Applications and Phenomenology

QFT II - Weeks 3 & 4

1. Leptonic Decays of Hadrons: from $\pi \to \ell\, \nu$ to $B \to \ell\, \nu$

QFT in Hadron Decays. Decay Constants. Helicity Suppression in the SM.

2. On the Structure and Unitarity of the CKM Matrix

The CKM Matrix. The GIM Mechanism. CP Violation. The Unitarity Triangle.

3. Introduction to the "Flavour Anomalies": Semi-Leptonic Decays

$B \to D^{(*)}\, \ell\, \nu$. The Spectator Model. Form Factors. Heavy Quark Symmetry.

$B \to K^{(*)}\, \ell^+\, \ell^-$. FCNC. Aspects beyond tree level. Penguins. The OPE.

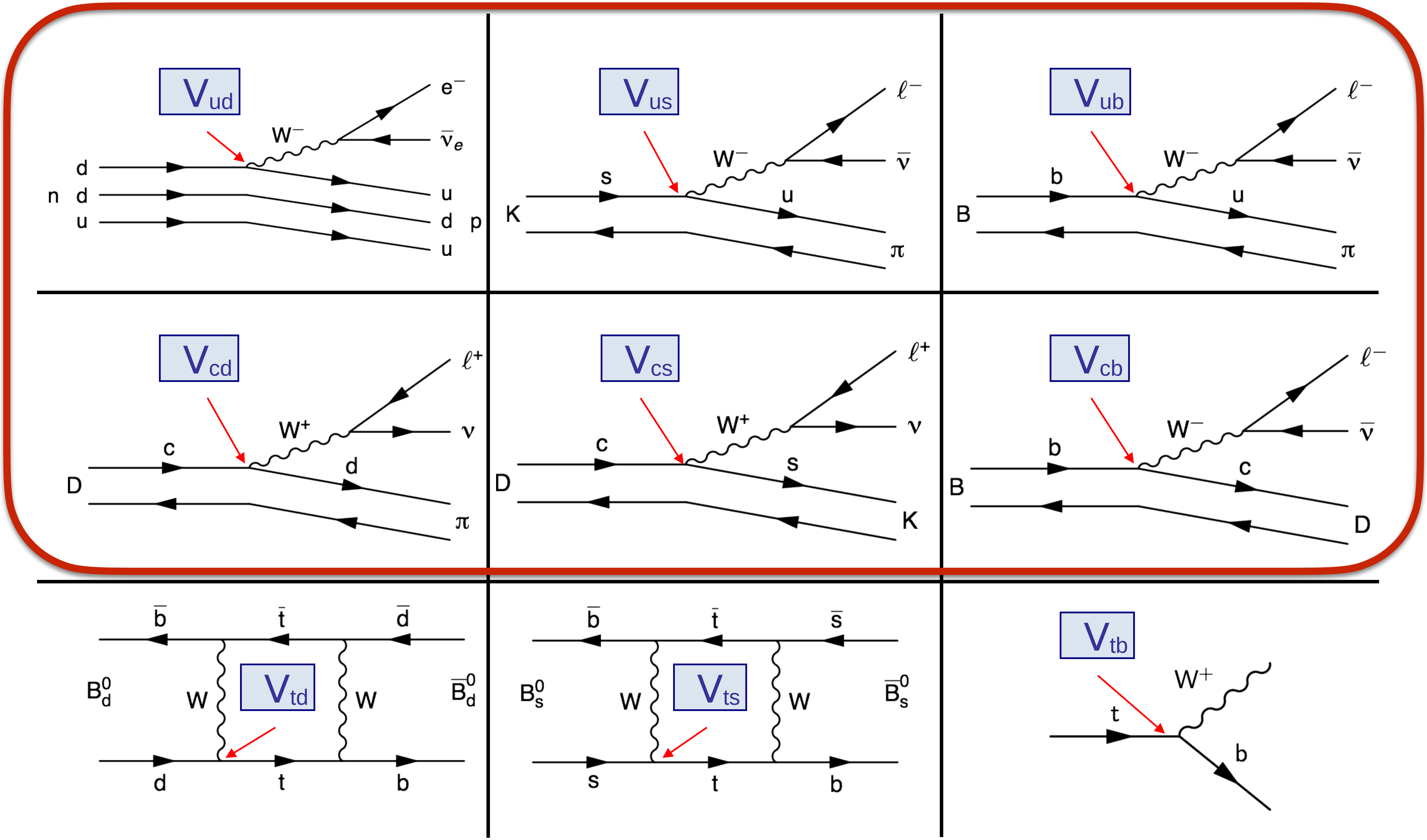
4. Introduction to Radiative Corrections: $B \to \mu\, \nu\, \gamma$

The (infrared) pole structure of gauge field theory amplitudes.
Collinear and Infrared Safety.

