Recent Developments in Vincia & Pythia

Peter Skands — U of Oxford & Monash U.

1. Perturbative Uncertainties (in Showers)
2. Sector Showers & NNLO Matching
3. EW Showers and Resonance Decays
4. From Showers to Jets: Colour Confusion

… including some questions for discussion …

Note: see talk by Silvia (Monday) for $N^{(n)}LL$ showers (PanScales, Alaric, etc)

Standard for Shower Uncertainties: Renormalization-scale variations

Example: PYTHIA's DGLAP-based shower

$$|M_{n+1}|^2 \sim \sum_{i \in \text{partons}} \underbrace{\frac{\alpha_s^{\text{MC}}(\mu_i^2)}{4\pi}}_{\mu_i^2 \propto p_{\perp i}^2} \underbrace{\mathscr{C}_i}_{\substack{2C_F \text{ for quark,} \\ C_A \text{ for gluon}}} \underbrace{\left(\frac{P_i(z)}{Q_i^2} \right)}_{\substack{\text{DGLAP Splitting Kernel} \\ \text{(Or dipole/antenna/...)}}} |M_n|^2 \underbrace{\Delta_n(t_n, t_{n+1})}_{\text{Sudakov factor}}$$

t is the shower evolution/ ordering variable

Varying μ_i only induces terms proportional to the shower splitting kernels

Actual higher-order MEs **also have:**

- **Non-singular terms** (dominate far from singular limits),
- **Non-trivial colour factors** outside collinear limits,
- **Higher-order log terms** not captured exactly by $\Delta_n(t_n, t_{n+1})$

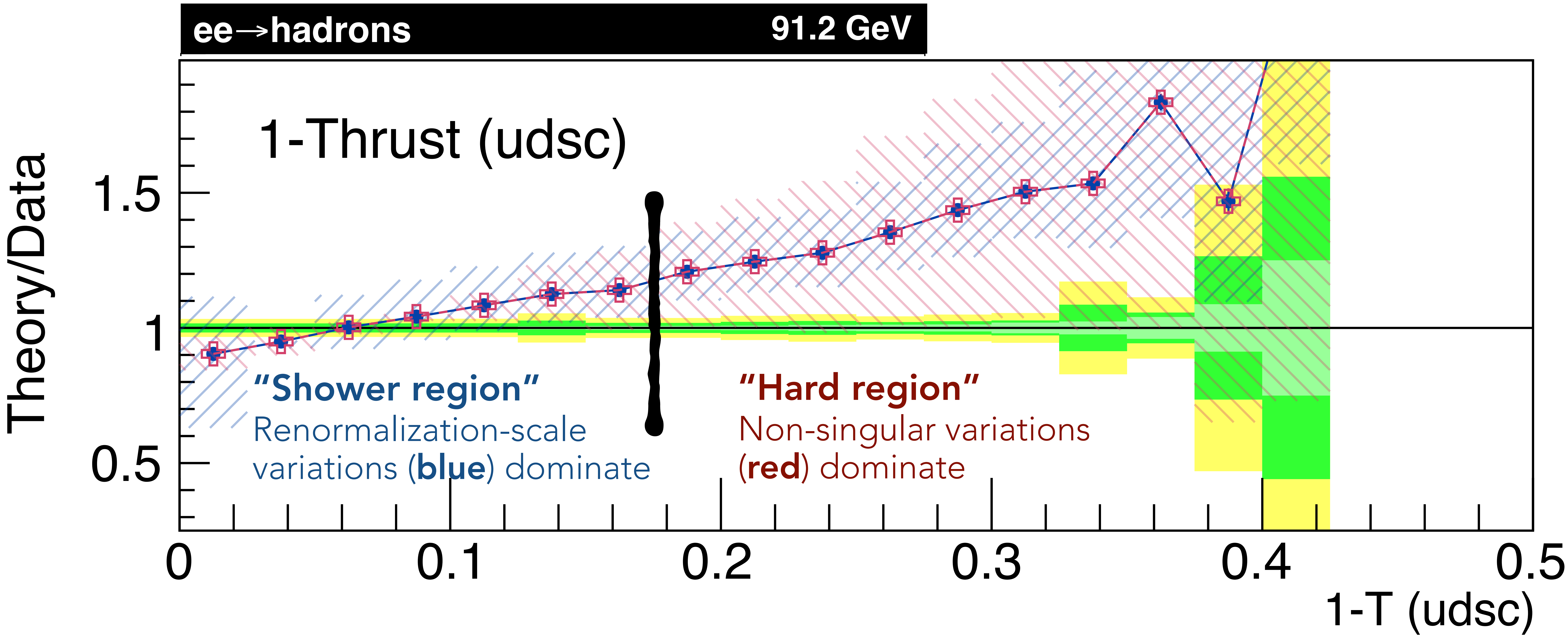
} *Vary μ_R **and** these*

[Hartgring, Laenen, PS JHEP 10 (2013) 127]

Non-Singular Variations: Example

Example from Mrenna & PS, "Automated Parton-Shower Variations in Pythia 8", PRD 94 (2016) 7

Can vary **renormalisation-scale** and **non-singular terms** independently

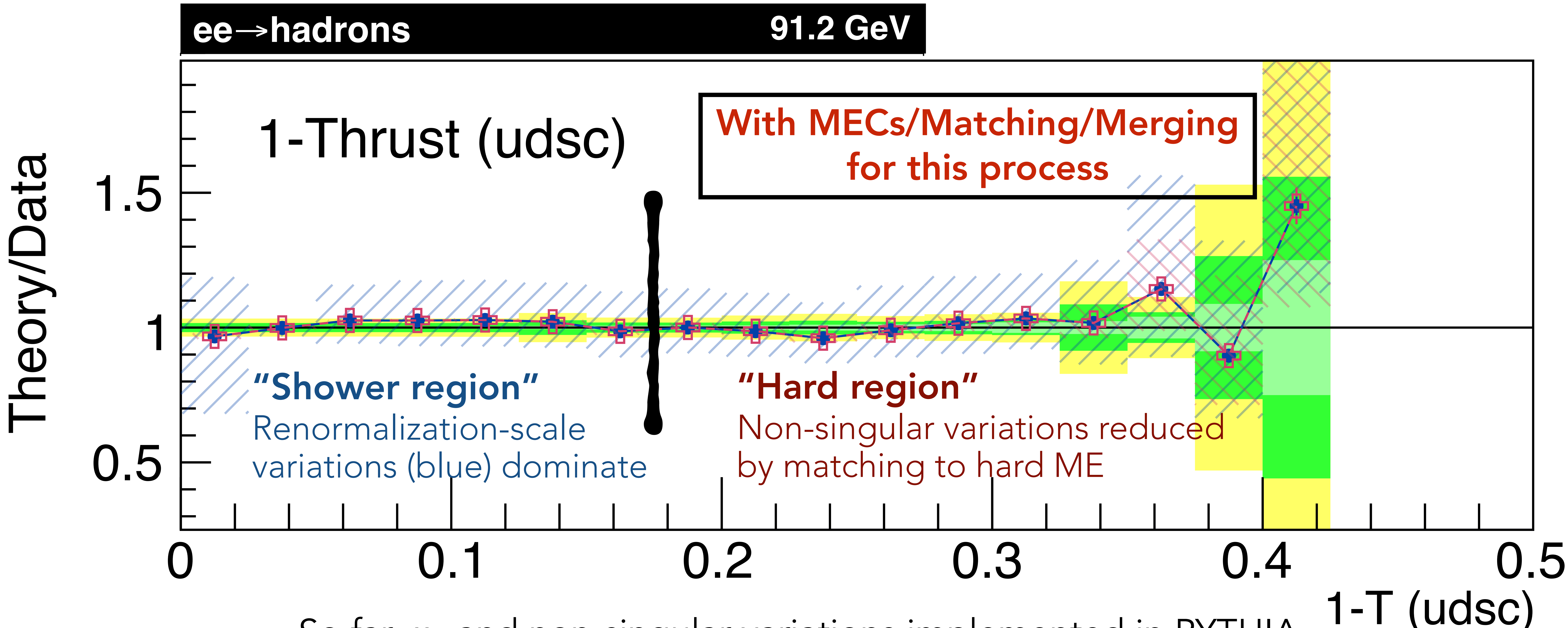


Note: ME corrections were switched off for illustration here. Would reduce **red** band, but not **blue**.

(Non-Singular Variations: Effect of Matching to Matrix Elements)

Example from Mrenna & PS, "Automated Parton-Shower Variations in Pythia 8", [PRD 94 (2016) 7]

Can vary **renormalisation-scale** and **non-singular terms** independently

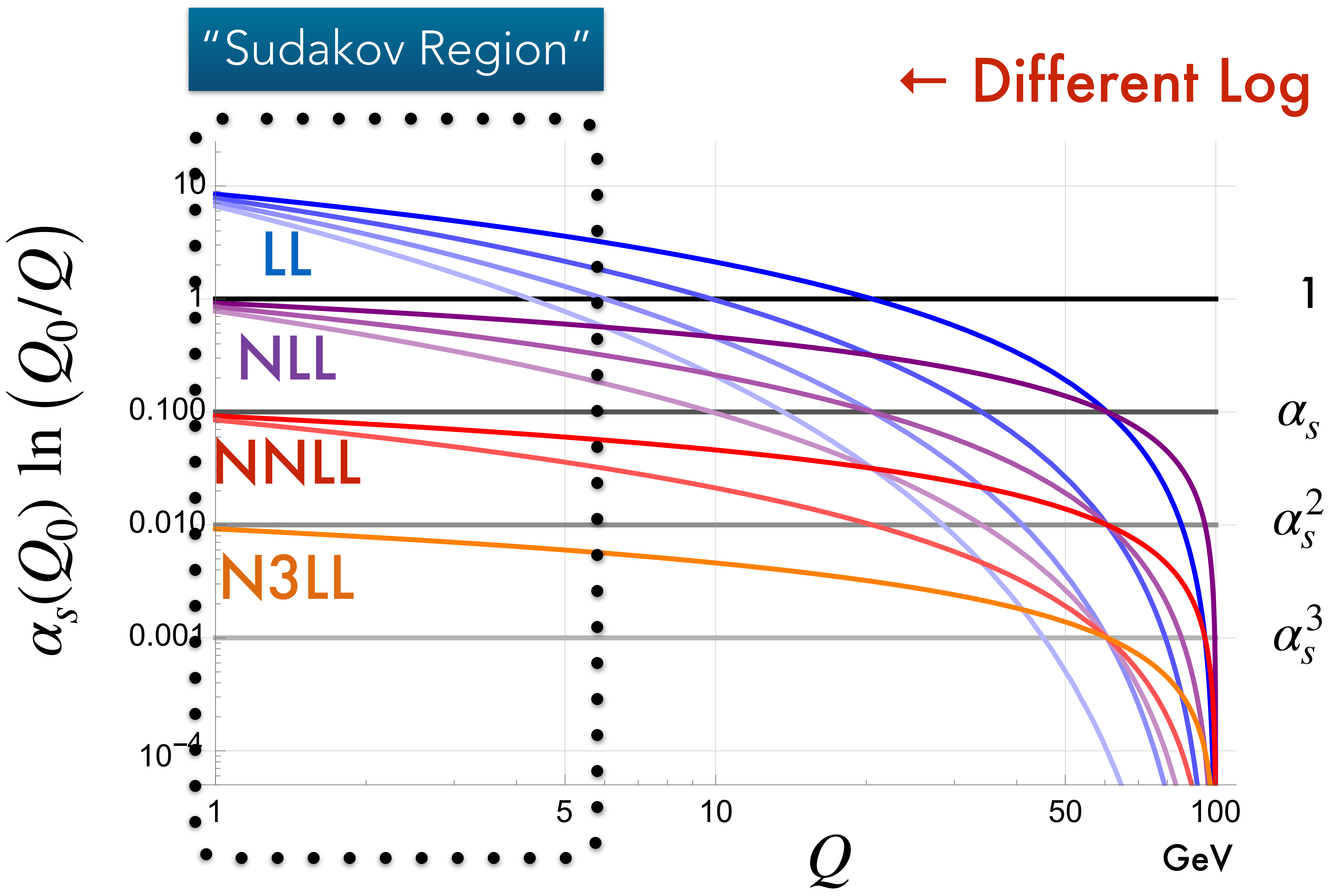


So far, μ_R and non-singular variations implemented in PYTHIA

Being re-implemented in VINCIA. Plan to add colour and Sudakov variations as well.

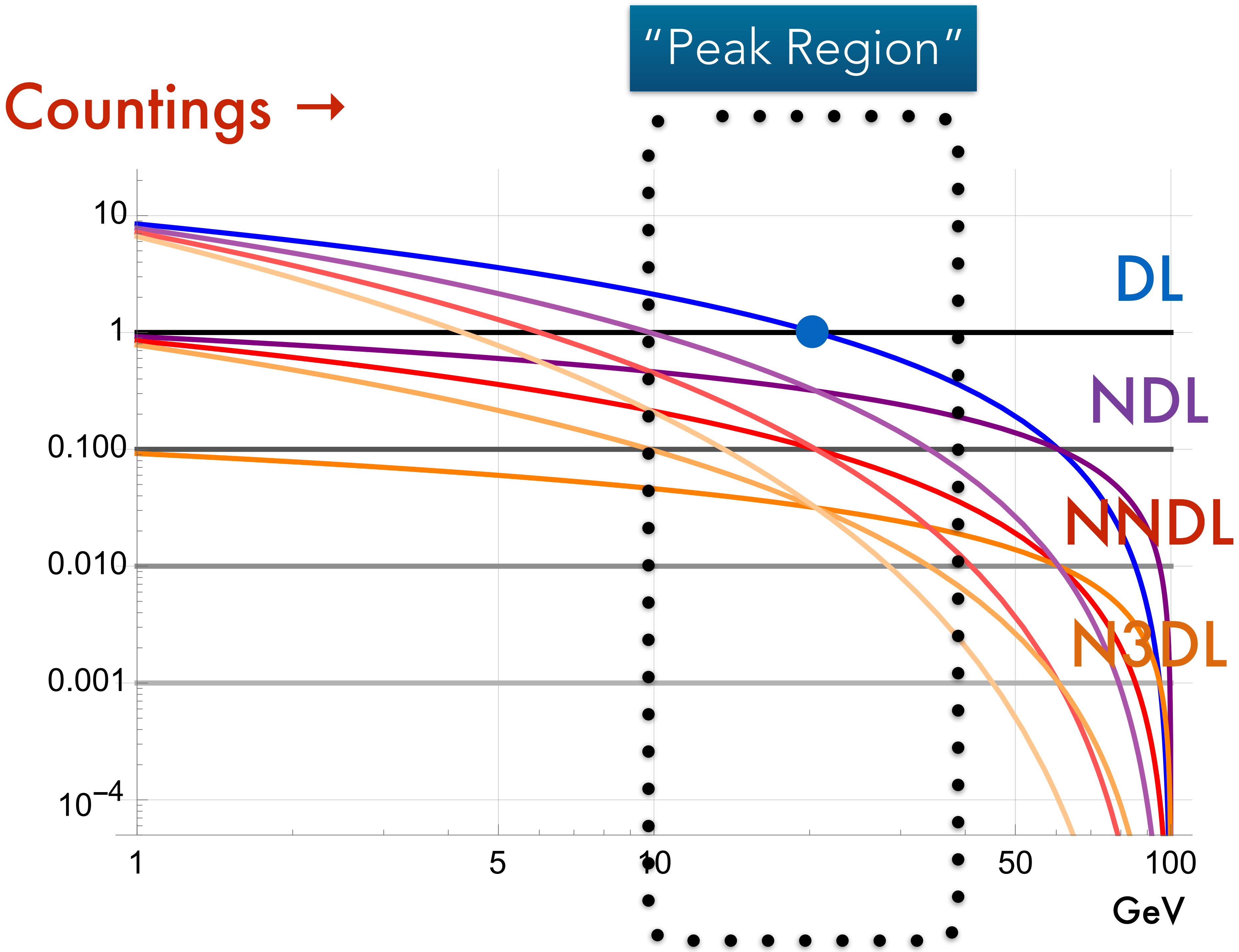
Schematic Example: starting scale Q_0 = 100 GeV

← Different Log Countings →



Conventional ("Caesar-style") log counting

Based on $\alpha_s L \sim 1$



Exponentiated "double-log" counting

Based on $\alpha_s L^2 \sim 1$

❷ Sector Showers in VINCIA

PS & Villarejo JHEP 11 (2011) 150
Brooks, Preuss, PS JHEP 07 (2020) 032

VINCIA's shower is unique in being a "Sector Shower"

Partition N-gluon Phase Space into N "sectors" (using step functions).

