# QFT with Hadrons
## Introduction to B Physics

### 1. Leptonic Decays of Hadrons: from $\pi \to \ell\nu$ to $B \to \ell\nu$

*QFT in Hadron Decays. Decay Constants. Helicity Suppression in the SM.*

### 2. On the Structure and Unitarity of the CKM Matrix

*The CKM Matrix. The GIM Mechanism. The Unitarity Triangle.*

### 3. Semi-Leptonic Decays and the "Flavour Anomalies"

*$B \to D^{(*)} \ell \nu$. The Spectator Model. Form Factors. Heavy Quark Symmetry.*

→ *$B \to K^{(*)} \ell^+ \ell^-$. FCNC. Aspects beyond tree level. Penguins. The OPE.* ***Data.***



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# From $m_W$ to $m_b$

## What does "running" of the Wilson coefficients mean, and what consequences does it have?

Matrix Equation: $C_i(\mu) = \sum_j U_{ij}(\mu, m_W) C_j(m_W)$

U: "Evolution Matrix"

**QCD corrections ➤ Large logs** & **operator mixing** (U is not diagonal)

Examples:

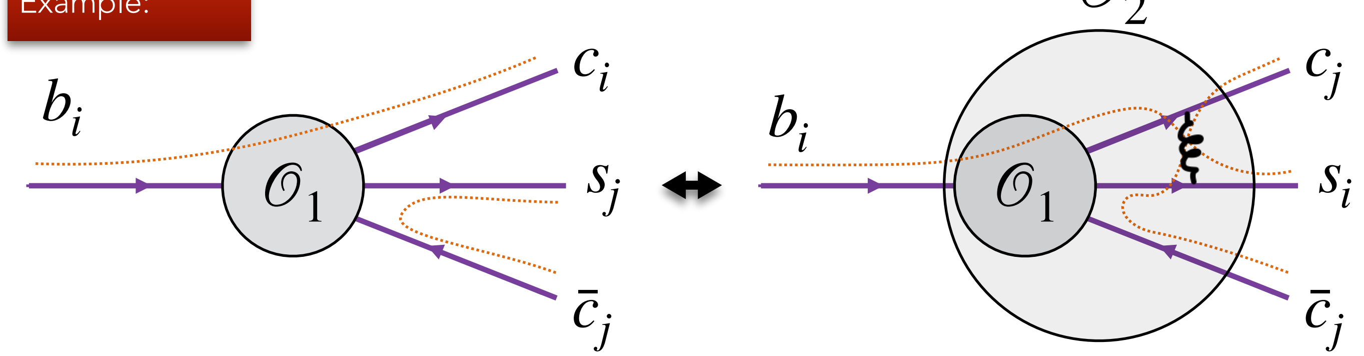
$$C_1(\mu) = 1 + \frac{3}{N_c}\frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2)\,,$$

$$C_2(\mu) = -3\,\frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2)\,.$$

Expansion parameter is not really $\alpha_s$
but $\alpha_s \ln(m_W^2/\mu^2)$

**Large for $\mu \sim m_b \ll m_W$**

Example:




## The "Renormalisation Group Method": sums $\left(\alpha_s \ln(m_W/\mu)\right)^n$

$U_{ij}$ obtained by solving differential equation ("RGE") analogous to that for other running couplings:

$$\frac{dC_i}{d\ln\mu} = \gamma_{ij}\, C_j$$

The kernels, $\gamma_{ij}$, are called the *"matrix of anomalous dimension"*

See, e.g., M. Schwarz "Quantum Field Theory and the Standard Model", chp.23
Buchalla, Buras, Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125

# Quark-Level Matrix Element

E.g., Buchalla, Buras, Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125

**For now, all we shall care about is that the $C_i(m_b)$ have been calculated in the theoretical literature with high precision**

Not just for SM, but for many scenarios of physics BSM as well.

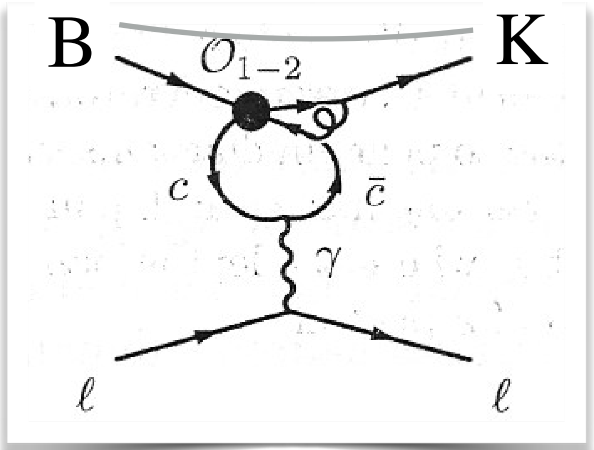
E.g., SUSY: Ali, Ball, Handoko, Hiller, hep-ph/9910221

$$\mathcal{M}(b \to s\ell^+\ell^-) = \frac{G_F\sqrt{\alpha}}{2\pi} V_{ts}^* V_{tb} \Big[ C_{9V}(m_b)[\bar{s}\gamma^\mu \frac{1}{2}(1-\gamma_5)b][\bar{\ell}\gamma_\mu \ell] + C_{10A}(m_b)[\bar{s}\gamma^\mu \frac{1}{2}(1-\gamma_5)b][\bar{\ell}\gamma_\mu\gamma_5\ell] - 2\frac{m_b}{m_B} C_{7\gamma}(m_b)[\bar{s}i\sigma^{\mu\nu}\frac{q_\nu}{q^2}\frac{1}{2}(1+\gamma_5)b][\bar{\ell}\gamma_\mu\ell] \Big]$$