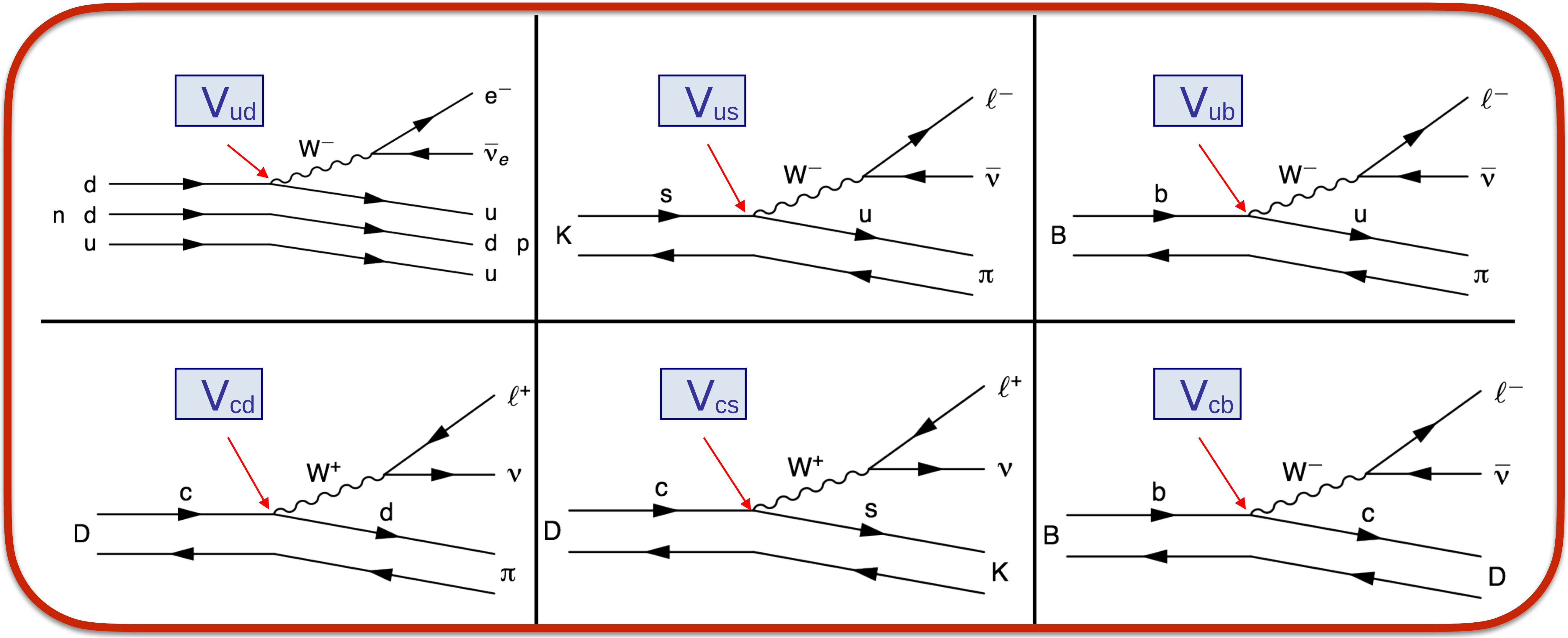
Peter Skands
Monash University — 2020

Now, we move on to:

Semi-Leptonic Decays of Hadrons



Semi-Leptonic Decays of Hadrons



Simplifying factors:

These are all tree-level diagrams, in which one of the quarks acts as a pure "spectator".

There is only one hadron in the the final state

Should be possible to write the amplitude as a lepton current interacting (via a virtual W) with the "active" quark, embedded in a "hadronic current"

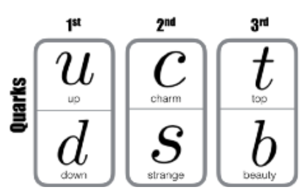
Cabibbo Favoured vs Cabibbo Suppressed

Which is Cabibbo Favoured vs Cabibbo Suppressed? And why.