Each sector corresponds to one specific gluon being the "softest" in the event — the one you would cluster if you were running a jet algorithm (ARCLUS)

Inside each sector, **only a single kernel is allowed to contribute** (the most singular one)!

Sector Kernel = the eikonal for the soft gluon and its collinear DGLAP limits for $z > 0.5$.

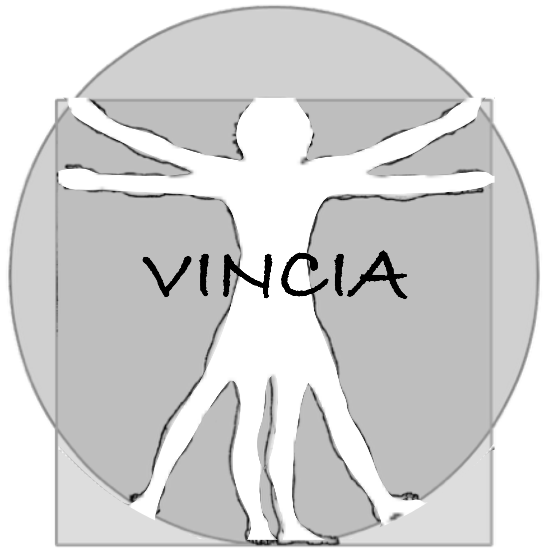
➜ Unique properties: shower operator becomes **bijective** and is a true **Markov chain**

The crucial aspect:

Only a single history contributes to each phase-space point !

$\Longrightarrow$ **Factorial growth of number of histories reduced to constant!**

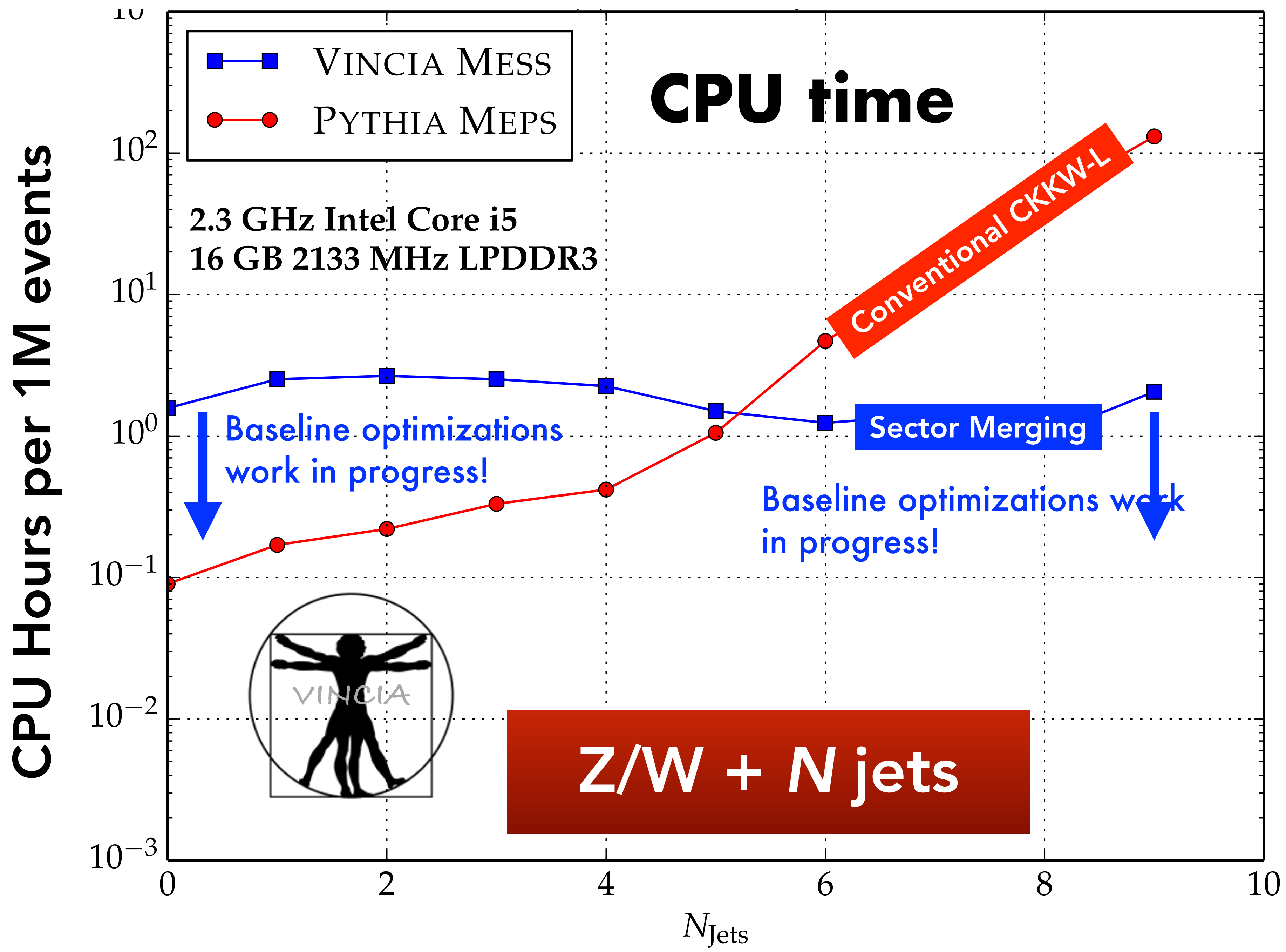
(And the number of sectors only grows linearly with the number of gluons)

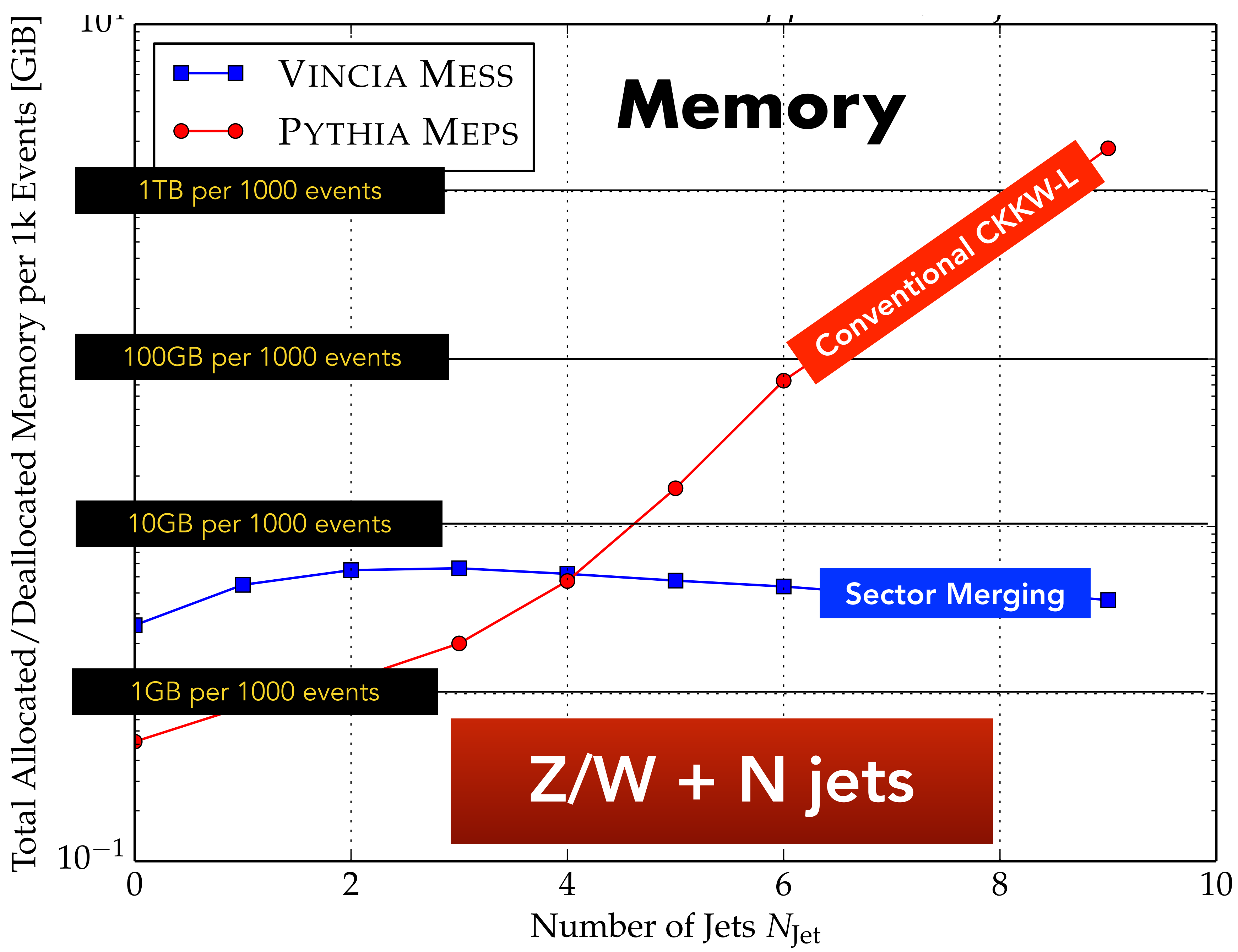


($g \to q\bar{q}$ ➜ leftover factorial in number of *same-flavour* quarks; not a big problem)

Sectorized CKKW-L Merging publicly available from Pythia 8.306

Brooks & Preuss (2021) "Efficient multi-jet merging with the VINCIA sector shower"





Extensions now pursued:

Sectorized **matching at NNLO** (proof of concepts in arXiv:2108.07133 & arXiv:2310.18671)

Sectorized **iterated tree-level ME corrections** (demonstrated in PS & Villarejo arXiv:1109.3608)

Sectorized **multi-leg merging at NLO** (active research grants, with **C. Preuss, Wuppertal**)

Sectorized Matching at NNLO (in VINCIA)

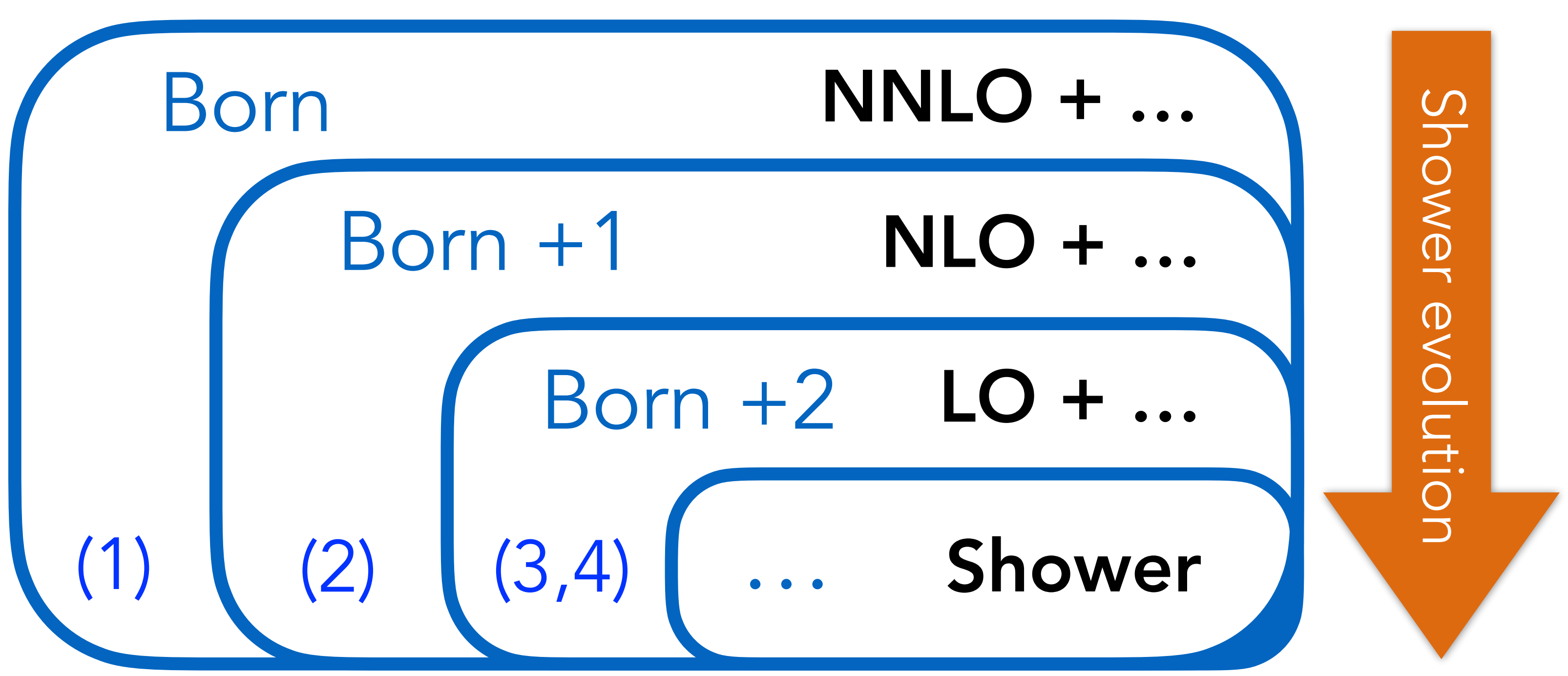
Idea: harness the power of showers as efficient phase-space generators

a.k.a. **"ME Corrections"** Sjöstrand et al. (1986, 2001); Giele, Kosower, PS (2011); Lopez-Villarejo, PS (2011)

a.k.a. **"Forward-Branching"** PS generation Weinzierl, Kosower (1999); Draggiotis, v. Hameren, Kleiss (2000); Figy, Giele (2018)

Conventional Fixed-Order phase-space generation (eg VEGAS)

Nested phase-space generation in a Shower Markov Chain



Need:

(1) Born-local NNLO K-factors: $k_{\mathrm{NNLO}}(\Phi_2)$

(2) NLO MECs in the first $2 \mapsto 3$ shower branching: $w^{\mathrm{NLO}}_{2\mapsto 3}(\Phi_3)$

(3) LO MECs for second (iterated) $2 \mapsto 3$ shower branching: $w^{\mathrm{LO}}_{3\mapsto 4}(\Phi_4)$

(4) Direct $2 \mapsto 4$ branchings for unordered sector with LO MECs: $w^{\mathrm{LO}}_{2\mapsto 4}(\Phi_4)$

① Weight each Born-level event by local *K*-factor

Campbell, Hoeche, Li, Preuss, PS (2023)

$$k_{\mathrm{NNLO}}(\Phi_2) = 1 + \frac{\mathrm{V}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}_\mathrm{S}^{\mathrm{NLO}}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{VV}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}_\mathrm{T}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}_\mathrm{S}(\Phi_2)}{\mathrm{B}(\Phi_2)}$$
$$+ \int \mathrm{d}\Phi_{+1} \left[\frac{\mathrm{R}(\Phi_2, \Phi_{+1})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{S}^{\mathrm{NLO}}(\Phi_2, \Phi_{+1})}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{RV}(\Phi_2, \Phi_{+1})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{T}(\Phi_2, \Phi_{+1})}{\mathrm{B}(\Phi_2)} \right]$$
$$+ \int \mathrm{d}\Phi_{+2} \left[\frac{\mathrm{RR}(\Phi_2, \Phi_{+2})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{S}(\Phi_2, \Phi_{+2})}{\mathrm{B}(\Phi_2)} \right]$$

Fixed-Order Coefficients:

Loops \ Legs	0	1	2
0	B	R	RR
1	V	RV	
2	VV		

Subtraction Terms (not tied to shower formalism):

Loops \ Legs	0	1	2
0	0	S^{NLO}	S
1	I_S^{NLO}	T	
2	I_S, I_T		

Note: requires "Born-local" NNLO subtraction terms. Currently only for simplest cases.