Next: add **perturbative contributions from other operators**

Then: add non-perturbative effects of **hadronic resonances**

Finally: form factors ➡ **hadronic matrix elements**

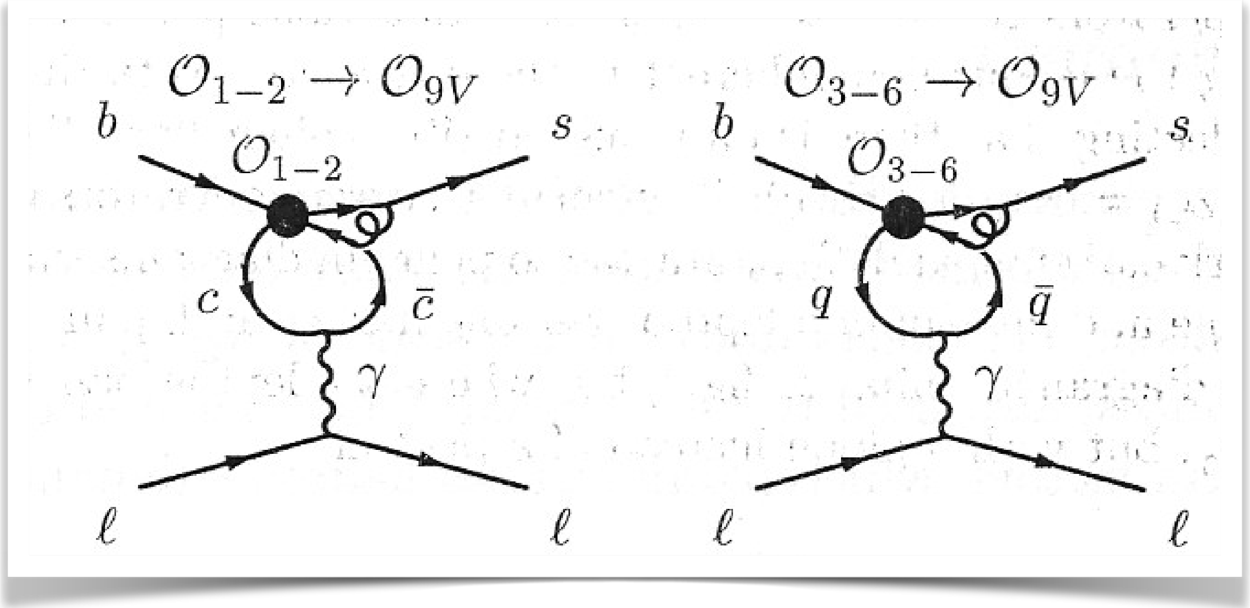


## Additional Contributions to O9:

W-exchange $O_{1,2}$ : $c\bar{c}$ pairs

QCD penguins $O_{3\text{-}6}$ : $q\bar{q}$ pairs (u,d,s,c,b)

Buras, M. Münz, Phys. Rev. D52 (1995) 186.
Misiak, Nucl. Phys. B393 (1993) 23; +err. Ibid. B439 (1995) 461




$$C_{9V} \to C_9^{\text{eff}}(q^2) = C_9 + g_c(q^2; C_{1-6}) + g_b(q^2; C_{3-6}) + g_{uds}(q^2; C_{3-4}) + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6)$$

Recall: $q^2 = (p_B - p_K)^2 = (p_{\ell^+} + p_{\ell^-})^2$

**"Loop functions"**

contain $\ln m_c^2/m_b^2$, $\ln q^2/m_b^2$, $\ln \mu^2/m_b^2$

Large at low $q^2$

also contain imaginary parts for $q^2 > 4m_q^2$

Perturbative calculation is presumably not valid.

**Main worry is $g_c$** since it gets contributions from the O(1) $C_1$ coefficient

Corresponds to on-shell quarks ➤ can propagate over long distances

Question: what do you call a $c\bar{c}$ pair with $q^2 \sim 4m_c^2$, in a spin-1 state?