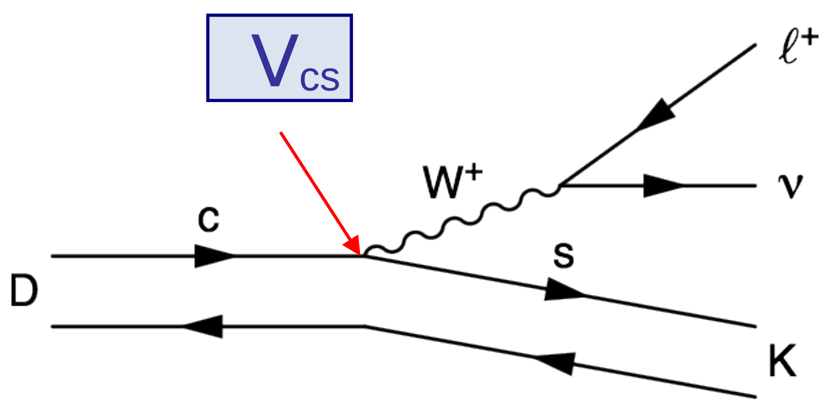
$D^0 = |c\bar{u}>$

$K^- = |s\bar{u}>$

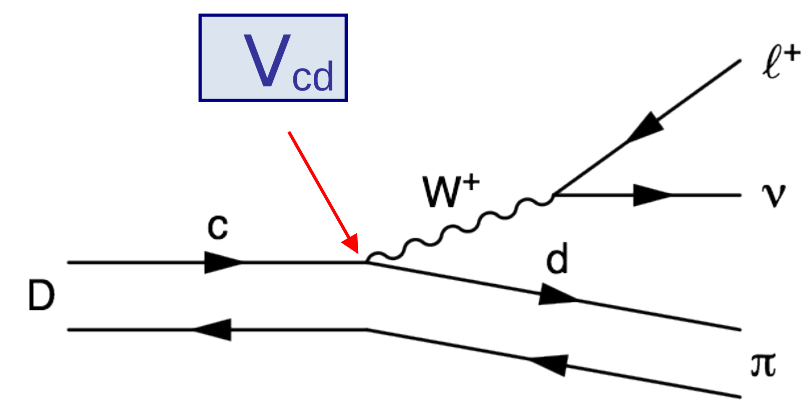
$\pi^- = |d\bar{u}>$



$D^0 \to K^- \ell^+ \nu$



$D^0 \to \pi^- \ell^+ \nu$

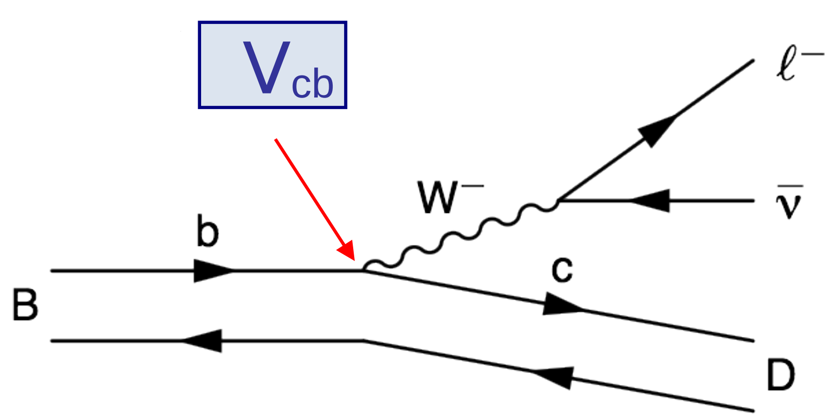


Which is CKM Favoured vs CKM Suppressed? And why.

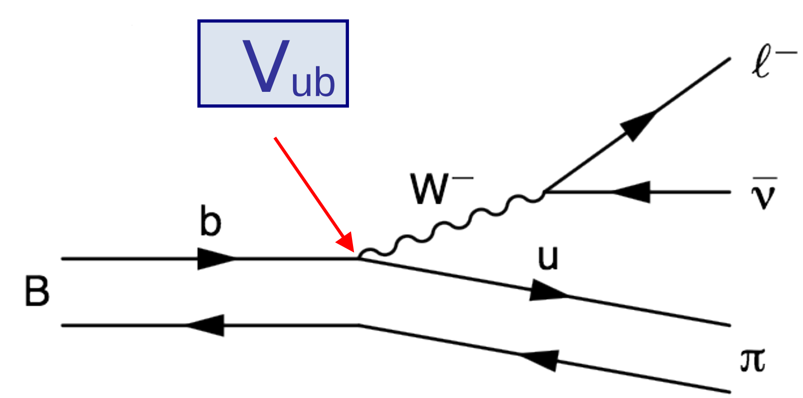
$B^- = |b\bar{u}>$

➠ Our case study.
Has gotten attention recently, as part of the "flavour anomalies".