Interested in discussing & exploring connections with local subtraction schemes

②, ③, ④ Shower Markov chain with Second-Order Corrections

Key aspect
up to matched order, include **process-specific** **NLO corrections** into shower evolution:

② correct first branching to exclusive ($< t'$) NLO rate: [Hartgring, Laenen, PS (2013)]

Born → Born + 1
Sudakov Factor

$$\Delta^{\mathrm{NLO}}_{2\mapsto3}(t_0, t') = \exp\left\{ -\int_{t'}^{t_0} \mathrm{d}\Phi_{+1}\, \mathrm{A}_{2\mapsto3}(\Phi_{+1})\, w^{\mathrm{NLO}}_{2\mapsto3}(\Phi_2, \Phi_{+1}) \right\}$$

③ correct second branching to LO ME: [Giele, Kosower, PS (2011); Lopez-Villarejo, PS (2011)]

Born + 1 → Born + 2
Sudakov Factor

$$\Delta^{\mathrm{LO}}_{3\mapsto4}(t', t) = \exp\left\{ -\int_{t}^{t'} \mathrm{d}\Phi'_{+1}\, \mathrm{A}_{3\mapsto4}(\Phi'_{+1})\, w^{\mathrm{LO}}_{3\mapsto4}(\Phi_3, \Phi'_{+1}) \right\}$$

④ add direct $2 \mapsto 4$ branching and correct it to LO ME: [Li, PS (2017)]

Born → Born + 2
Sudakov Factor

$$\Delta^{\mathrm{LO}}_{2\mapsto4}(t_0, t) = \exp\left\{ -\int_{t}^{t_0} \mathrm{d}\Phi^{>}_{+2}\, \mathrm{A}_{2\mapsto4}(\Phi_{+2})\, w^{\mathrm{LO}}_{2\mapsto4}(\Phi_2, \Phi_{+2}) \right\}$$

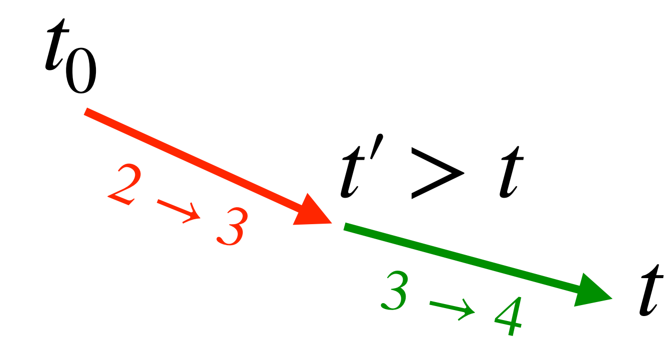
⇒ entirely based on **MECs** and **sectorisation**

⇒ **by construction**, expansion of extended shower **matches** NNLO singularity structure

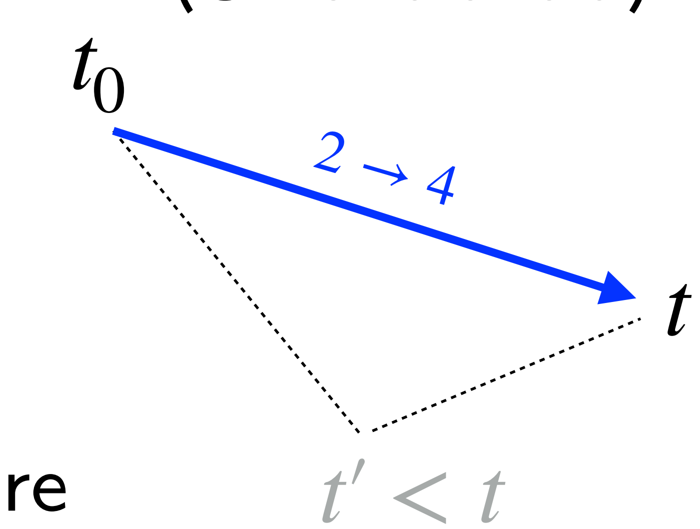
But shower kernels **do not** define **NNLO subtraction terms**[1] (!)

VINCIA NNLO

Iterated:
(Ordered)



Direct:
(Unordered)

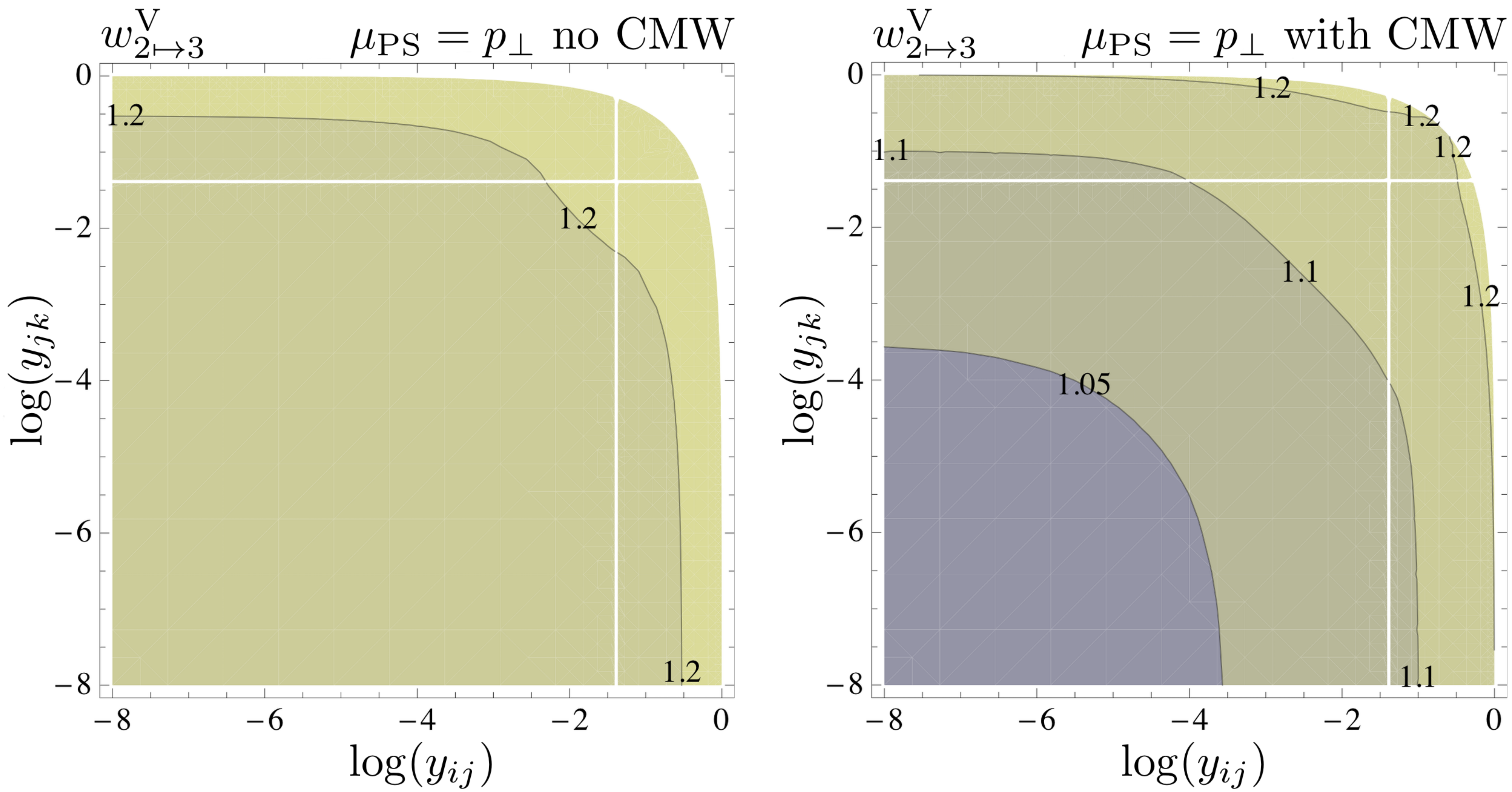


[1] This would be required in an "MC@NNLO" scheme, but difficult to realise in antenna showers.

Size of the Real-Virtual Correction Factor (②)

$$w^{\mathrm{NLO}}_{2\mapsto 3} = w^{\mathrm{LO}}_{2\mapsto 3}\left(1 + w^{\mathrm{V}}_{2\mapsto 3}\right)$$

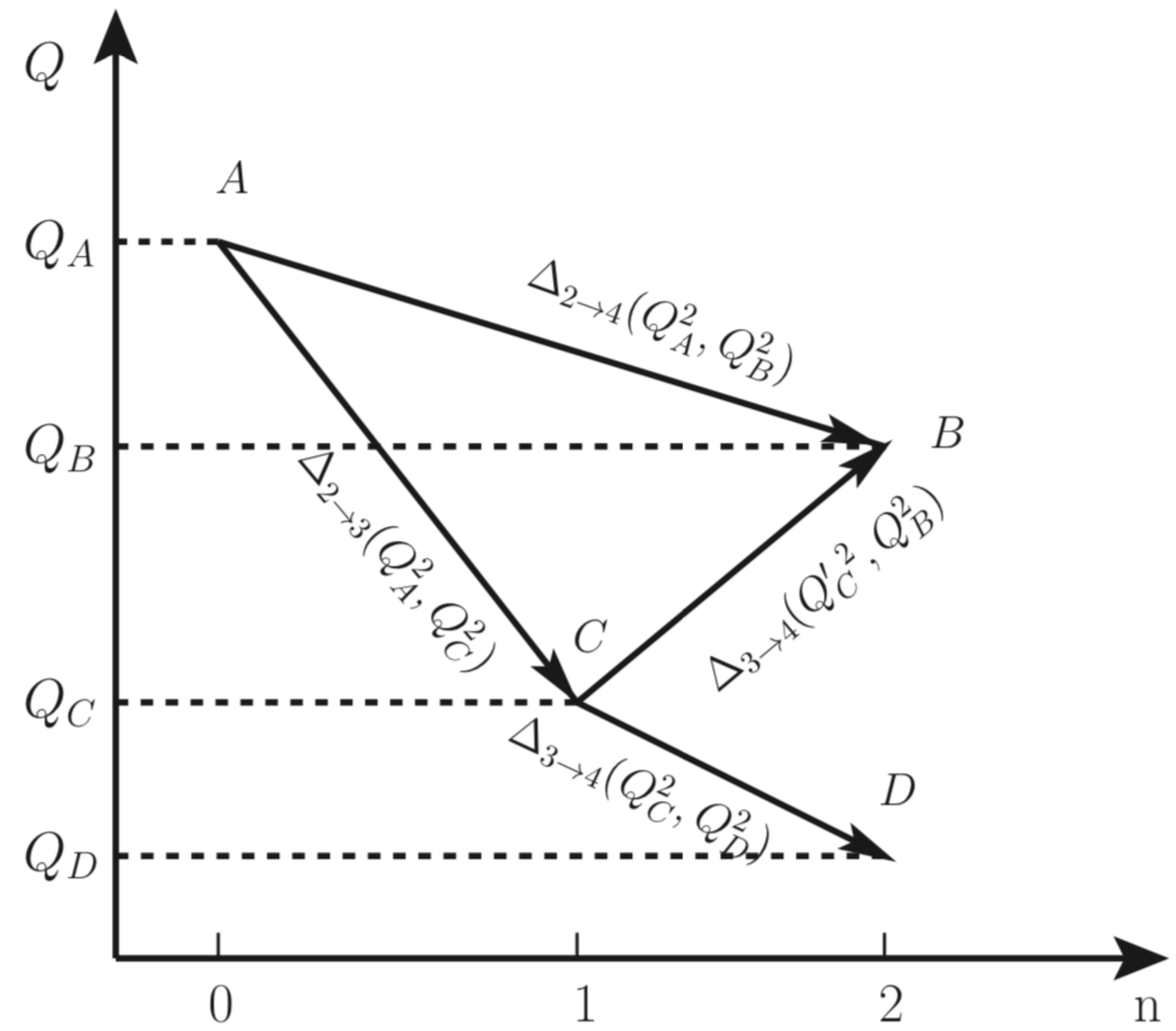
studied **analytically** in detail for $Z \to q\bar{q}$ in Hartgring, Laenen, PS JHEP 10 (2013) 127



⇒ now: **generalisation** & **(semi-)automation** in VINCIA in form of NLO MECs

A priori, direct $2 \mapsto 4$ and iterated $2 \mapsto 3$ branchings **overlap in ordered** region.

In **sector showers**, iterated $2 \mapsto 3$ branchings are **always strictly ordered**.



Divide double-emission phase space into **strongly-ordered** and **unordered** region:
[Li, Skands 1611.00013]

$$\mathrm{d}\Phi_{+2} = \underbrace{\mathrm{d}\Phi_{+2}^{>}}_{\text{u.o.}} + \underbrace{\mathrm{d}\Phi_{+2}^{<}}_{\text{s.o.}}$$

$\mathrm{d}\Phi_{+2}^{<}$: **single-unresolved** limits $\Rightarrow$ iterated $2 \mapsto 3$
$\mathrm{d}\Phi_{+2}^{>}$: **double-unresolved** limits $\Rightarrow$ direct $2 \mapsto 4$

Restriction on double-branching phase space enforced by additional veto:

$$\mathrm{d}\Phi_{+2}^{>} = \sum_j \theta\left(p_{\perp,+2}^2 - \hat{p}_{\perp,+1}^2\right) \Theta_{ijk}^{\mathrm{sct}} \, \mathrm{d}\Phi_{+2}$$

Preview: VINCIA NNLO+PS for $H \to b\bar{b}$

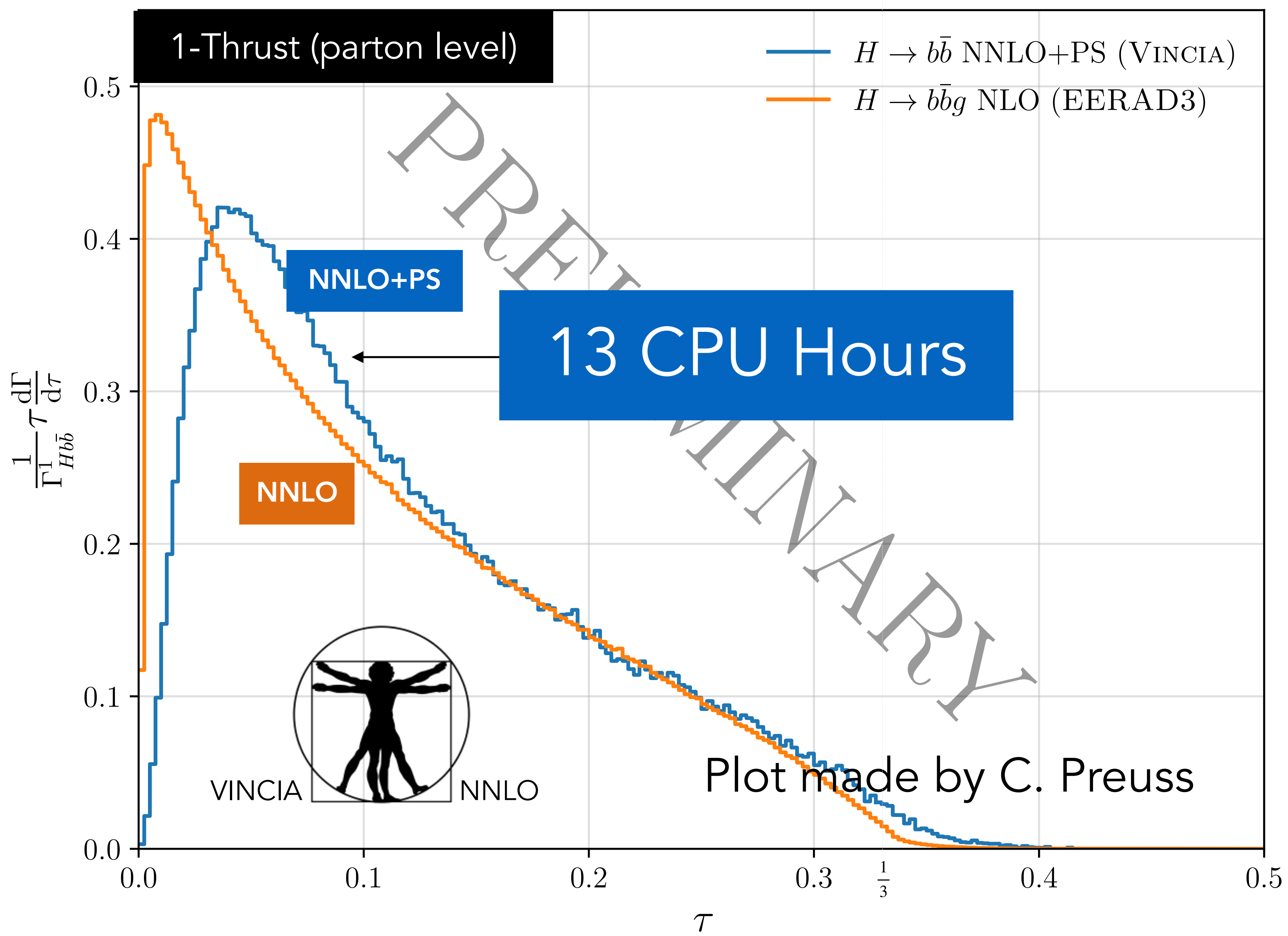
Coloretti, Gehrmann-de Ridder, Preuss, JHEP 06 (2022) 009

Fixed-Order Reference = EERAD3 NLO $H \to b\bar{b}g$: already quite optimised

Uses analytical MEs, "folds" phase space to cancel azimuthally antipodal points, and uses antenna subtraction (→ smaller # of NLO subtraction terms than Catani-Seymour or FKS).