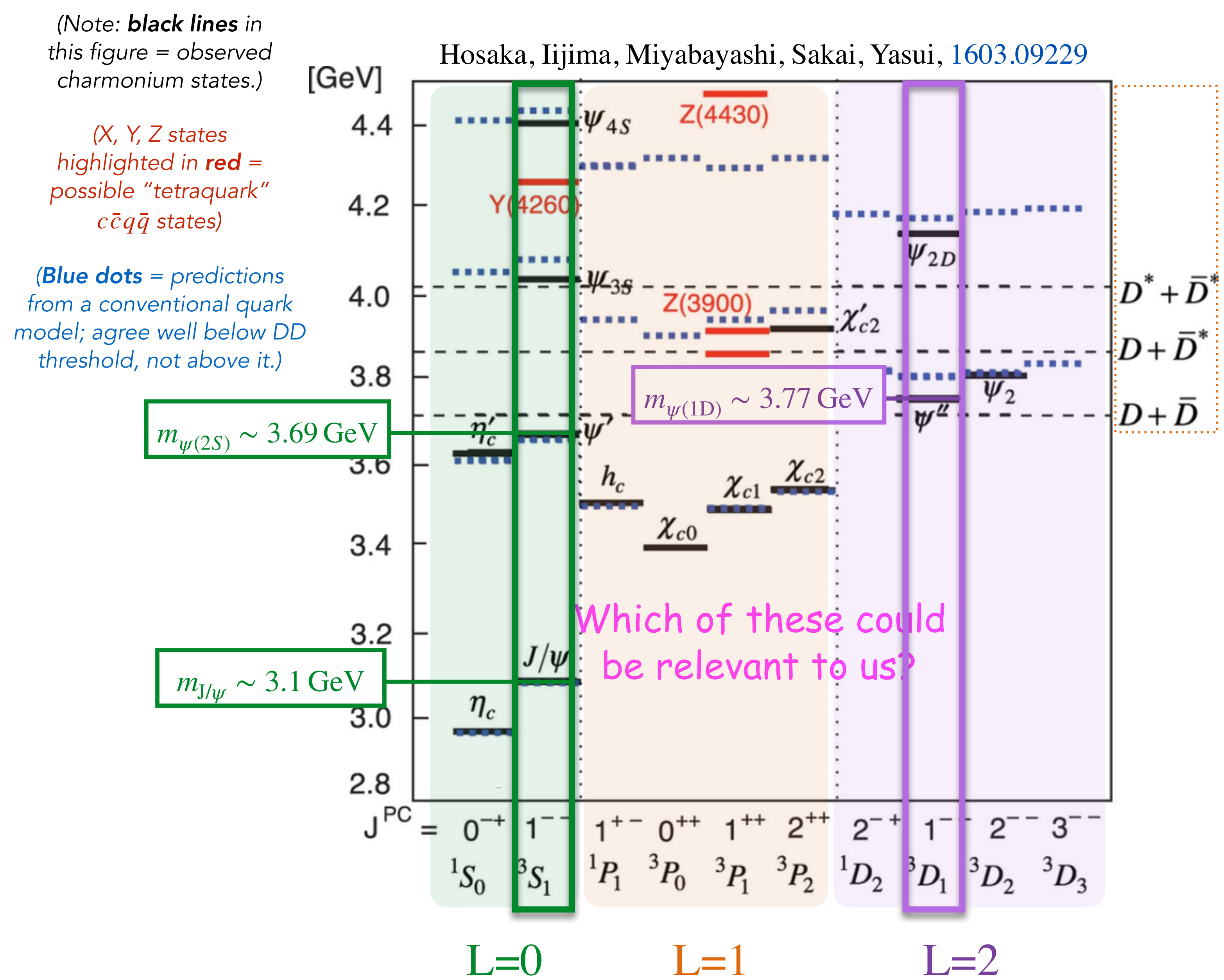
**Note also:** $C_{7\gamma} \to C_7^{\text{eff}} = C_{7\gamma} + C_5/3 - C_6$

(*in the scheme used by Buras, Fleischer, hep-ph/9704376)