$B \to D \ell \nu$



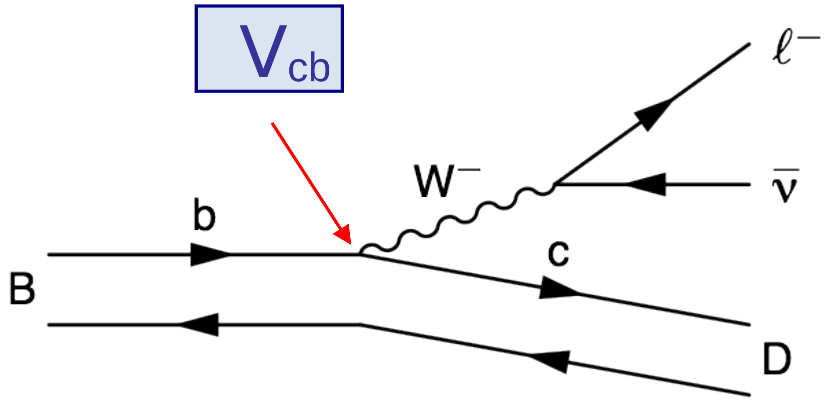
$B \to \pi \ell \nu$



Starting Point for B→Dℓν: The **Spectator** Model

Unlike B→ℓν, this is **not** an annihilation

Looks like a **weak decay of the heavy quark**, accompanied by a non-interacting **spectator**:



Suggests a simple starting point for semi-leptonic decays:

Assume the quark(s) which accompany the heavy quark play **no role**.

$$\mathscr{L} = -\frac{G_F}{\sqrt{2}} V_{cb} \, [\bar{c}\gamma^\rho(1-\gamma^5)b] \, [\bar{\ell}\gamma_\rho(1-\gamma_5)\nu_\ell]$$

Can give some insights (e.g., lepton spectrum) but is not a precision tool.

B→Dℓν with **Hadronic Effects**

Can promote the spectator model's quark-level matrix element to a hadronic one by sandwiching it between initial and final hadronic states:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cb} \left\langle D(p_D) \,|\, \bar{c}\gamma^\rho(1-\gamma^5)b \,|\, B(p_B) \right\rangle \; [\bar{\ell}\gamma_\rho(1-\gamma_5)\nu_\ell]$$

Both B and D are pseudoscalars. To construct a vector, must use L=1 ⇒ negative parity ⇒ Axial part does not contribute.

$$= \frac{G_F}{\sqrt{2}} V_{cb} \left\langle D(p_D) \,|\, \bar{c}\gamma^\rho b \,|\, B(p_B) \right\rangle \; [\bar{\ell}\gamma_\rho\nu_\ell]$$

Unlike for pion decay, we have two (independent) momenta here, p_B and p_D ⇒ a priori two Lorentz-covariant combinations

$$= \frac{G_F}{\sqrt{2}} V_{cb} \left[f_+(q^2)(p_B + p_D)^\rho \; + f_-(q^2)(p_B - p_D)^\rho \right] \; [\bar{\ell}\gamma_\rho\nu_\ell]$$

f_+ and f_- are called **Form Factors**

They depend on $q^2 = (p_B - p_D)^2 = p_W^2 = (p_\ell + p_{\bar{\nu}})^2$ = Momentum Transfer

(Alternative Parameterisation)

We wrote:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cb} \left[f_+(q^2)(p_B + p_D)^\rho + f_-(q^2)(p_B - p_D)^\rho\right] [\bar{\ell}\gamma_\rho \nu_\ell]$$

Another common parametrisation [Wirbel, Stech, Bauer, Z.Phys. C29 (1985) 637] is to write in terms of a "Transverse" F_0 and a "Longitudinal" F_1 form factor:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cb} \left[F_1(q^2)\left(p_B + p_D - \frac{m_B^2 - m_D^2}{q^2} q\right)^\rho + F_0(q^2)\frac{m_B^2 - m_D^2}{q^2} q^\rho\right] [\bar{\ell}\gamma_\rho \nu_\ell]$$

with $F_1(0) = F_0(0)$ and $q = p_B - p_D$

Thus: $f_+ = F_1$

$f_- = (F_0 - F_1)(m_B^2 - m_D^2)/q^2$

Exercise: prove this

Note: for decays involving **vector mesons**, polarisations ε^μ ⇒ more form factors.

Looks like we went from bad to worse?

Our ignorance about non-perturbative physics is now cast as two whole functions.