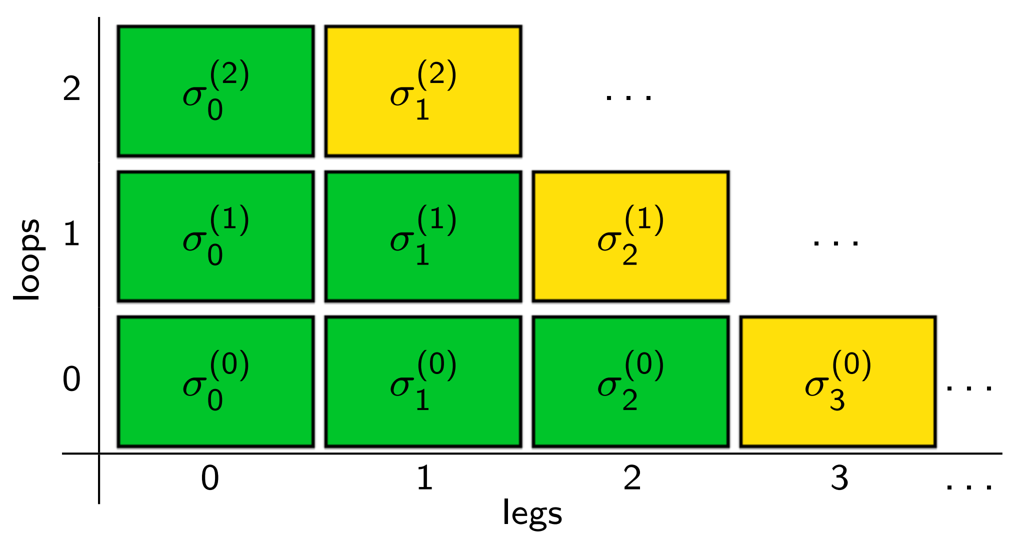
VINCIA NNLO+PS: shower as phase-space generator: efficient & no negative weights

➤ **Looks ~ 5 x *faster* than EERAD3** (for similar unweighted stats) + is matched to shower ⟹ includes resummation; can calulate any IR safe observable; can be hadronised → IR sensitive observables, etc.



Note:

NNLO accuracy in $H \to 2j$ implies **NLO correction in first emission** and **LO correction in second emission.**



So for Thrust, NNLO $H \to b\bar{b}$ is effectively

NLO for $\tau < 1/3$

LO for $\tau > 1/3$

Proof of concepts done for H & Z → 2

Work remains to extend to pp, ep, and ee → $n \geq 3$

(& on marrying this formalism with N^{n}LL accuracy)

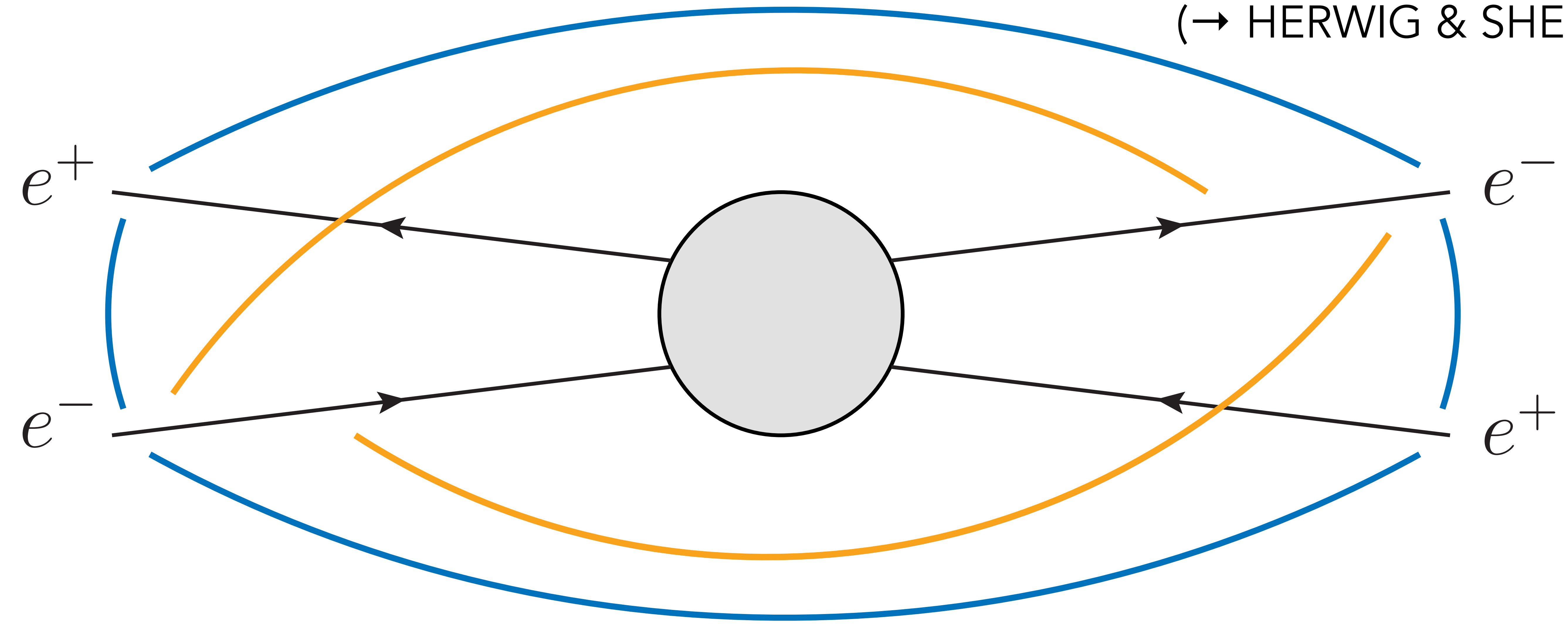
Main component: soft photon emission

[Dittmaier, 2000] $$|M_{n+1}(\{p\},p_j)|^2 = -8\pi\alpha \sum_{x,y}^{n} \sigma_x Q_x \sigma_y Q_y \frac{s_{xy}}{s_{xj}s_{yj}} |M_n(\{p\})|^2$$

Example: Quadrupole final state (4-fermion: $e^+e^+e^-e^-$)

Opposite-charge pairs ➤ positive terms

Same-charge pairs ➤ negative terms ← **Not well suited for showers** (→ HERWIG & SHERPA use YFS)

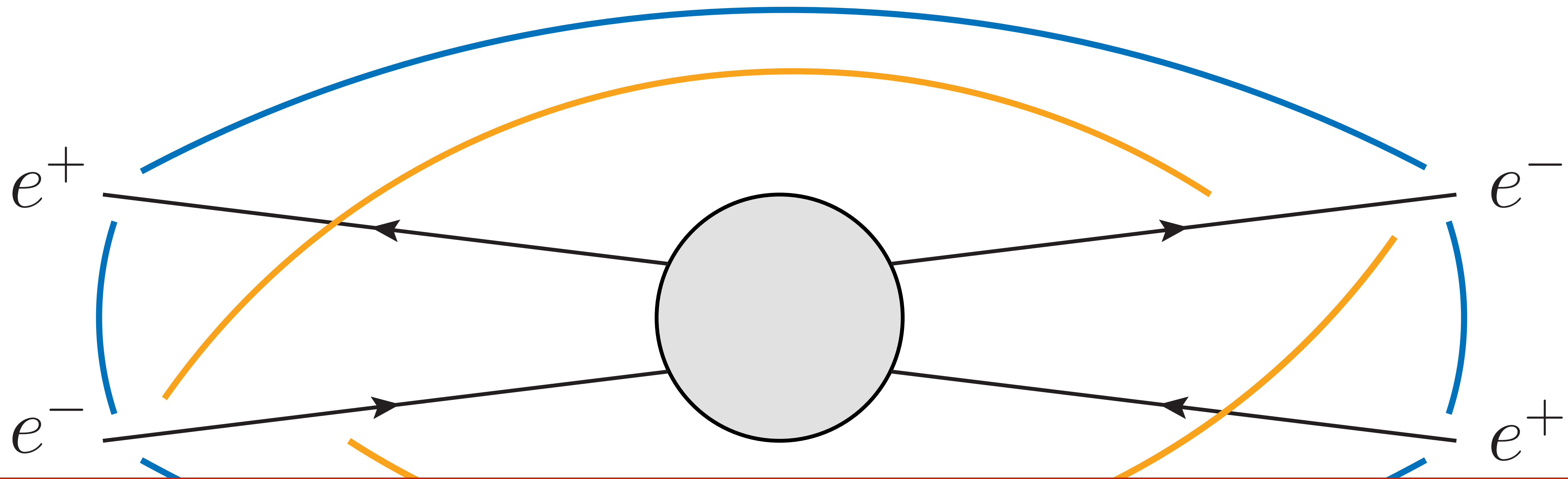


QED Multipole Showers in VINCIA

Sectorize QED phase space: for each possible photon emission kinematics p_γ, find the 2 charged particles with respect to which that photon is softest ➤ "Dipole Sector"

Use dipole-antenna *kinematics* for that sector, but sum **all** the positive and negative *antenna* terms (w spin dependence) to find **coherent emission *probability > 0***

$\Longrightarrow$ QED shower with **full soft multipole coherence and DGLAP collinear limits and no negative weights** [Kleiss & Verheyen (2017); PS & Verheyen (2020)]



Available in PYTHIA 8; directly applicable also to $e^+e^- \to Z/\gamma^* \to f\bar{f}$ and $e^+e^- \to W^+W^- \to 4f$
Also accounts for **initial-final interference** via **interleaved resonance decays**; discussed later

Example of QED multipole interferences

High-mass Drell-Yan

$u\bar{u} \to Z/\gamma^* \to e^+e^-$

$m^2_{ee} > 1$ TeV, $p_{\perp,e} > 25$ GeV and $|\eta_e| < 3.5$

$p_{\perp,\gamma} > 0.5$ GeV and $|\eta_\gamma| < 3.5$

PYTHIA

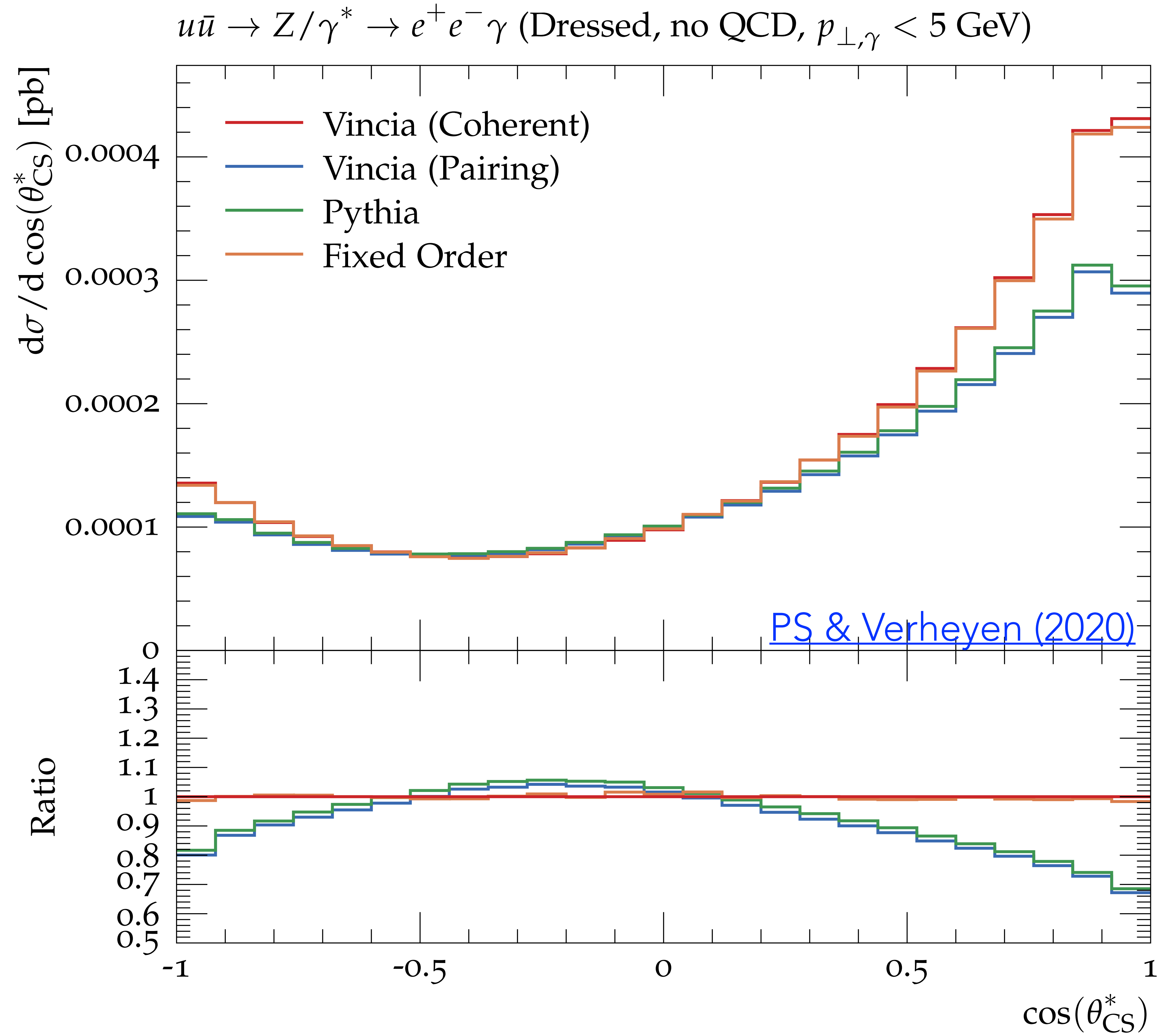
Factorizes $u\bar{u}$ and e^+e^- radiation

VINCIA

1) **Coherent** = full multipole treatment

2) **Pairing** ~ PYTHIA: only consider "maximally screening" charge pairs; no genuine multipole effects

Next: QED matrix-element corrections & applications to QED corrections in B decays



$$\cos\theta^*_{CS} = 2\frac{p^z_{ee}}{|p^z_{ee}|}\frac{p^+_{e^+}p^-_{e^-} - p^-_{e^+}p^+_{e^-}}{m_{ee}\sqrt{m^2_{ee}+p^2_{\perp,ee}}},$$