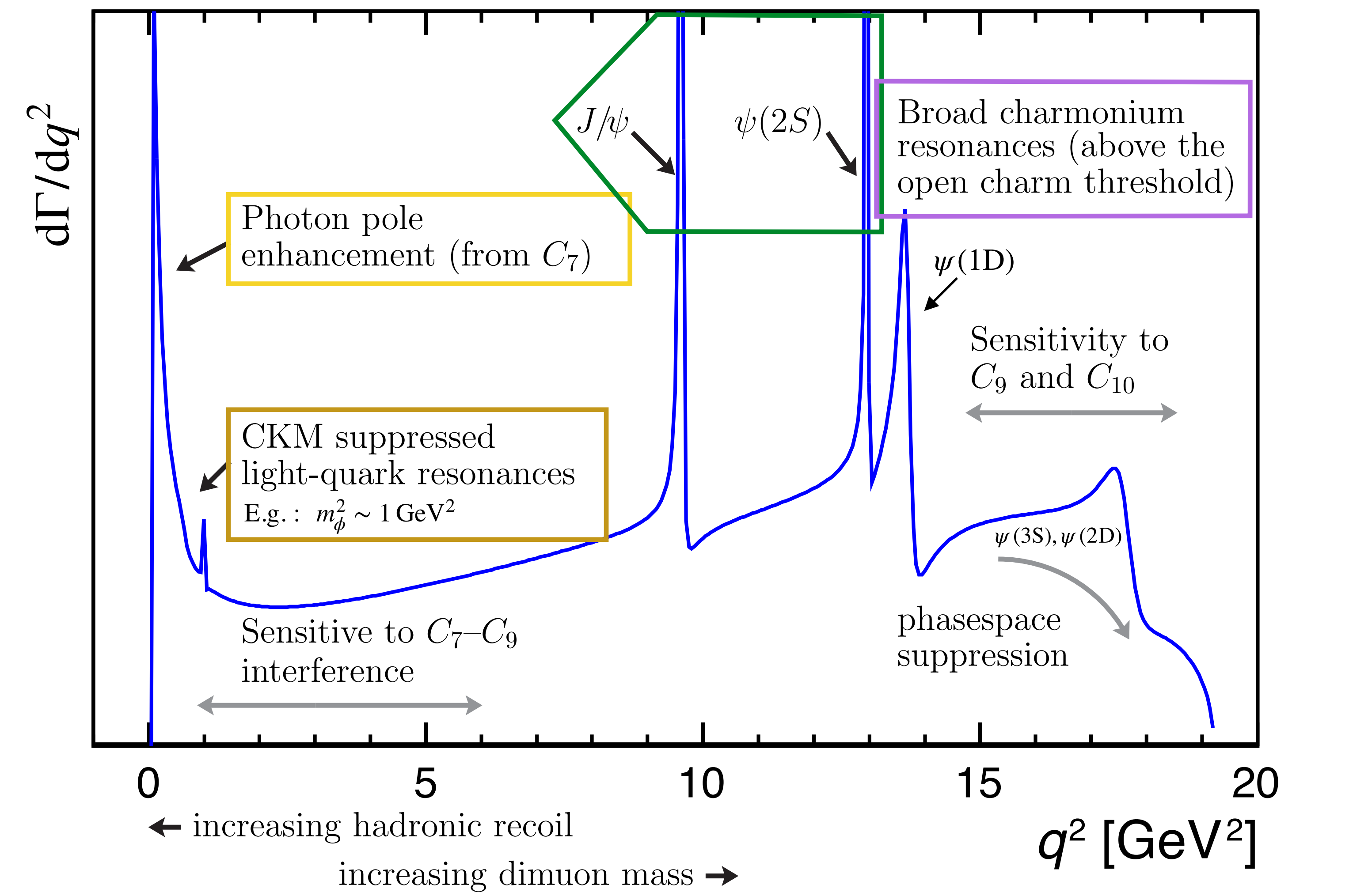
## Which $c\bar{c}$ states are there?

Are they important?

**Yes:** in resonant region(s), process is really $B \to J/\psi K$, followed by $J/\psi \to \ell^+\ell^-$.




Cartoon adapted from Blake, Lanfranchi, Straub, 1606.00916




(can add resonances with Breit-Wigner functions + "non-factorizable contributions" in $C_9^{\text{eff}}$)

Note: the dilepton $q^2$ spectrum is still **relatively clean below the J/psi**

# (Non-Factorizable Contributions?)

## We so far did not consider multi-hadronic final states

But that is effectively what the $B \to J/\psi K$ intermediate states are.

**Non-factorisable contributions**: general problem in **multi-hadronic processes**.

## The factorisation ansatz

Including the $J/\psi$ and other $c\bar{c}$ (henceforth $\psi_n$) states as Breit-Wigner distributions in $C_9^{\rm eff}$, we are effectively *factoring* the process, into $B \to K$ transition $\otimes$ $\psi_n$ creation (& decay):

$$\left\langle K \ell^+\ell^- \middle| \hat{H} \middle| B \right\rangle \underset{\rm Res.}{\approx} \left\langle \ell^+\ell^- \middle| \hat{H} \middle| \psi_n \right\rangle \left\langle \psi_n K \middle| \hat{H} \middle| B \right\rangle \underset{\rm Fact.}{\approx} \left\langle \ell^+\ell^- \middle| \hat{H} \middle| \psi_n \right\rangle \left\langle \psi_n \middle| \hat{H} \middle| 0 \right\rangle \left\langle K \middle| \hat{H} \middle| B \right\rangle$$

(The creation & decay amplitudes for $\psi_n$ are proportional to the $\psi_n$ decay constant.)

Ignores any crosstalk between the $J/\psi$ and $B \to K$ currents.

## Non-factorizable contributions

Long-distance interactions between the (hadronic) $J/\psi$ and $B \to K$ currents.

Beyond the scope of this course

## We are now ready to look at the hadron-level matrix element

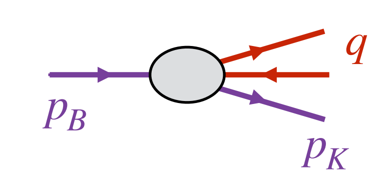
$$\mathcal{M}(B \to K\ell^+\ell^-) = \frac{G_F\alpha}{\sqrt{2}\pi} V_{tb}V_{ts}^* \Big[ C_9^{\rm eff} \big\langle K(p_K) \big| \bar{s}\gamma^\mu(1-\gamma_5)b \big| B(p_B) \big\rangle \, [\bar{\ell}\gamma_\mu\ell]$$

$$+ C_{10A} \big\langle K(p_K) \big| \bar{s}\gamma^\mu(1-\gamma_5)b \big| B(p_B) \big\rangle \, [\bar{\ell}\gamma_\mu\gamma_5\ell]$$

$$-2\frac{m_b}{m_B} C_7^{\rm eff} \big\langle K(p_K) \big| \bar{s}i\sigma^{\mu\nu}\frac{q_\nu}{q^2}(1+\gamma_5)b \big| B(p_B) \big\rangle \, [\bar{\ell}\gamma_\mu\ell] \Big]$$