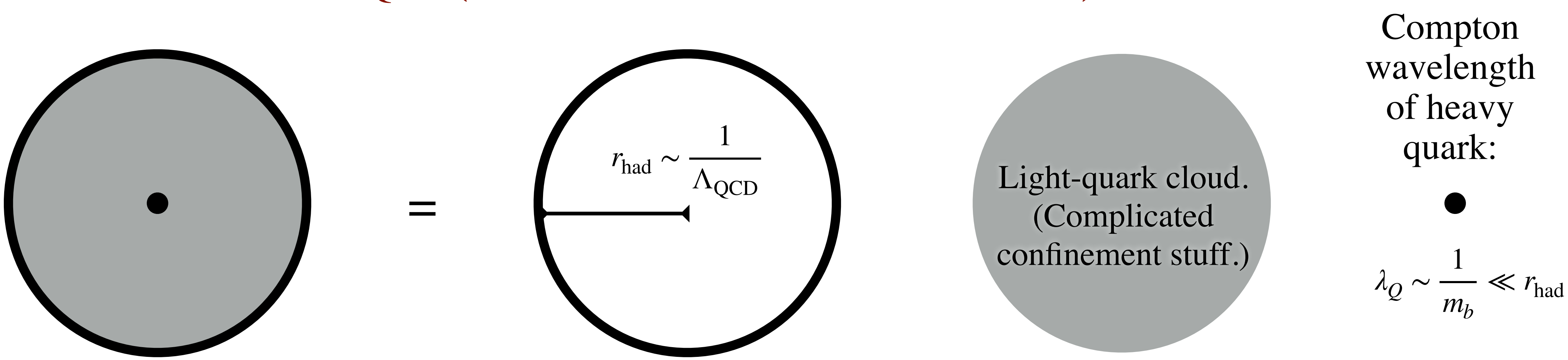
How can we learn anything (precise) from this?

Frustrating when the process *looks* so simple …

Let's take a second look at the problem, physicist style:

The B meson is a *heavy-light system;*

$m_b \sim 4$ GeV $\gg \Lambda_{QCD}$ (confinement scale ~ 200 MeV)



➤ Large **separation of scales**!

Heavy Quark Symmetry

Soft gluons exchanged between the heavy quark and the light constituent cloud can only resolve distances much larger than $\lambda_Q \sim 1/m_Q$

- **In limit $m_Q \to \infty$, the light degrees of freedom:**

  Are blind to the flavour (mass) and spin of the heavy quark.

  Experience only the colour field of the heavy quark (which extends over distances large compared with $1/m_Q$)

- **If we swap out the heavy quark Q by one with a different mass and/or spin, the light cloud would be the same.**

  $\Rightarrow$ Relations between B, D, B^*, and D^*, and between Λ_b and Λ_c.

  For finite m_Q, these relations are only **approximate.**

  Deviations from exact heavy-quark symmetry: "*symmetry breaking corrections*"

  Can be organised systematically in powers of $\alpha_s(m_Q)$ (perturbative) and $1/m_Q$ (non-perturbative) in a formalism called **HQET** (heavy-quark effective theory).

Physics of heavy-quark symmetry

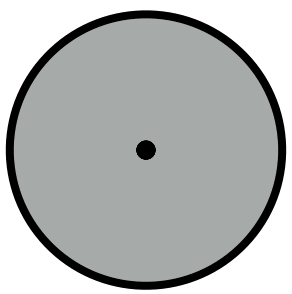
Isgur & Wise , Phys. Lett. B 232 (1989) 113; Phys. Lett. B 237 (1990) 527

Before we consider decays, consider just elastic scattering* of a B meson

Induced by giving a **kick** to the *b* quark at time t_0:

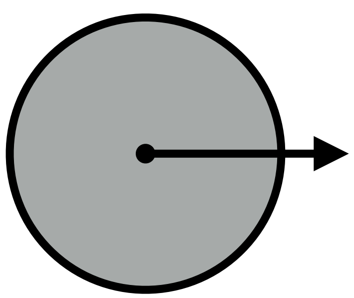
***Elastic Scattering:** means B meson does not break up.

Illustrations and physics arguments inspired by the BaBar Physics Book.

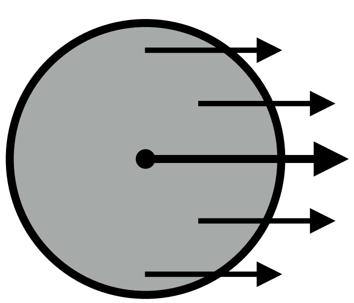


Before t_0: light degrees of freedom orbit around the heavy quark, which acts as a static source of colour.

On average, *b* quark and *B* meson have same velocity, *v*.



At t_0: instantaneously replace colour source by one moving at velocity *v*' (possibly with a different spin).



After t_0: If *v*=*v*' (spectator limit), nothing happens; light degrees of freedom have no way of knowing anything changed.

But if *v*≠*v*', *the light cloud will need to be rearranged (sped up), to form a new B meson moving at velocity v'*.

➤ Form-factor suppression. (Large Δv ⇒ **elastic** transition less likely.)

Elastic Form Factor of a Heavy Meson (Isgur-Wise Function)

Isgur & Wise , Phys. Lett. B 232 (1989) 113; Phys. Lett. B 237 (1990) 527

In limit $m_b \to \infty$, form factor can only depend on the difference between v and v':

Lorentz invariance ☛ use the **relative boost** between the rest frames of the initial- and final-state mesons.