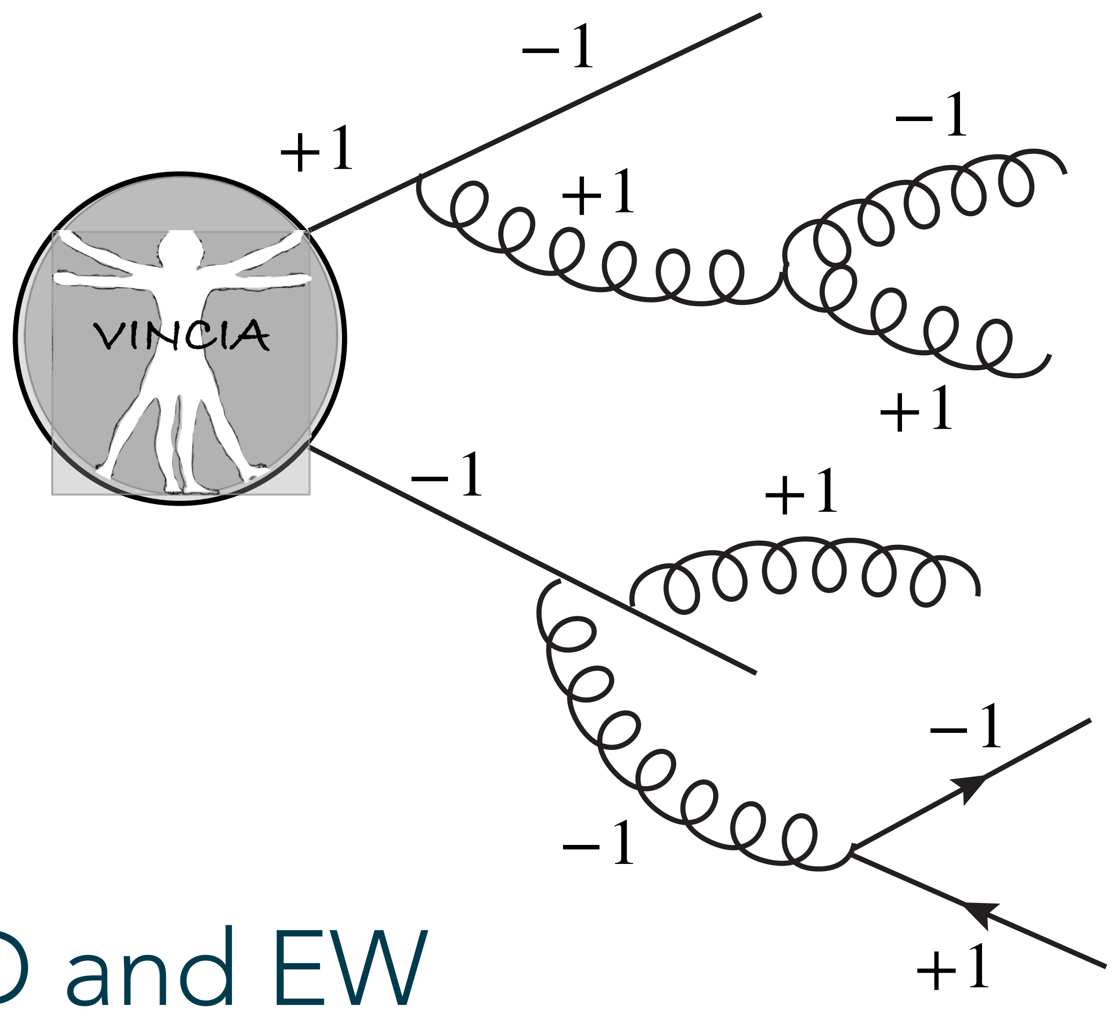
Angle between the incoming quark and the outgoing electron in the Collins-Soper frame, using longitudinal boost of ee pair as stand-in for ambiguous quark direction

Weak Showers

Real corrections: **EW gauge bosons, tops, Higgs part of jets**

Virtual corrections: **Universal incorporation of Sudakov logs** $\frac{\alpha}{\pi} \ln^2(s/Q^2_{\mathrm{EW}})$

Features of VINCIA's EW Shower [Brooks, PS, Verheyen (2022)]



- Chiral → **Helicity showers** Larkoski, Lopez-Villarejo, PS (2013); Fischer, Lifson, PS (2017)
- EW-scale mass corrections & exact massive phase spaces
- Longitudinal polarisations / Goldstone bosons
- Treatment of neutral boson interference
- Overlap vetos to eliminate double-counting between QCD and EW
- Resonance-decay like branchings → **Interleaved Resonance Decays**

Caveat: Our EW antenna functions constructed from collinear limits (~DGLAP)

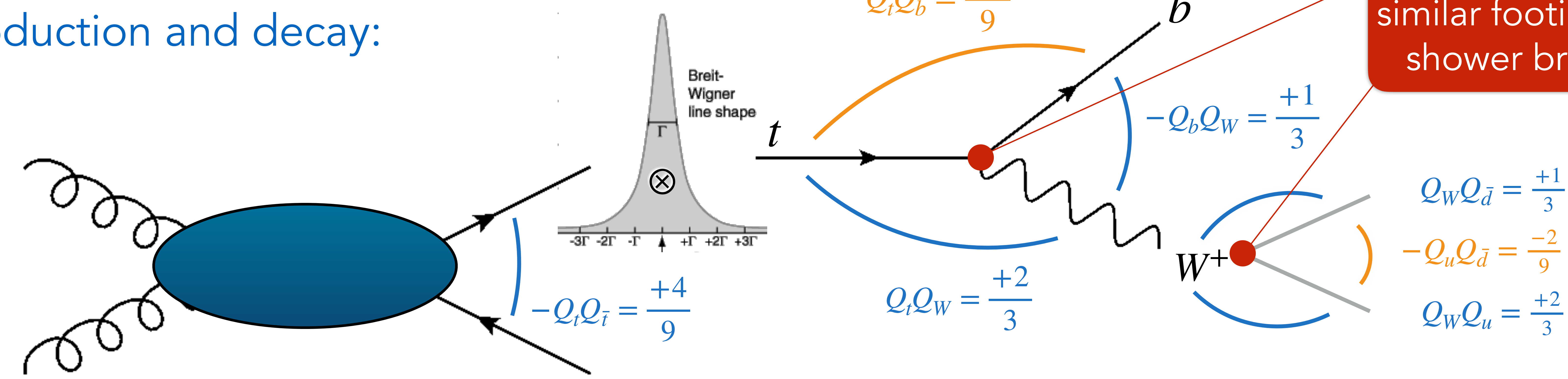
Soft multipole coherence so far only for pure QED, not full EW

Narrow-Width Limit ⇔ Conventional "sequential" treatment

Treat each decay (sequentially) as if alone in the universe

Observation: these are also EW vertices.
➤ Treat decays on similar footing as other shower branchings.

Example: QED radiation in $t\bar{t}$ production and decay:



Beyond Narrow-Width Limit:

Expect interferences to become important for $E_\gamma \lesssim \Gamma_t$ (and $E_\gamma \lesssim \Gamma_W$)

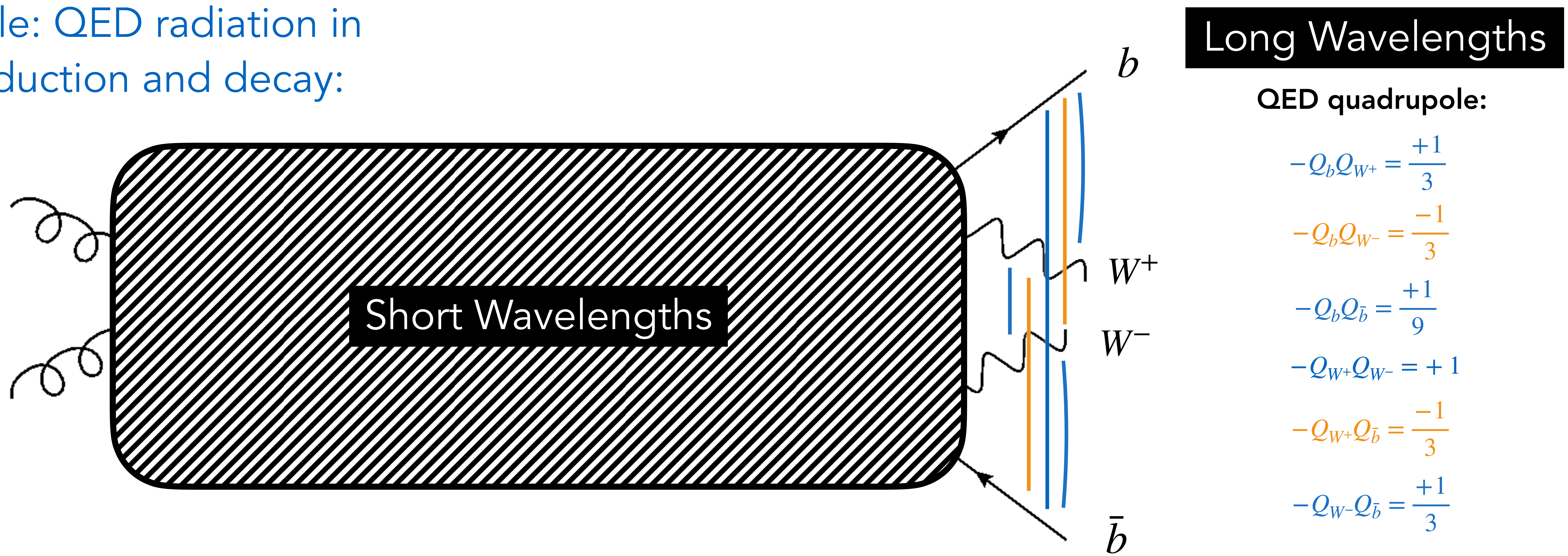
(Note: for charged resonances, VINCIA utilises unique coherent "**resonance-final**" antenna patterns with **global recoil** [Brooks, PS (2019)])

Physics Motivation for Interleaved Resonance Decays

Long-wavelength radiation should not be able to resolve short-lived intermediate states

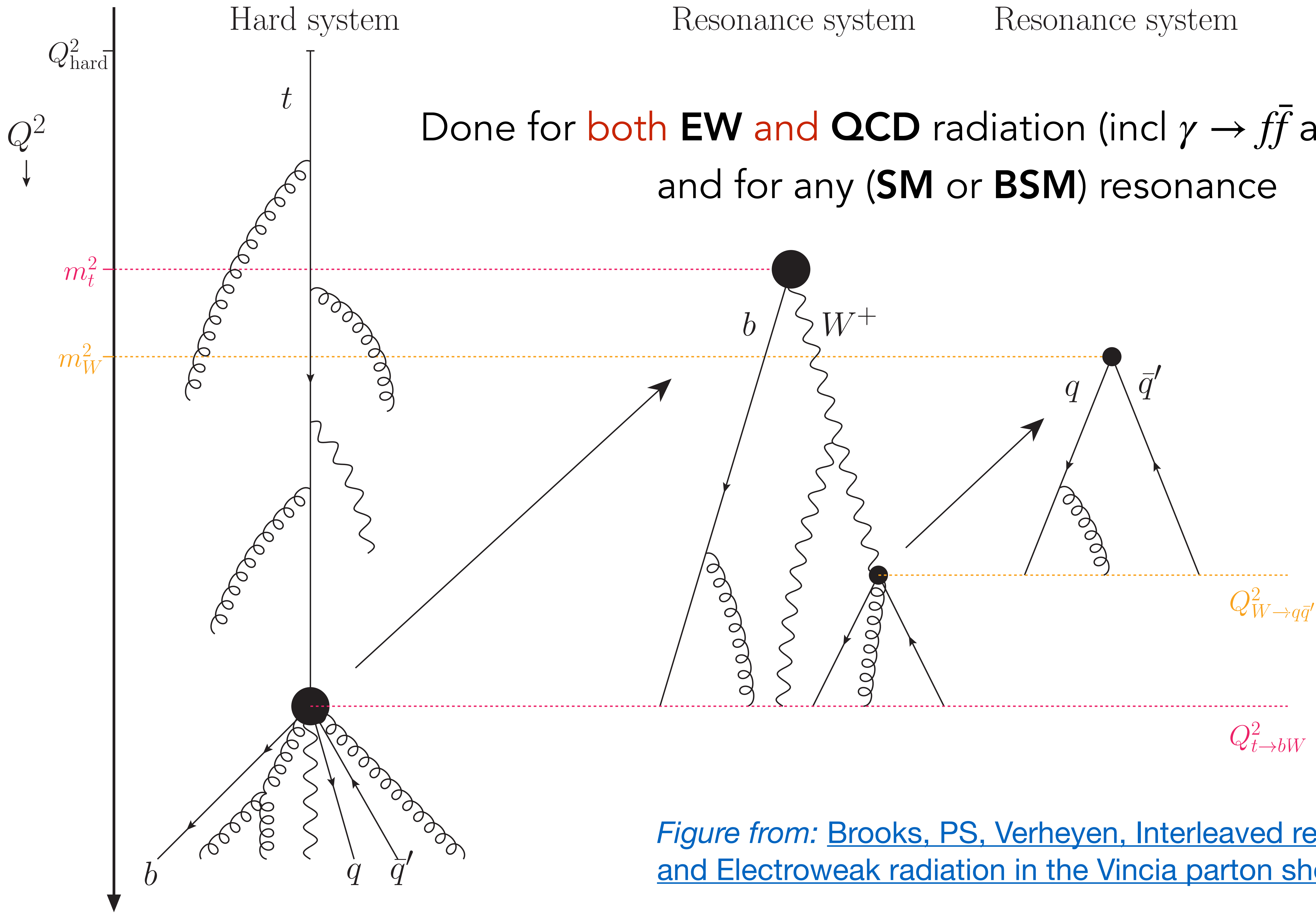
For **long wavelengths** $\lambda \gtrsim (\hbar c)/\Gamma$ expect interferences (& recoils) *between* decays

Example: QED radiation in $t\bar{t}$ production and decay:



Affects radiation spectrum, for energies $E_\gamma \lesssim \Gamma$

+ Interferences and recoils *between* systems => ***non-local BW modifications***



Done for both **EW** and **QCD** radiation (incl $\gamma \to f\bar{f}$ and $g \to q\bar{q}$)
and for any (**SM** or **BSM**) resonance

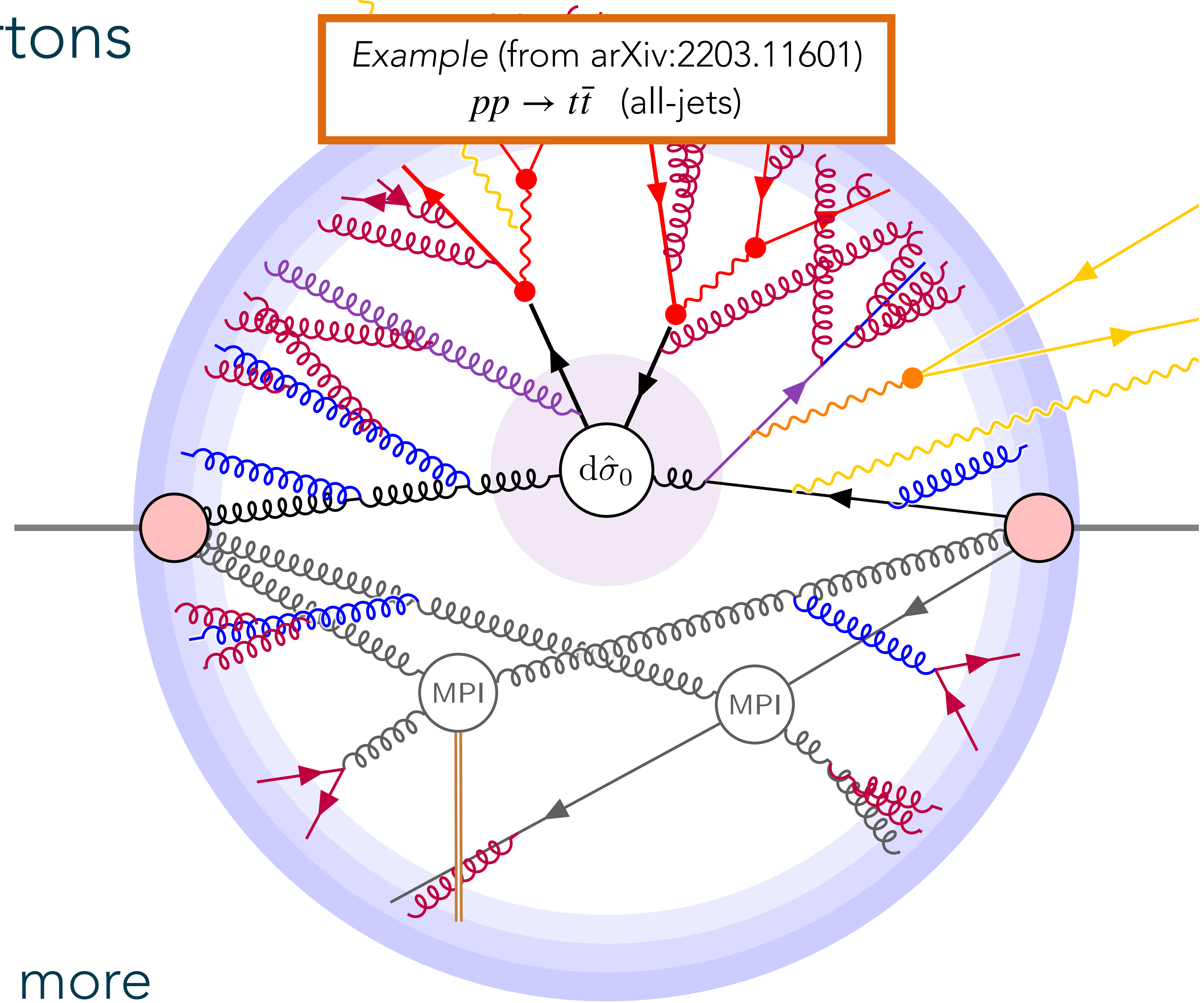
Figure from: Brooks, PS, Verheyen, Interleaved resonance decays and Electroweak radiation in the Vincia parton shower (2022)

High-energy pp collisions — with ISR, Multi-Parton Interactions, and Beam Remnants

Final states with **very many** coloured partons

With significant overlaps in phase space

Who gets confined with whom?