## Similarly to $B \to D\ell\nu$, axial part does not contribute in $B \to K\ell^+\ell^-$.

But we do need a magnetic form factor, due to the $C_7$ contribution.




$$\big\langle K(p_K) \big| \bar{s}\gamma^\mu(1-\gamma_5)b \big| B(p_B) \big\rangle = f_+(q^2)\big(p_B + p_K\big)^\mu + f_-(q^2)(p_B - p_K)^\mu$$

(Recall: $p_B - p_K = q$)

$$\big\langle K(p_K) \big| \bar{s}i\sigma^{\mu\nu}\frac{q_\nu}{q^2}(1+\gamma_5)b \big| B(p_B) \big\rangle = \frac{f_T(q^2)}{m_B + m_K}\big(q^2(p_B+p_K)^\mu - (m_B^2 - m_K^2)q^\mu\big)$$

K is not a "heavy-light" system ($\Lambda_{QCD}/m_s \sim 1$) ➜ cannot play Isgur-Wise trick; have to keep both $f_+$ and $f_-$

## Main method is called "Light Cone Sum Rules" (LCSR)

The ones below are admittedly rather old; from hep-ph/9910221

$$F(\hat{s}) = F(0)\exp(c_1\hat{s} + c_2\hat{s}^2 + c_3\hat{s}^3).$$

| Central | $f_+$ | $f_0$ | $f_T$ |
|---|---|---|---|
| $F(0)$ | 0.319 | 0.319 | 0.355 |
| $c_1$ | 1.465 | 0.633 | 1.478 |
| $c_2$ | 0.372 | −0.095 | 0.373 |
| $c_3$ | 0.782 | 0.591 | 0.700 |

| Max | $f_+$ | $f_0$ | $f_T$ |
|---|---|---|---|
| $F(0)$ | 0.371 | 0.371 | 0.423 |
| $c_1$ | 1.412 | 0.579 | 1.413 |
| $c_2$ | 0.261 | −0.240 | 0.247 |
| $c_3$ | 0.822 | 0.774 | 0.742 |

| Min | $f_+$ | $f_0$ | $f_T$ |
|---|---|---|---|
| $F(0)$ | 0.278 | 0.278 | 0.300 |
| $c_1$ | 1.568 | 0.740 | 1.600 |
| $c_2$ | 0.470 | 0.080 | 0.501 |
| $c_3$ | 0.885 | 0.425 | 0.796 |

(and there are corresponding ones for $B \to K^*$)

# The B → K ℓ+ ℓ- Decay Distribution

## Squared matrix element + trace algebra

$$\overline{|\mathcal{M}|}^2 = \frac{G_F^2\alpha^2}{4\pi^2}|V_{ts}^*V_{tb}|^2 D(q^2)\left(\lambda(m_B^2, m_K^2, q^2) - u^2\right)$$

**Exercise:** do the steps

*Hint: use advantage of OPE basis: operators are orthogonal. E.g., axial and vector currents can't interfere.*

With $D(q^2) = \left|C_9^{\text{eff}}(q^2)\,|f_+(q^2) + \frac{2m_b}{m_B + m_K}C_7^{\text{eff}} f_T(q^2)\right|^2 + |C_{10A}|^2 f_+(q^2)^2$

And $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ac, \quad u \equiv 2p_B\cdot(p_{\ell^+} - p_{\ell^-})$

Note: we assumed lepton mass vanishes ➜ **no dependence** on $f_-$ any more!

## Phase Space

Useful Trick: factor 1 → 3 phase space into two 1 → 2 ones using

$$\int d^4q\,\delta^{(4)}(q - p_1 - p_2) = 1$$

**Exercise:** starting from the standard form of dLIPS for a 1 → 3 decay, show that :

$$\frac{d\Gamma_{B\to K\ell^+\ell^-}}{dq^2\,du} = \frac{\overline{|\mathcal{M}|}^2}{2^9\pi^3 m_B^3}$$

# What does data say?

Here just looking at LHCb measurements; From talk by E. Graverini, BEACH 2018
Additional measurements by BaBar and Belle not shown.