Using $v^\mu = \dfrac{p^\mu}{m_b}$ and $v'^\mu = \dfrac{p'^\mu}{m_b}$ the relative boost is $\gamma = \boxed{v \cdot v'} \geq 1$

Exercise: prove this

➤ In this limit, a dimensionless probability amplitude $\xi(\gamma)$ describes the transition amplitude. *(ξ is called the Isgur-Wise function.)*

➤ The hadronic matrix element can be written as:

$$\left\langle \bar{B}(p') \,|\, \bar{b}_{p'} \gamma^\mu b_p \,|\, \bar{B}(p) \right\rangle = \xi(\gamma)(p + p')^\mu$$

ξ is the elastic form factor of a heavy meson. **Only depends on** $\gamma = v.v'$, not m_B.
Constraint: at γ=1 (zero momentum transfer), current conservation ⇒ $\xi(1)=1$

Question: why is $\xi(1)=1$ intuitive?

Implications

Using heavy-quark symmetry, we can replace the b quark in the final-state meson by a c quark:

$$\left\langle \bar{D}(v') \,|\, \bar{c}_{v'}\gamma^\mu b_v \,|\, \bar{B}(v) \right\rangle = \sqrt{m_B m_C}\, \xi(v \cdot v')\, (v + v')^\mu$$

Writing it terms of velocities, v and v', instead of momenta

(This corresponds to the field definitions in HQET)

Same Isgur-Wise functions!

Compare with the general expression from before:

$$\mathscr{M} = \frac{G_F}{\sqrt{2}} V_{cb} \left[f_+(q^2)(p_B + p_D)^\rho \; + f_-(q^2)(p_B - p_D)^\rho \right] \; [\bar{\ell}\gamma_\rho \nu_\ell]$$

$\Rightarrow$ the functions f_+ and f_- are not independent. Both are related to ξ.

Assignment Problem 3: derive expressions for $f_+(\xi)$ and $f_-(\xi)$

The Partial Widths

In the limit that $m_b, m_c \gg \Lambda_{QCD}$, the differential semileptonic decay rates become:

$$\frac{d\Gamma(\overline{B} \to D\,\ell\,\overline{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (m_B + m_D)^2\, m_D^3\, (w^2-1)^{3/2}\, \xi^2(w)\,,$$

$$\frac{d\Gamma(\overline{B} \to D^*\ell\,\overline{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (m_B - m_{D^*})^2\, m_{D^*}^3\, \sqrt{w^2-1}\, (w+1)^2 \times \left(1 + \frac{4w}{w+1}\, \frac{m_B^2 - 2w\, m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}\right) \xi^2(w)$$

… in terms of the "recoil variable" $w = v \cdot v'$

(Similar expressions can be derived for semi-leptonic $\Lambda_b \to \Lambda_c \ell \bar{\nu}$ or $\bar{B} \to D^{**} \ell \bar{\nu}$
Different clouds so different Isgur-Wise functions ξ.)

Reminder: **corrections** from **finite m_Q** (breaking of heavy quark symmetry).

Perturbative: order $\alpha_s^n(m_Q)$

Non-perturbative: order $(\Lambda_{QCD}/m_Q)^n$ analysed in **HQET** (effective QFT with velocity-dependent Q fields, expansion in powers of $1/m_Q$ starting from $m_Q \to \infty$)

Determination of $|V_{cb}|$ from $\bar{B} \to D^* \ell \bar{\nu}$

An important result in HQET is "Luke's Theorem"

The leading **$1/m_Q$ correction** to $\bar{B} \to D^* \ell \bar{\nu}$ **vanishes at zero recoil** (not true for $\bar{B} \to D \ell \bar{\nu}$).

We write:

$$\frac{d\Gamma(\overline{B} \to D^* \ell \,\overline{\nu})}{dw} = \frac{G_F^2}{48\pi^3} (m_B - m_{D^*})^2 \, m_{D^*}^3 \, \sqrt{w^2 - 1} \, (w+1)^2 \times \left(1 + \frac{4w}{w+1} \frac{m_B^2 - 2w \, m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}\right) |V_{cb}|^2 \, \mathcal{F}^2(w)$$

Coincides with the Isgur-Wise function up to small symmetry-breaking corrections

Idea is to measure the product $|V_{cb}|\mathcal{F}(w)$ as a function of w and then extrapolate to zero recoil, $w=1$ where the B and D* mesons have a common rest frame, and