"Parton Level"

(Event structure before confinement)

Each has a colour ambiguity $\sim 1/N_C^2 \sim 10\,\%$

E.g.: **random triplet** charge has 1/9 chance to be in **singlet** state with **random antitriplet**:

$$3 \otimes \bar{3} = 8 \oplus 1$$

$$3 \otimes 3 = 6 \oplus \bar{3} \quad ; \quad 3 \otimes 8 = 15 \oplus 6 \oplus 3$$

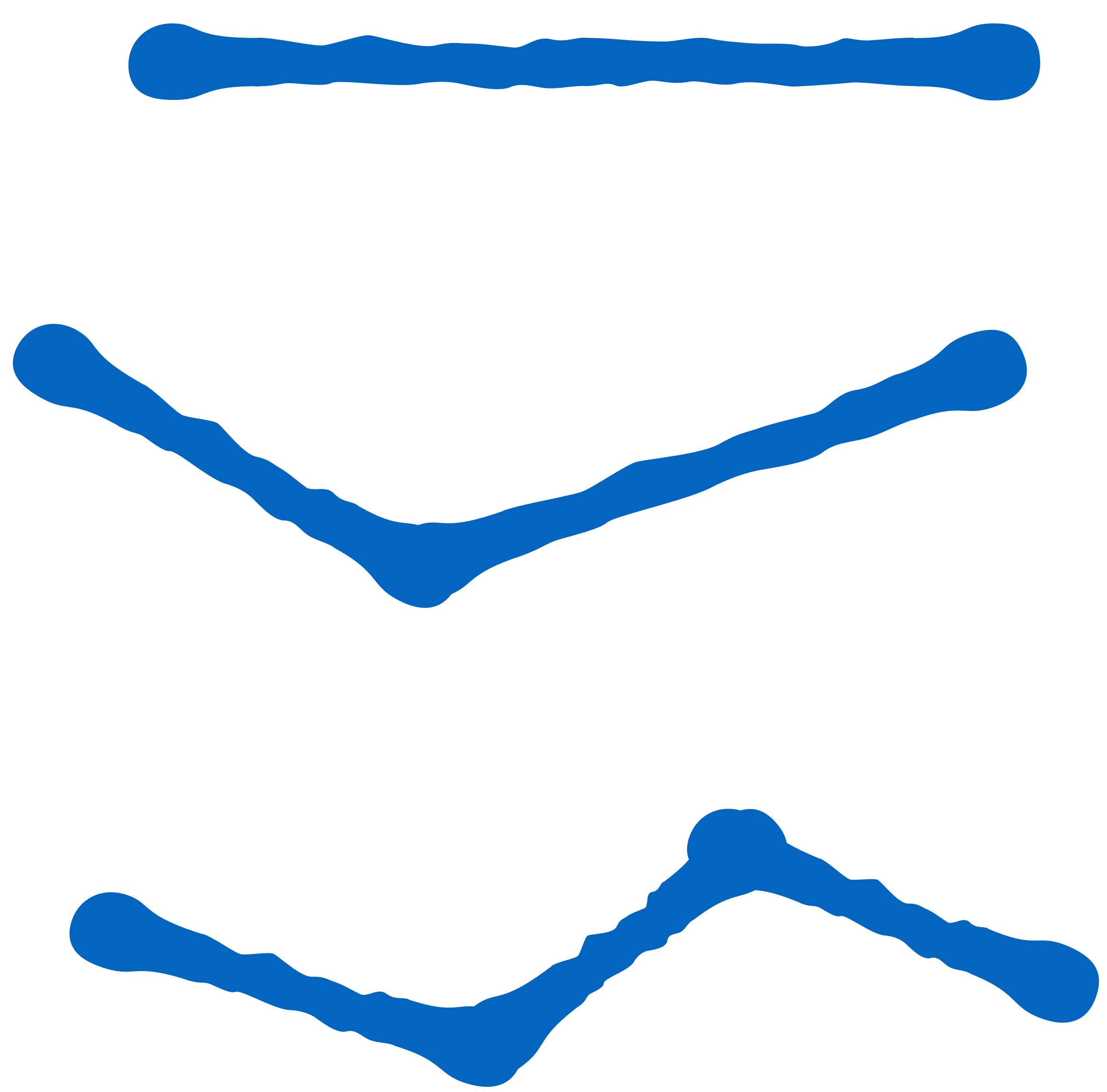
$$8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8_S \oplus 8_A \oplus 1$$

Many charges ➜ Colour Reconnections* (CR) more likely than not — "Colour Promiscuity!" [J. Huston]

**): in this context, QCD CR simply refers to an ambiguity beyond Leading N_C, known to exist. Note the term "CR" can also be used more broadly to incorporate further physics concepts.*

QCD Colour Reconnections ⟷ String Junctions

Open Strings

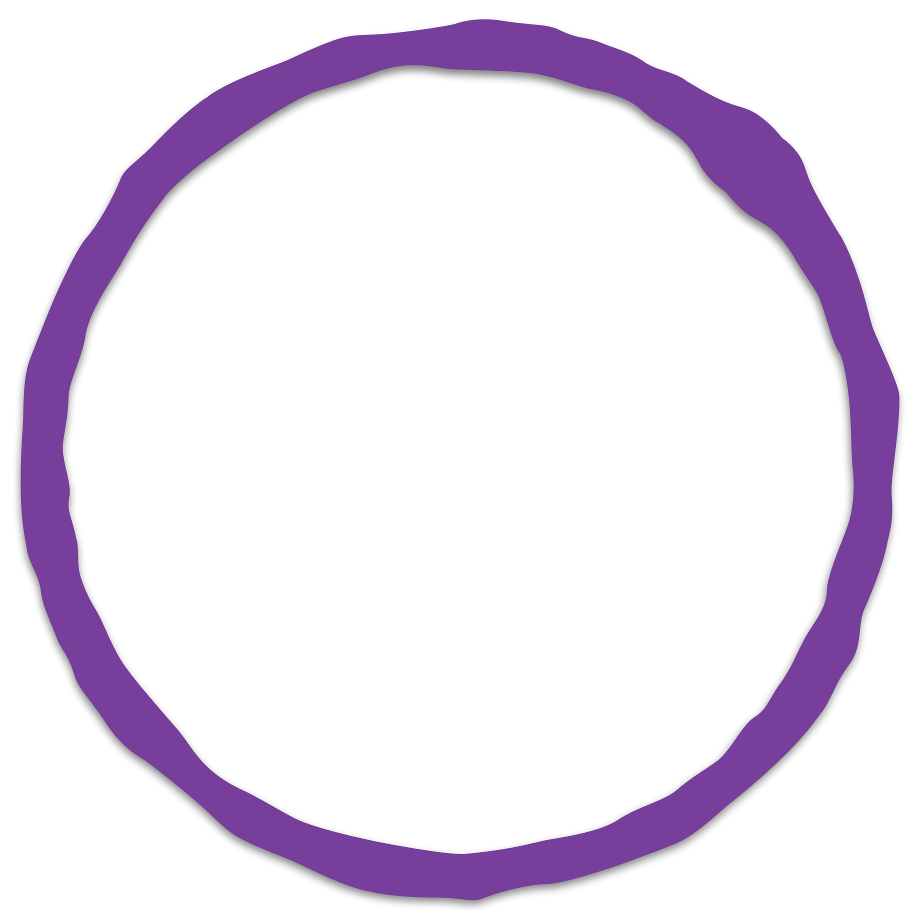


$q\bar{q}$ strings (with gluon kinks)

E.g., $Z \to q\bar{q}$ + shower

$H \to b\bar{b}$ + shower

Closed Strings

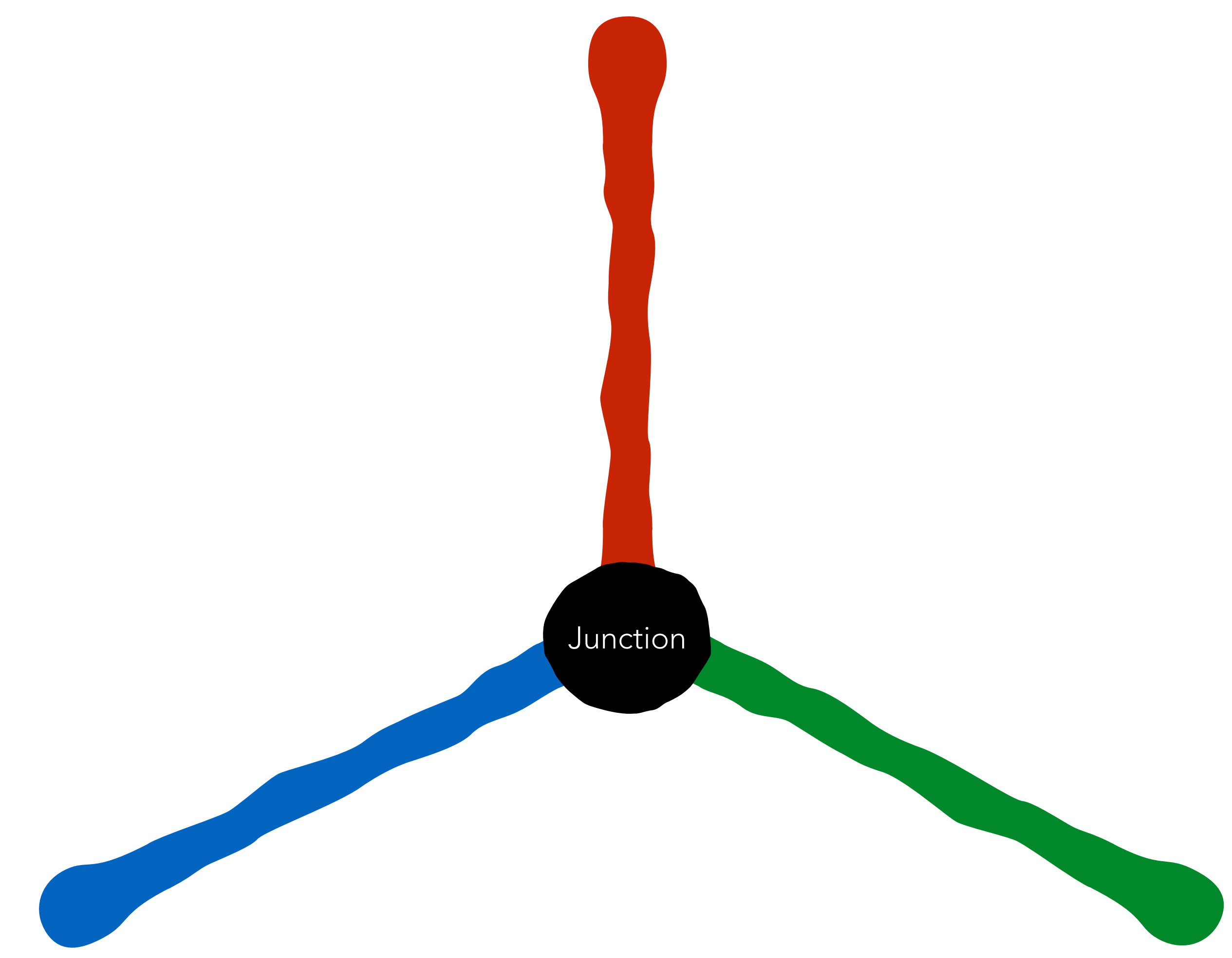


Gluon rings

E.g., $H \to gg$ + shower

$\Upsilon \to ggg$ + shower

SU(3) String Junction



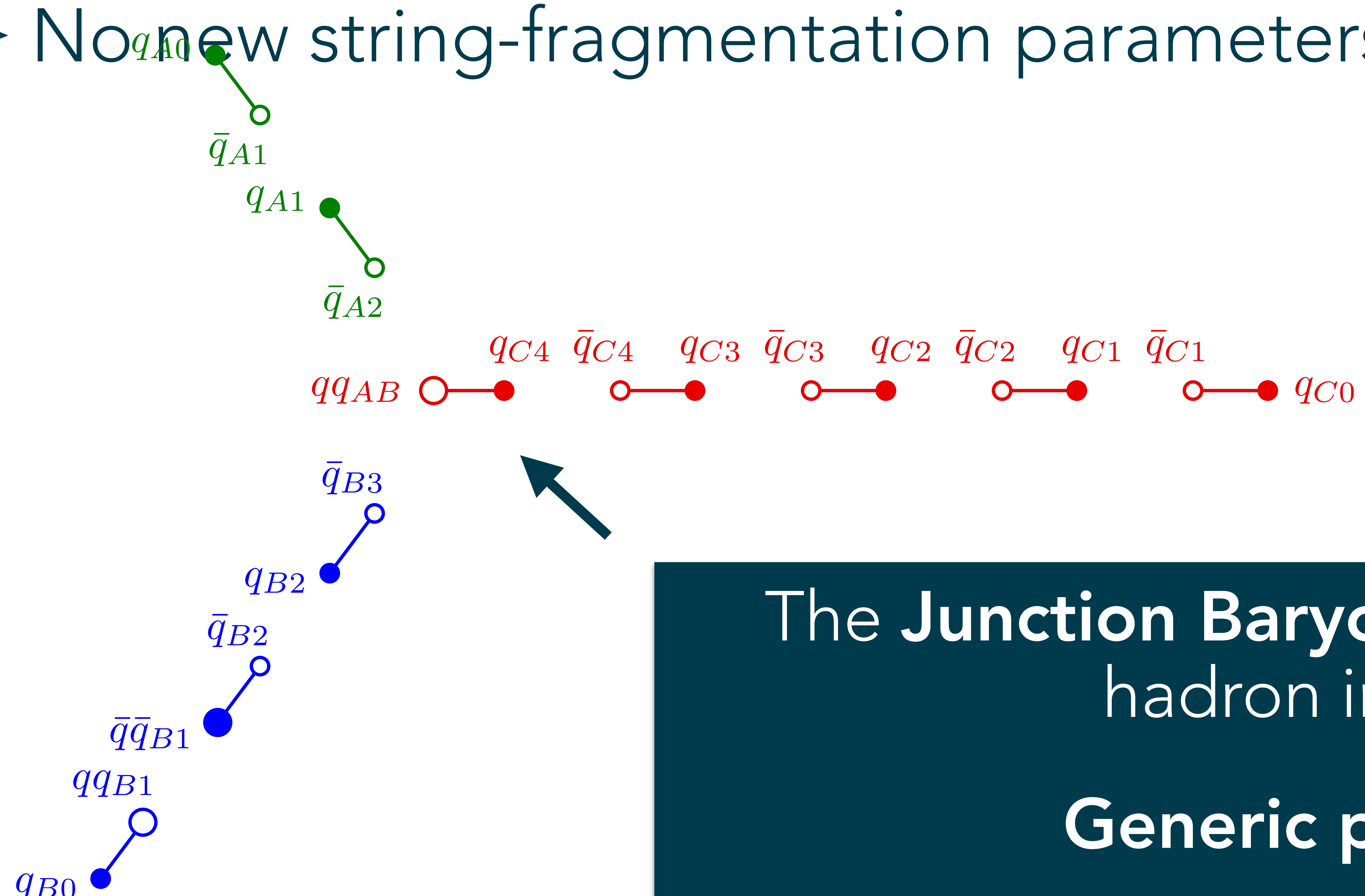
Open strings with $N_C = 3$ endpoints

E.g., Baryon-Number violating neutralino decay $\tilde{\chi}^0 \to qqq$ + shower

Assume Junction Strings have same properties as ordinary ones
(u:d:s, Schwinger p_T, etc)