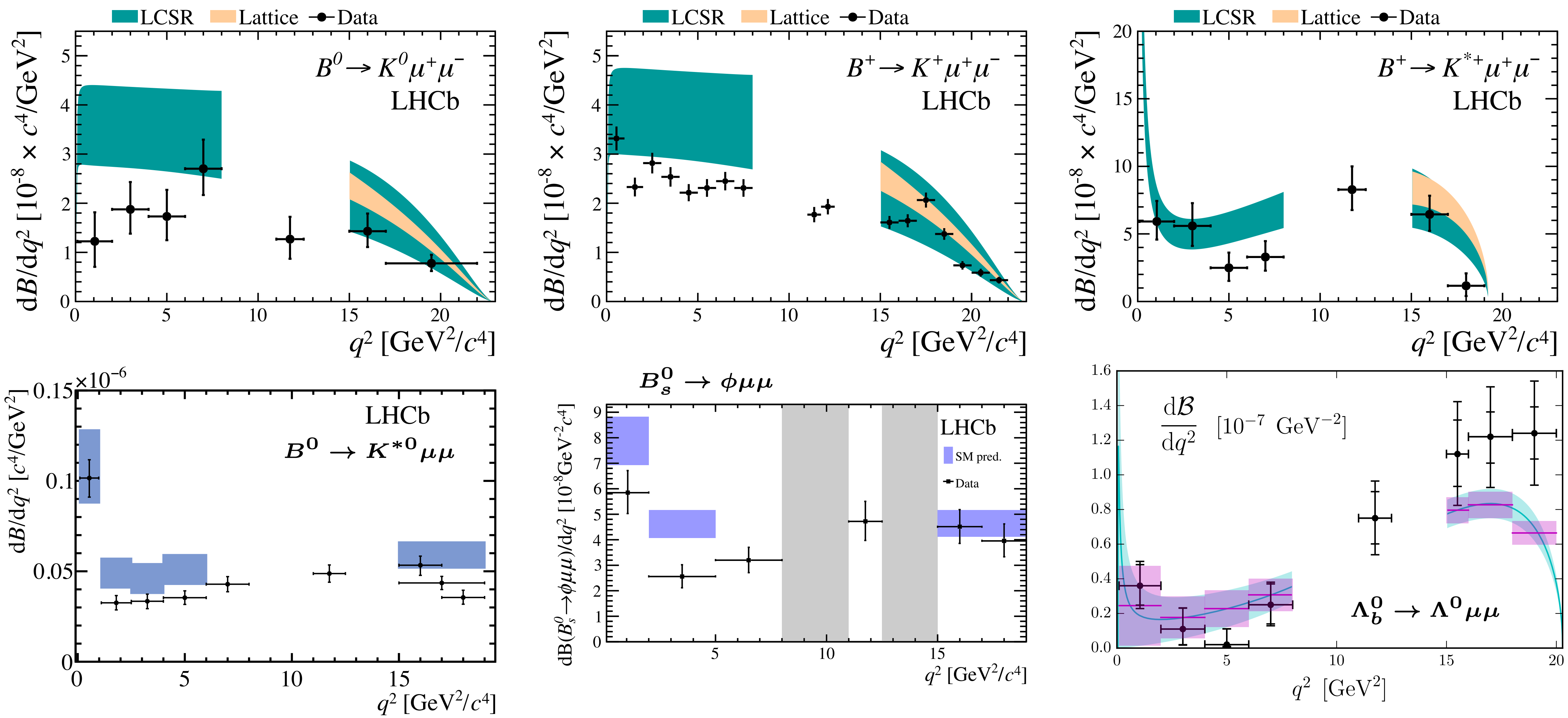



**Figure 3.** (Colours online) Differential branching fraction for various $b \to s\mu\mu$ transitions measured at LHCb, superimposed to SM predictions [2–5, 40].

For both the K and K* final states, the data is a bit on the low side (compared with SM)?