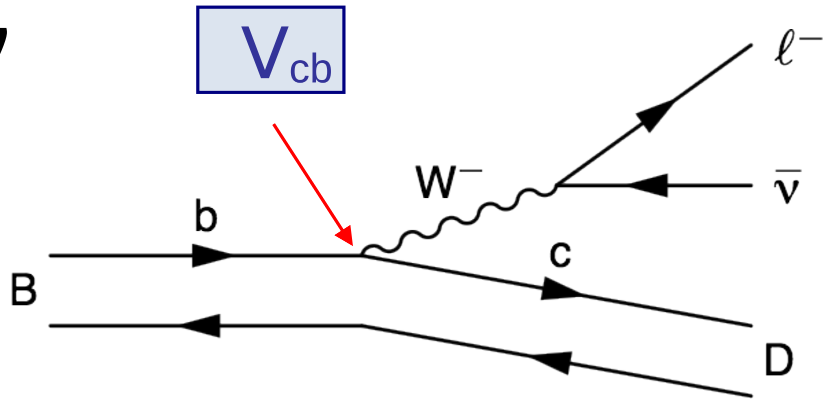
$$\mathcal{F}(1) = \eta_A \, \eta_{\rm QED} \left(1 + 0 \times \frac{\Lambda_{\rm QCD}}{m_Q} + \text{const} \times \frac{\Lambda_{\rm QCD}^2}{m_Q^2} + \ldots \right) \equiv \eta_A \, \eta_{\rm QED} \, (1 + \delta_{1/m^2})$$

QED
η ~ 1.007

Luke's Theorem

Perturbative QCD: renomalization of flavour-changing axial current at zero recoil η ~ 0.96

$B \to D^{(*)} \ell \nu$



First approximation: "spectator model"

The other quark is a pure "spectator"; plays no role; ignore it.

More realistic: embed quark-level amplitude inside hadronic one → **Form factors**

One form factor for each L.I. combination of relevant 4-vectors.

They parametrise the difference between spectator model (form factors =1) and real world.

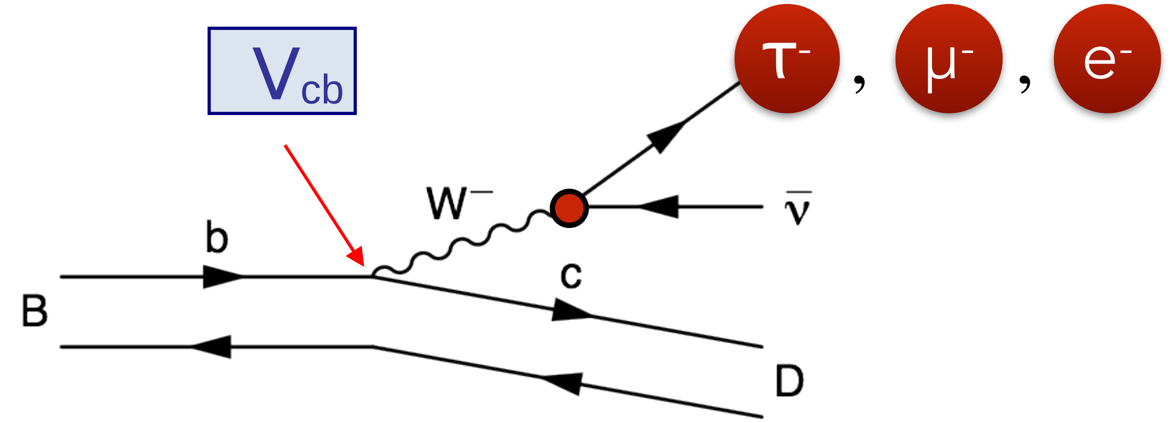
Use **Heavy Quark Symmetry**: exploit $m_Q \gg \Lambda_{QCD}$

Light-quark cloud insensitive to mass (and spin) of heavy quark: $B^{(*)}$ cloud ~ $D^{(*)}$ cloud.

Physics depends only on velocity change, L.I.: $w = v \cdot v'$, reflected by **Isgur-Wise function**
+ heavy-quark-symmetry-breaking corrections of order $(\alpha_s)^n$ and $(\Lambda/m_Q)^n$ ➠ **HQET**.

"Luke's Theorem": the leading $1/m_Q$ corrections are zero in $B \to D^* \ell \nu$ (but not in $B \to D \ell \nu$).