- No new string-fragmentation parameters

[Sjöstrand & **PS**, NPB 659 (2003) 243]
[+ J. Altmann & **PS**, in progress]



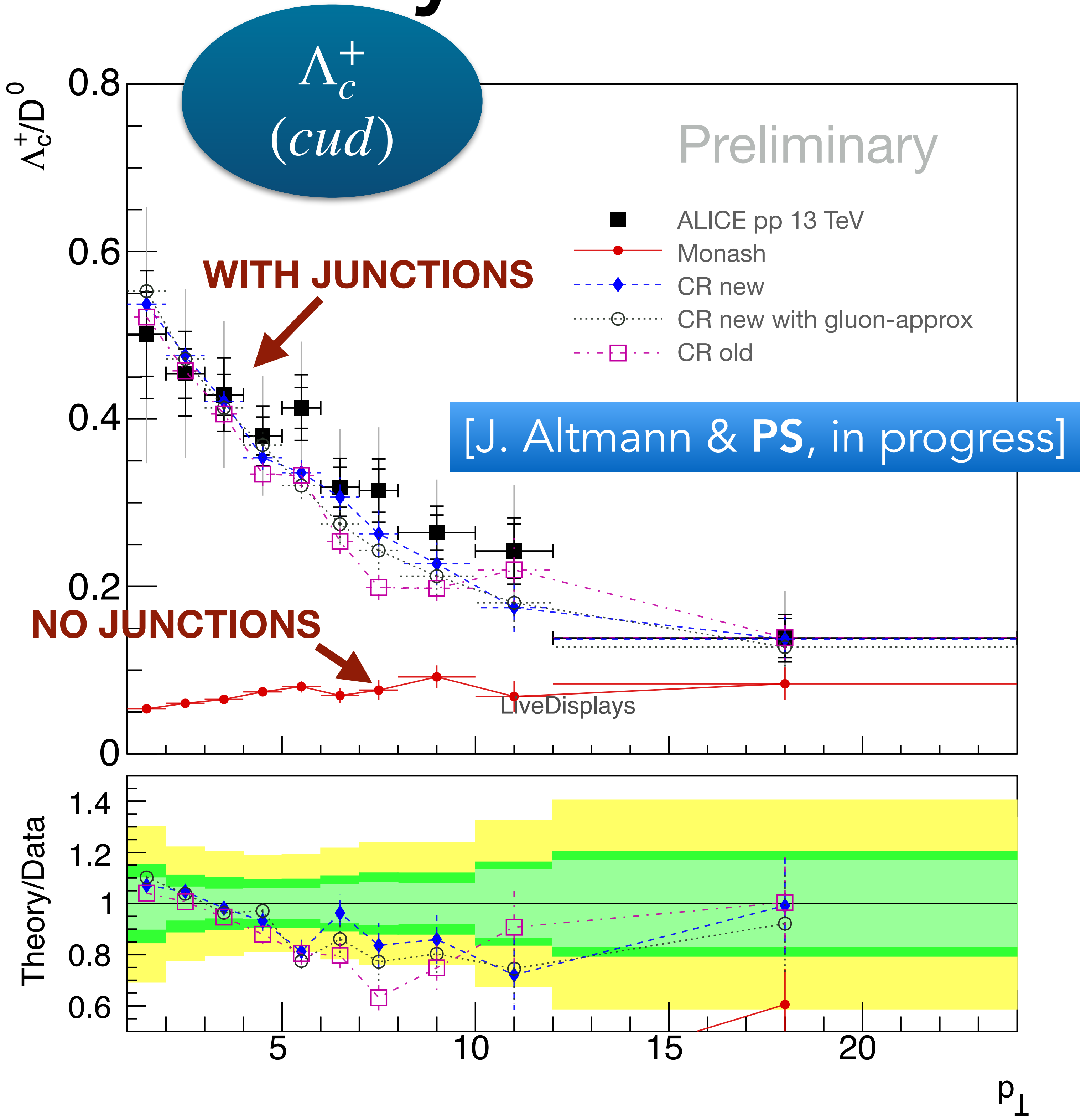
The **Junction Baryon** is the most "subleading" hadron in all three "jets".

Generic prediction: low p_T

A Smoking Gun for String Junctions: Baryon enhancements at low p_T

Confront with Measurements

LHC experiments report very large (factor-10) enhancements in heavy-flavour baryon-to-meson ratios at low p_T!



+ Lots of interesting new measurements showing changes in **strange** vs nonstrange strange **hadrons**

& evidence of **flow-like effects** in pp collisions → modifications to p_T spectra

Not reproduced by baseline string/cluster models

Very exciting! Lots of Activity

Particle Composition: Impact on Jet Energy Scale

ATLAS PUB Note

ATL-PHYS-PUB-2022-021

29th April 2022

Dependence of the Jet Energy Scale on the Particle Content of Hadronic Jets in the ATLAS Detector Simulation

The dependence of the ATLAS jet energy measurement on the modelling in Monte Carlo simulations of the particle types and spectra within jets is investigated. **It is found that the hadronic jet response, i.e. the ratio of the reconstructed jet energy to the true jet energy, varies by ~ 1–2% depending on the hadronisation model used in the simulation. This effect is mainly due to differences in the average energy carried by kaons and baryons in the jet.** Model differences observed for jets initiated by *quarks* or *gluons* produced in the hard scattering process are dominated by the differences in these hadron energy fractions indicating that **measurements of the hadron content of jets and improved tuning of hadronization models can result in an improvement in the precision of the knowledge of the ATLAS jet energy scale.**

Variation largest for gluon jets

- For E_T = **[30, 100, 200] GeV**
- Max JES variation = **[3%, 2%, 1.2%]**

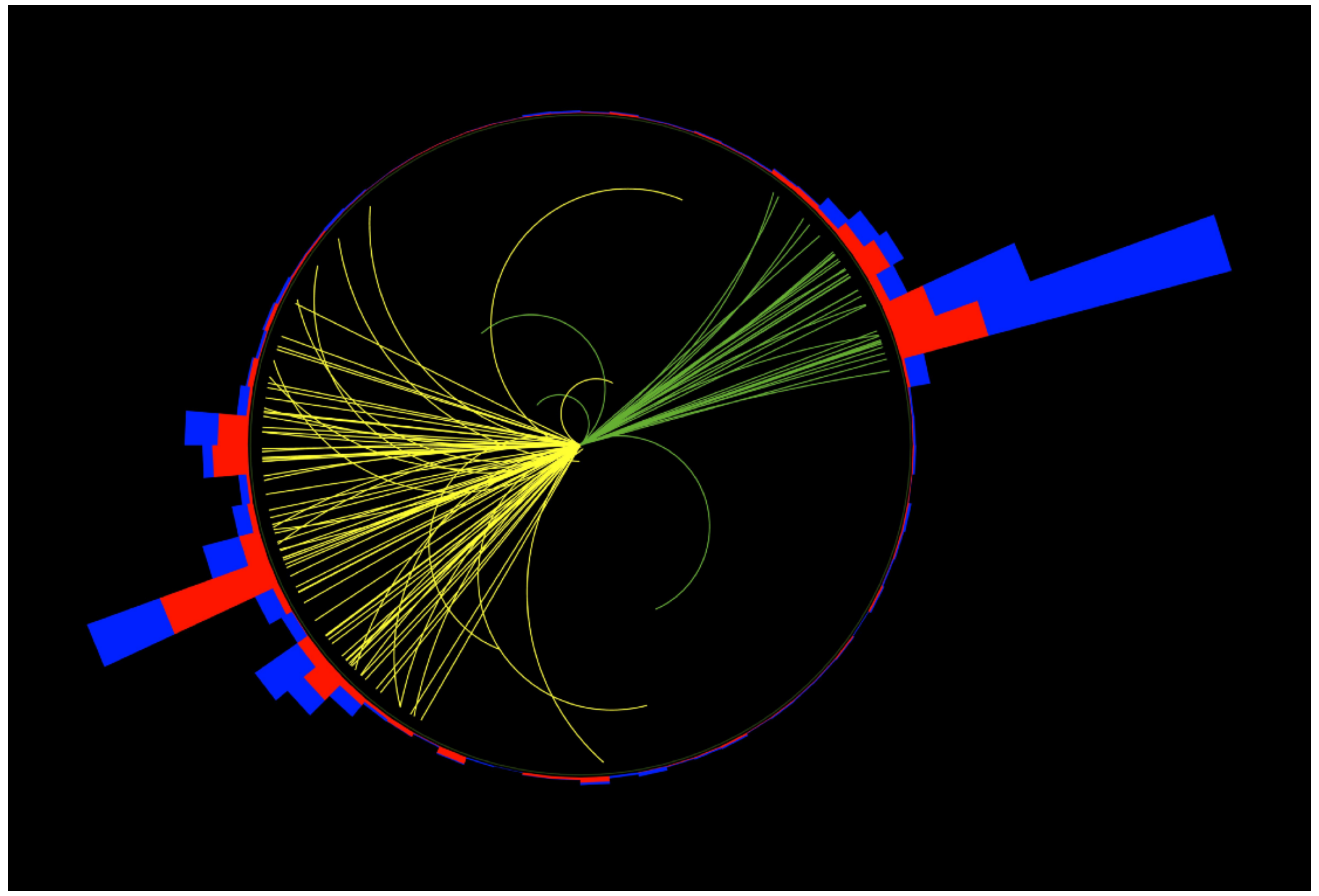
Fraction of jet E_T carried by baryons (and kaons) varies significantly

- Reweighting to force similar baryon and kaon fractions
- Max variation ➜ **[1.2%, 0.8%, 0.5%]**
- Significant potential for improved Jet Energy Scale uncertainties!

Motivates Careful Models & Careful Constraints

- Interplay with advanced UE models
- In-situ constraints from LHC data
- Revisit comparisons to LEP data

MC generators connect theory with experiment



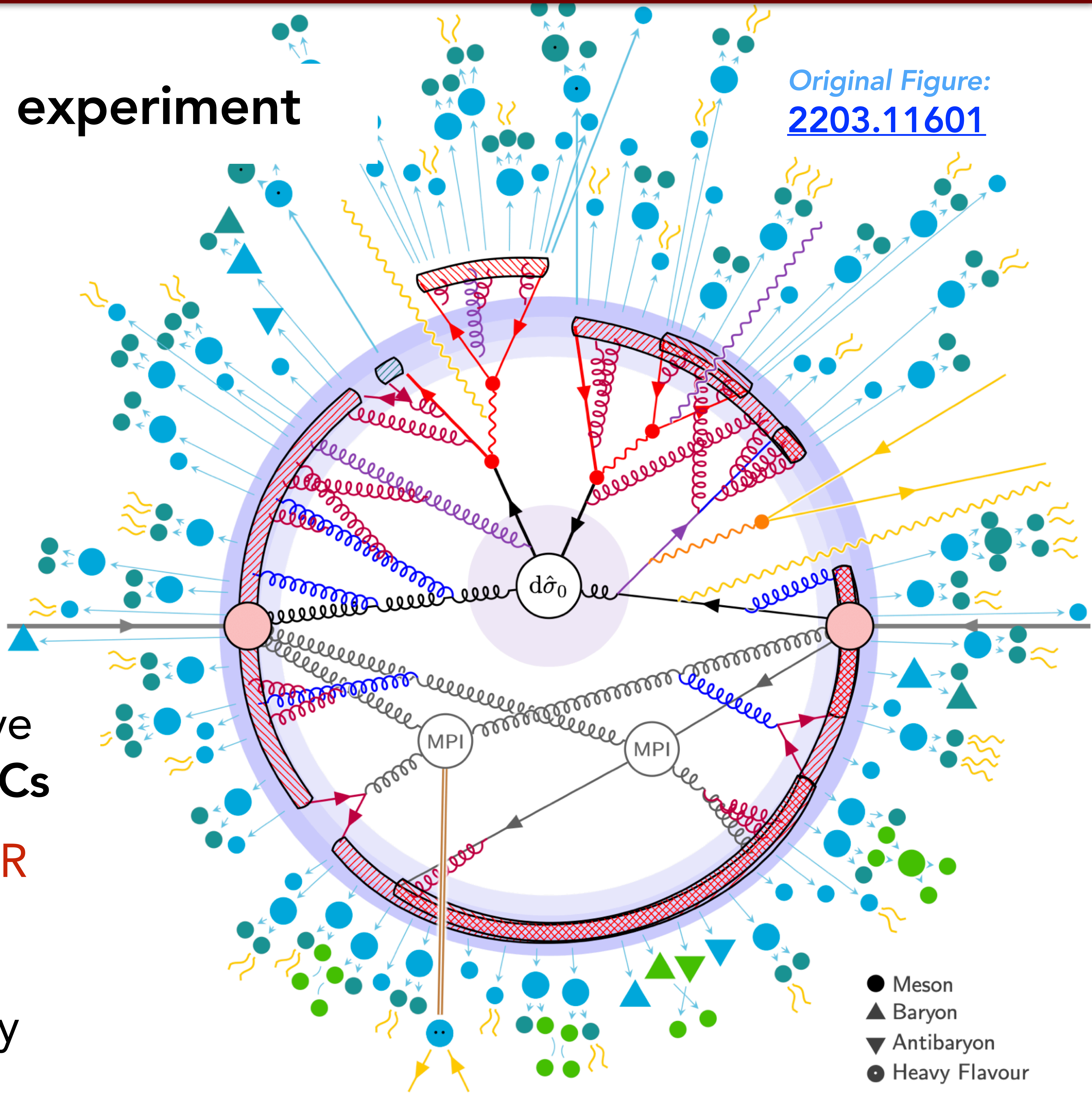
Entering era of percent-level perturbative accuracy, with **NNLO+N$^{(n)}$LL accurate MCs**