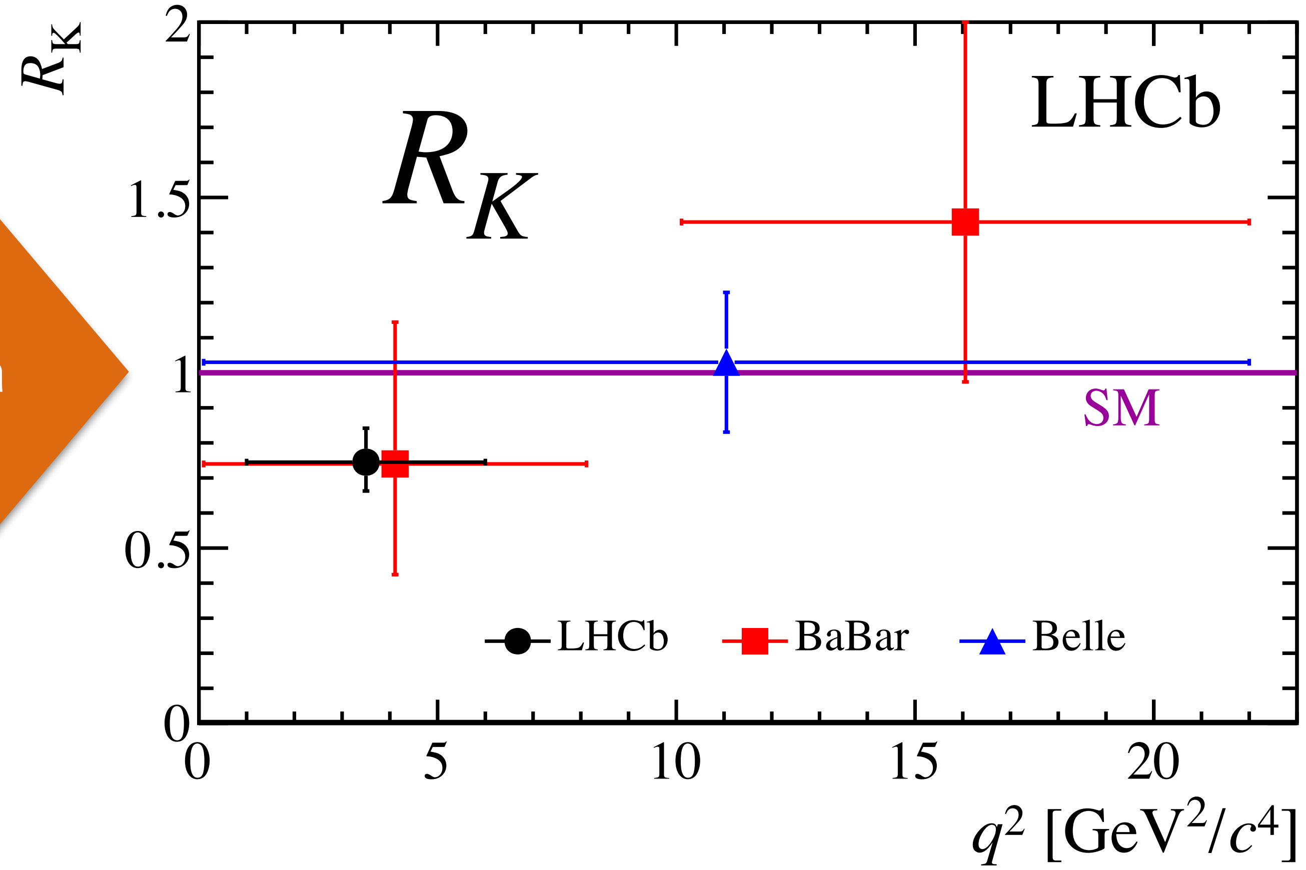
# The Flavour Anomalies Part 2

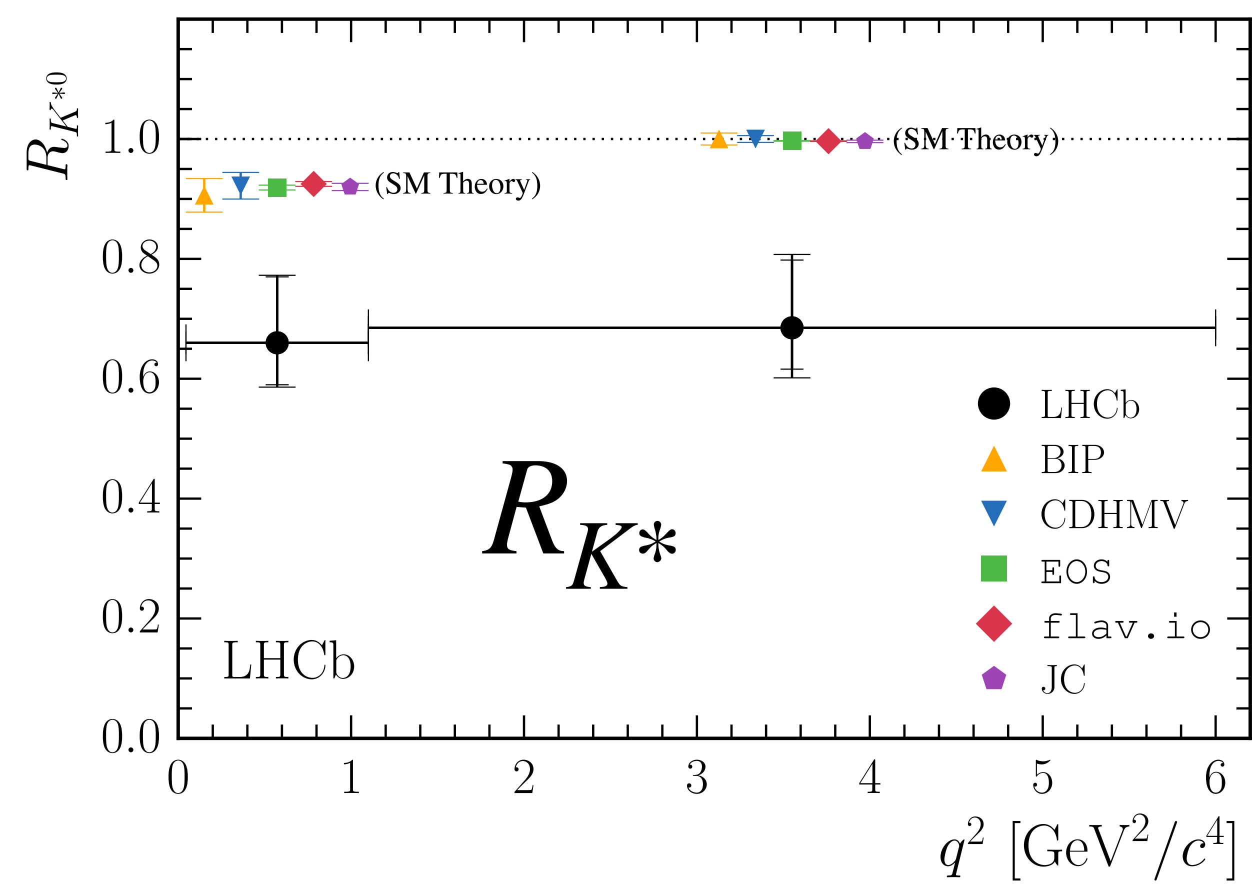
**Regardless of the complications in analysing these decays, we can again also use them as tests of lepton universality**

Now, form the two ratios: $$R_{K^{(*)}} \equiv \frac{\mathrm{Br}\left(B \to K^{(*)}\mu^+\mu^-\right)}{\mathrm{Br}\left(B \to K^{(*)}e^+e^-\right)}$$

Expect R = 1 in SM (the complicated stuff drops out in the ratio).

