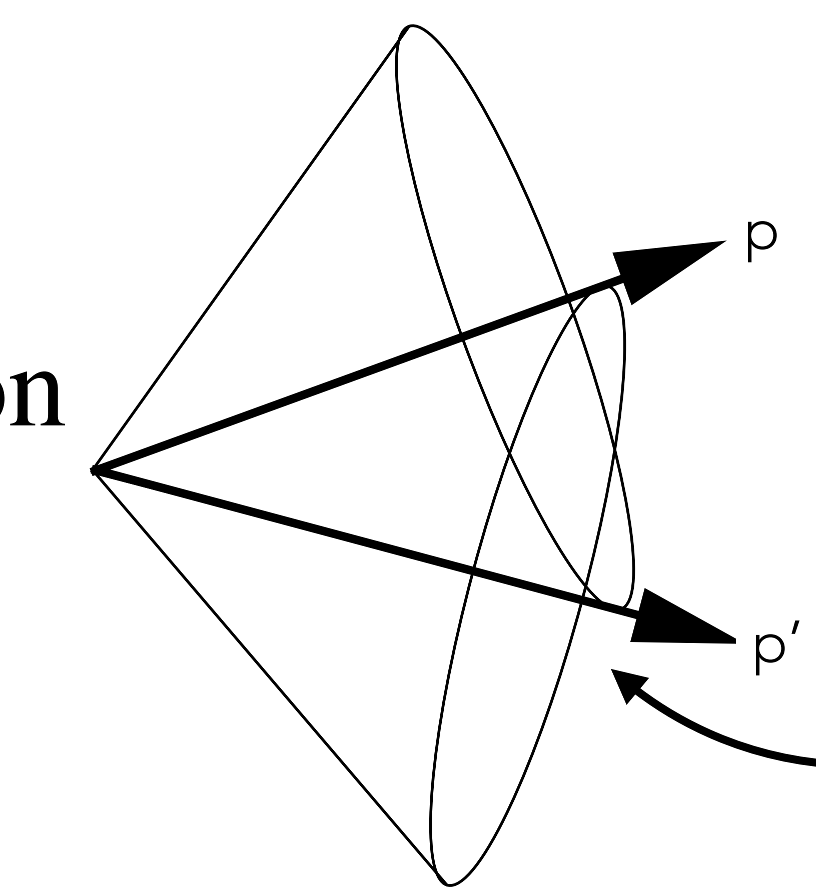
To do φ integral, separate the two collinear pole structures, by **adding and subtracting two terms**, then integrate **half** of the dipole factor with each combination, using:

$$\int_0^{2\pi}\frac{d\varphi_{kp}}{4\pi}\underbrace{\left(\frac{1-\cos\theta_{pp'}}{(1-\cos\theta_{kp})(1-\cos\theta_{kp'})}+\frac{1}{1-\cos\theta_{kp}}-\frac{1}{1-\cos\theta_{kp'}}\right)} = \frac{1}{2(1-\cos\theta_{kp})}\left(1+\frac{\cos\theta_{kp}-\cos\theta_{pp'}}{|\cos\theta_{kp}-\cos\theta_{pp'}|}\right)$$

Only divergent for $\theta_{kp}\to 0$, not for $\theta_{kp'}\to 0$ (and vice versa for the other half)

Already an interesting result!

Averaged over φ_k, there is **zero** soft radiation outside cone(s) with opening angle $\theta_{pp'}$



The **kp** term of the partitioned eikonal is only non-zero in a cone around p

The **kp'** term of the partitioned eikonal

Radiation before and after scattering adds **coherently**!