Apart from measuring V_{cb}, we can also use these decays to test “Lepton Universality”; compare different leptons:



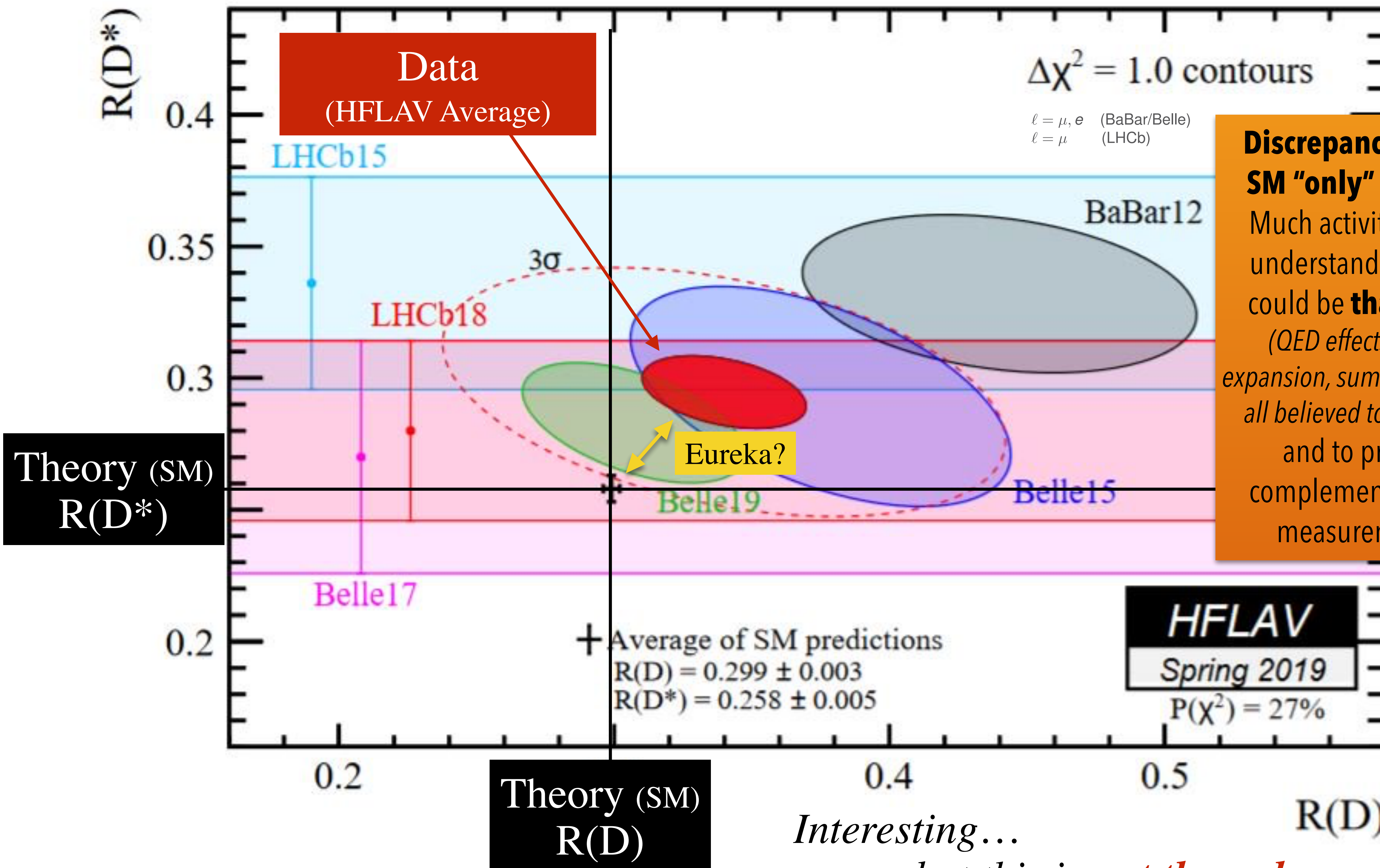
$m_B \sim 5.3\,\text{GeV}$ $\qquad$ $m_D \sim 1.9\,\text{GeV}$

The only difference are the lepton masses: $(m_\tau, m_\mu, m_e) \sim (1.8\,,\ 0.1\,, 0.0\,)\,\text{GeV}$

Form two ratios: $\quad R(D) = \dfrac{\text{BR}(B \to D\tau\nu)}{\text{BR}(B \to D\ell\nu)} \qquad R(D^*) = \dfrac{\text{BR}(B \to D^*\tau\nu)}{\text{BR}(B \to D^*\ell\nu)}$

Different masses ⇒ **Expect R≠1** but should be **well approximated** by **calculable functions** of the lepton masses; see eg the dΓ expressions we wrote down previously

What does the data say?



Discrepancies with SM "only" ~ 2-3σ.
Much activity now to understand if theory could be **that** wrong *(QED effects? HQET expansion, sum rules, lattice all believed to be small)* and to provide complementary exp measurements.

Interesting…
*…but this is **not the only anomaly**!*

Summary of Problems and Exercises for Self Study

Prove that $\gamma = v \cdot v'$

Prove the relation between (f_+,f_-) and (F_0,F_1)

You will present your progress on these in the next lesson and we will discuss any questions / issues you encounter.

Assignment Problem 1: B→τν

Assignment Problem 2: B→μν

Assignment Problem 3 : B→Dℓν