Data

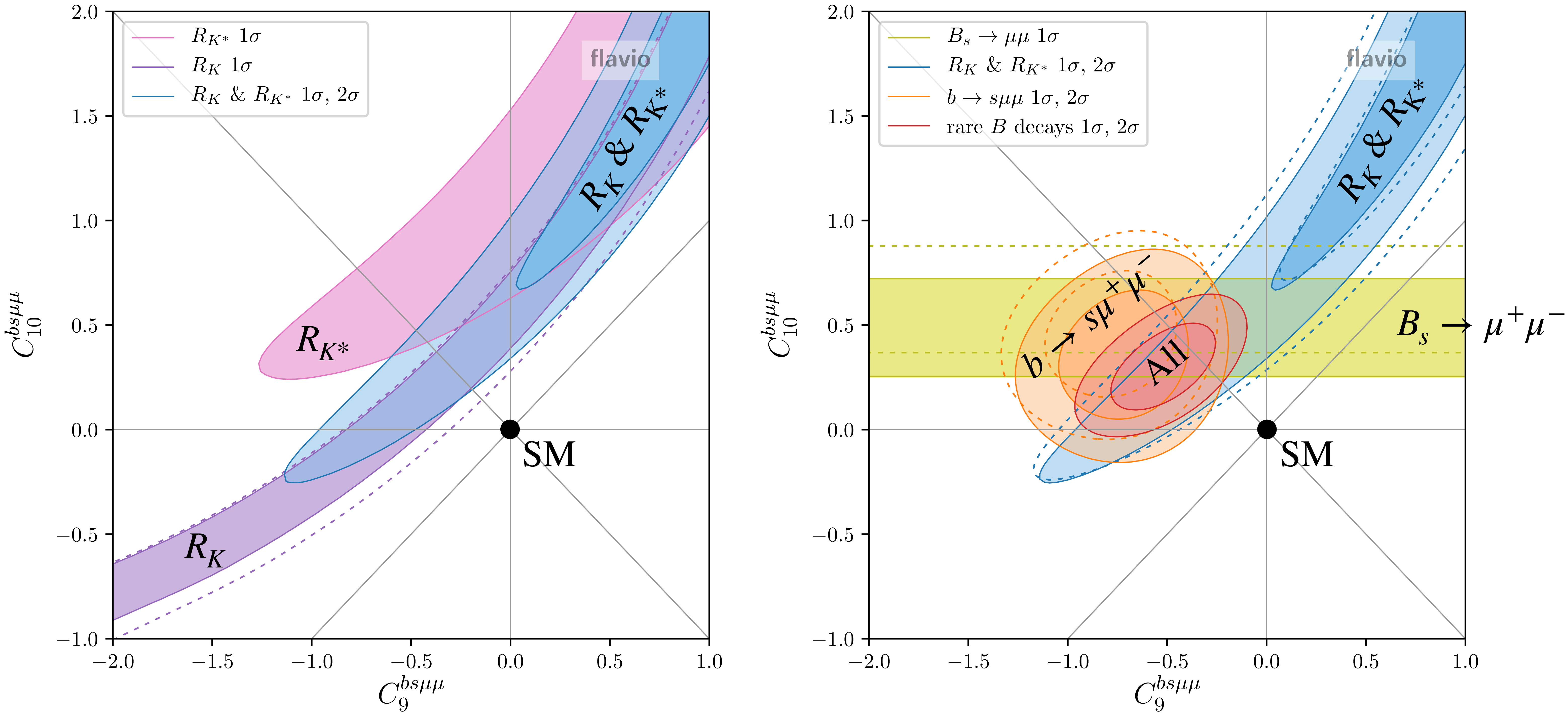



… Interesting …

# Representation in $C_9$ - $C_{10}$ space

From: Altmansshofer & Stangl, New physics in rare B decays after Moriond 2021, *Eur.Phys.J.C* 81 (2021) 10, 952 2103.13370 [hep-ph]




(Note: what is actually plotted here is the *difference* between the SM values of $C_i$ and the measured values; sometimes denoted $\Delta C_i$. Dashed lines show the status before 2021.)

# (What Approximations did we Make?)

## Top Quark Dominance

## Low-energy effective theory at quark level

Matched at finite loop order to full theory

Running at finite loop order from $m_W$ to $m_b$

Non-leptonic operators contributing to $C_7^{\mathrm{eff}}$ and $C_9^{\mathrm{eff}}$, but not $C_{10A}$

## Effect of intermediate c-cbar resonances

Non-factorizable contributions

Other hadronic states: light-quark resonances, open charm, … ?

## Form Factors

## QED Corrections at Hadronic Level?