+ much new work on hadronization & CR

Driven by **LHC physics program**

But ee often used as test bed ↔ synergy

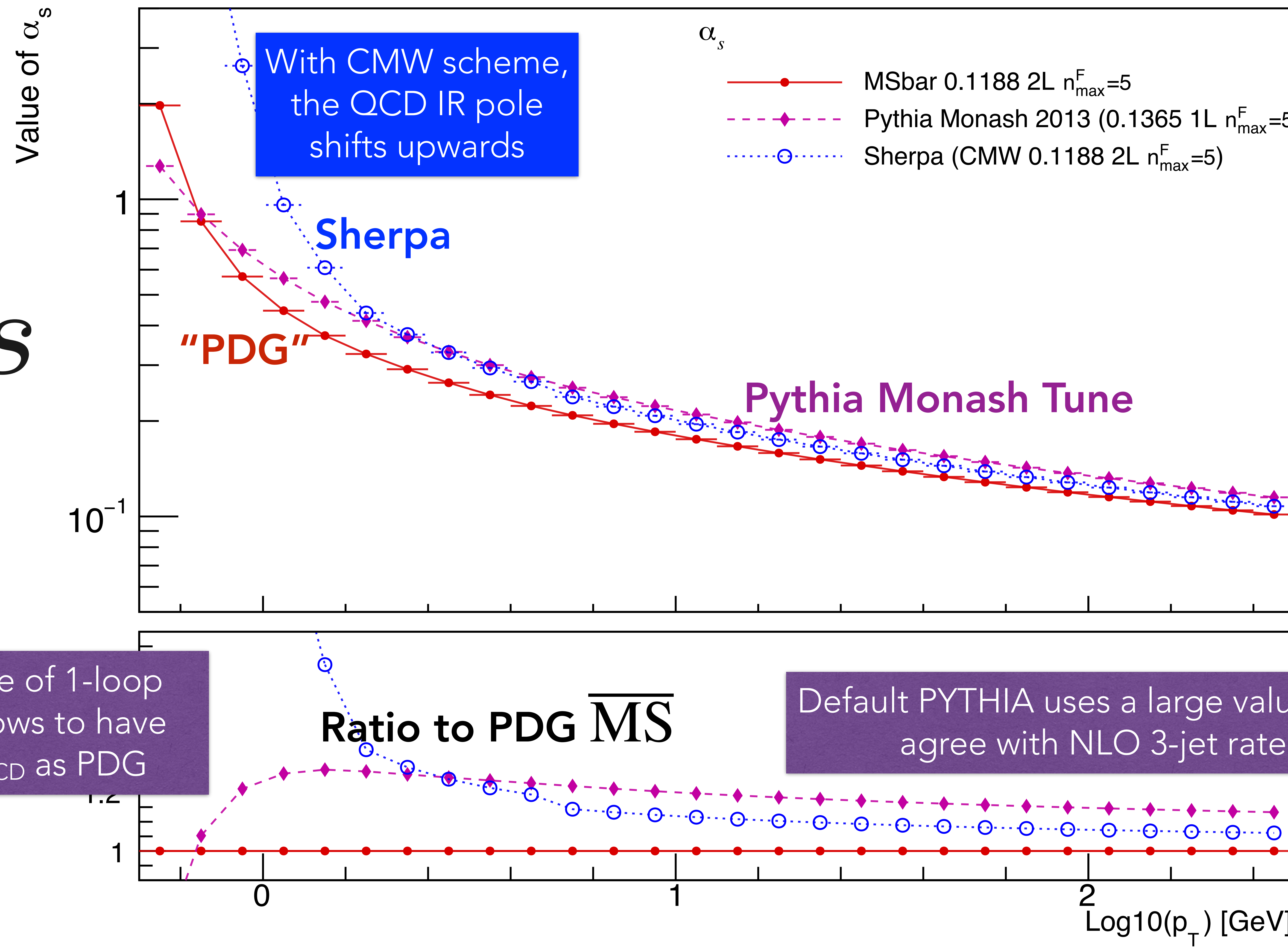
Original Figure: 2203.11601



Extra Slides

Note on Different alpha(S) Choices

α_s



Slower pace of 1-loop running allows to have similar Λ_{QCD} as PDG

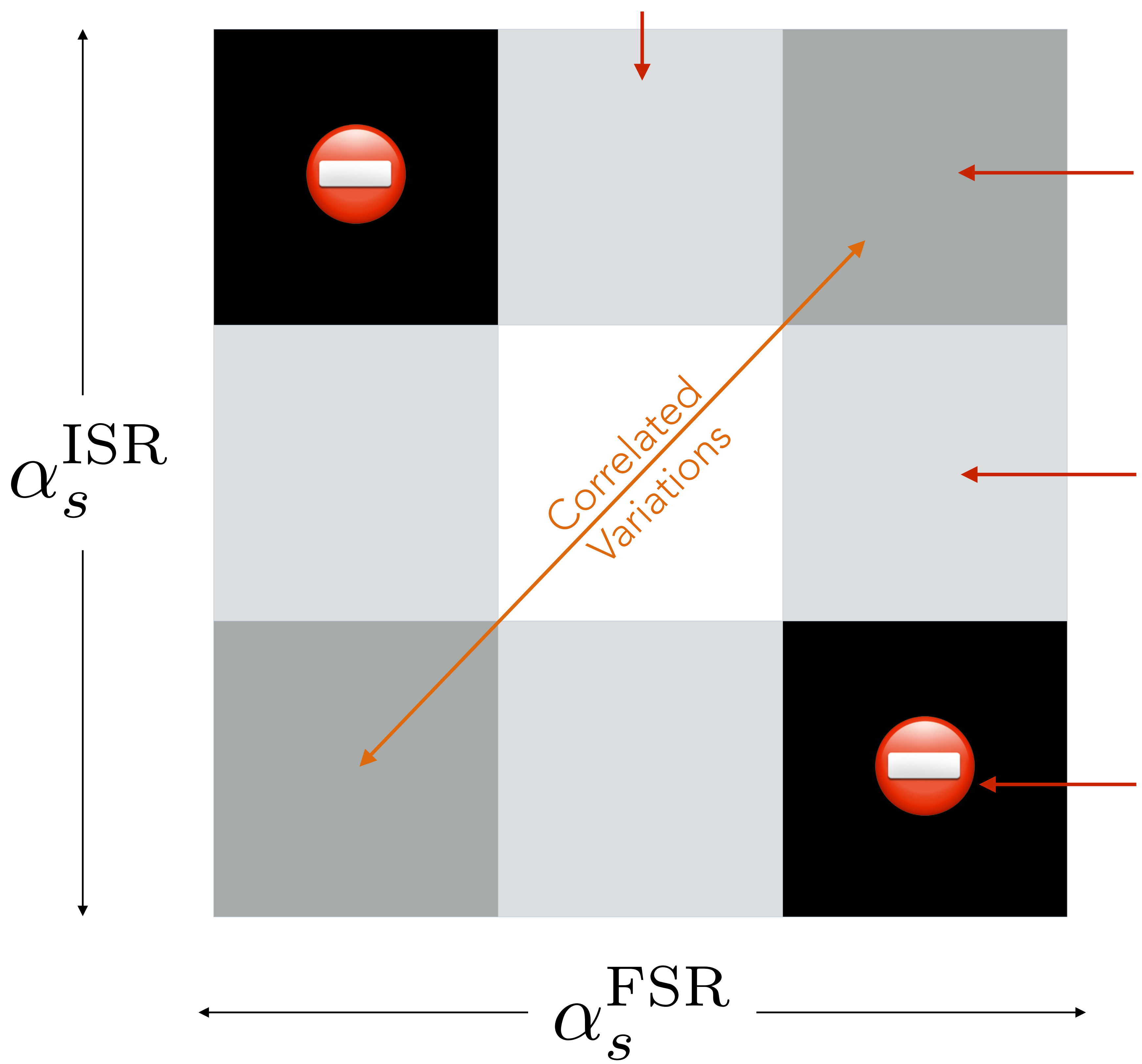
Default PYTHIA uses a large value of $\alpha_s(M_Z)$ to agree with NLO 3-jet rate at LEP

Correlated or Uncorrelated?

What I would do: **7-point variation** (resources permitting → use the automated bands?)

Increasing only ISR

➠ More H_T and N_{jets}; similar core jet shapes



Increasing both ISR and FSR

➠ More H_T in the events.

➠ More OOC loss (from FSR) but also more H_T and more hard ISR jet seeds → partial cancellation in N_{jets}?

Increasing only FSR

➠ More OOC loss (FSR jet broadening), acting on similar number of seed partons (no increase in ISR).

➠ Similar H_T

Increasing FSR, Decreasing ISR -> Exclude?

➠ Double counting? Fewer ISR partons, **and** more smearing of those that remain. *(Easy to rule out?)*

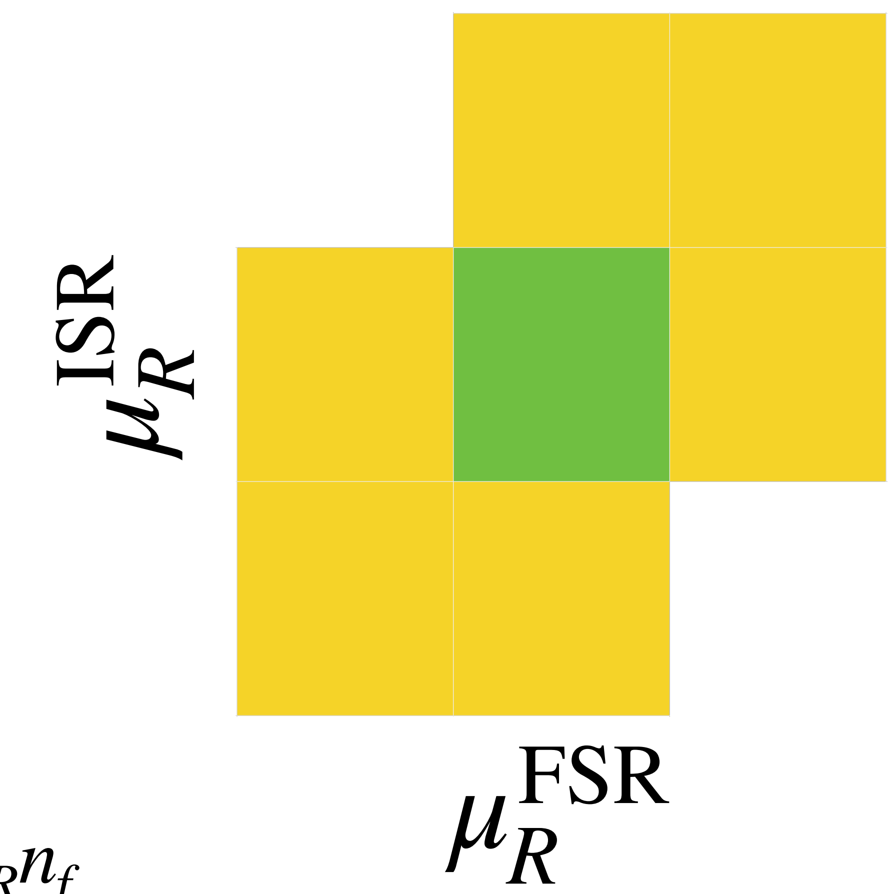
➠ Also from theoretical/mathematical point of view, the artificially induced discrepancy is now proportional to ln(16) = 2.8 instead of ln(4) = 1.4.

❶ Perturbative Uncertainties in Showers

First guess: renormalisation-scale variations,

$\mu_R^2 \to k_\mu \, \mu_R^2$, with constant $k_\mu \in [0.5, 2]$ or $[0.25, 4]$, …

+ e.g., do for ISR and FSR separately → **7-point variations** ⟶



Induces "nuisance" terms beyond calculated orders

Running of $\alpha_s(k\,\mu^2) \;=\; \alpha_s(\mu^2)\dfrac{1}{1 + b_0\alpha_s(\mu^2)\ln(k)}$ with $b_0 = \dfrac{11N_C - 4T_R n_f}{12\pi} \sim 0.6$

$\Longrightarrow$ ME proportional to $\alpha_s^n(\mu^2)\left(1 \;\pm\; \underbrace{b_0\,\alpha_s(\mu^2)\,\ln k^n}_{\text{variation}} + \;\ldots\right)$

I think many of us suspect this is unsatisfactory and unreliable

Problem: little guidance on what else to do …

Invitation for Discussions (after talk)

Issue #1: Multiscale Problems (e.g., a couple of bosons + a couple of jets)

Not well captured by **any** variation k_μ around any **single** scale

More of an issue for hard-ME calculations than for showers (which are intrinsically multiscale)

Best single-scale approximation = **geometric mean of relevant** (nested) **QCD scales**

My recommendation: vary which scales enter this geometric mean

Issue #2: Terms that are not proportional to the lower orders

Renormalization-scale variations always proportional to what you already:

$$\mu_R \text{ variations} \implies \mathrm{d}\sigma \ \rightarrow (1 \pm \Delta\alpha_s)\,\mathrm{d}\sigma$$

No new kinematic dependence

But full higher-order matrix elements will also contain **genuinely new terms** at each order, not proportional to previous orders:

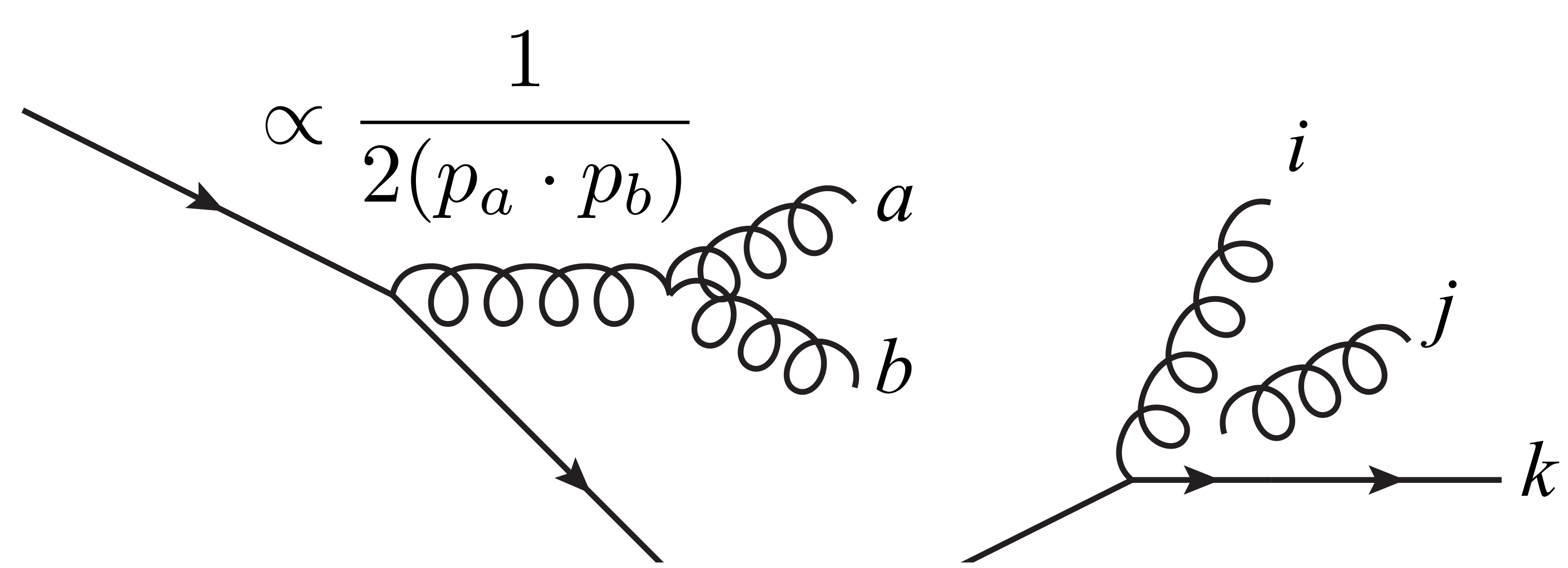
$$\text{More general} \implies \mathrm{d}\sigma \rightarrow \mathrm{d}\sigma \pm \Delta\mathrm{d}\sigma$$

Parton Showers: Theory

see e.g PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389

Most bremsstrahlung is driven by divergent propagators → simple structure

Mathematically, **gauge amplitudes factorize** in singular limits



Partons ab → **collinear**:

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \overset{a||b}{\to} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a\cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

$P(z)$ = **DGLAP splitting kernels**", with $z = E_a/(E_a+E_b)$

Gluon j → **soft**:

$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \overset{j_g\to 0}{\to} g_s^2 \mathcal{C} \frac{(p_i\cdot p_k)}{(p_i\cdot p_j)(p_j\cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

Coherence → Parton j really emitted by (i,k) "dipole" or **"antenna" (eikonal factors)**

These are the **building blocks of parton showers** (DGLAP, dipole, antenna, …)

(+ running coupling, unitarity, and explicit energy-momentum conservation.)

Scale Variations: How big?

What do parton showers do?

In principle, LO shower kernels proportional to α_s

Naively: do the analogous factor-2 variations of μ_{PS}.

There are at least 3 reasons this could be **too** conservative

1. **For soft gluon emissions,** we know what the NLO term is

   → even if you do not use explicit NLO kernels, you are effectively NLO (in the soft gluon limit) **if** you are coherent and use $\mu_{PS} = (k_{CMW}\ p_T)$, with 2-loop running and $k_{CMW} \sim 0.65$ (somewhat n_f-dependent). *[Though there are many ways to skin that cat; see next slides.]*

   Ignoring this, a **brute-force** scale variation **destroys** the NLO-level agreement.

2. Although hard to quantify, showers typically achieve better-than-LL accuracy by accounting for **further physical effects** like (E,p) conservation

3. We see empirically that (well-tuned) showers tend to stay inside the envelope spanned by factor-2 variations in **comparison to data**

(Illustration of the "Magic Trick")

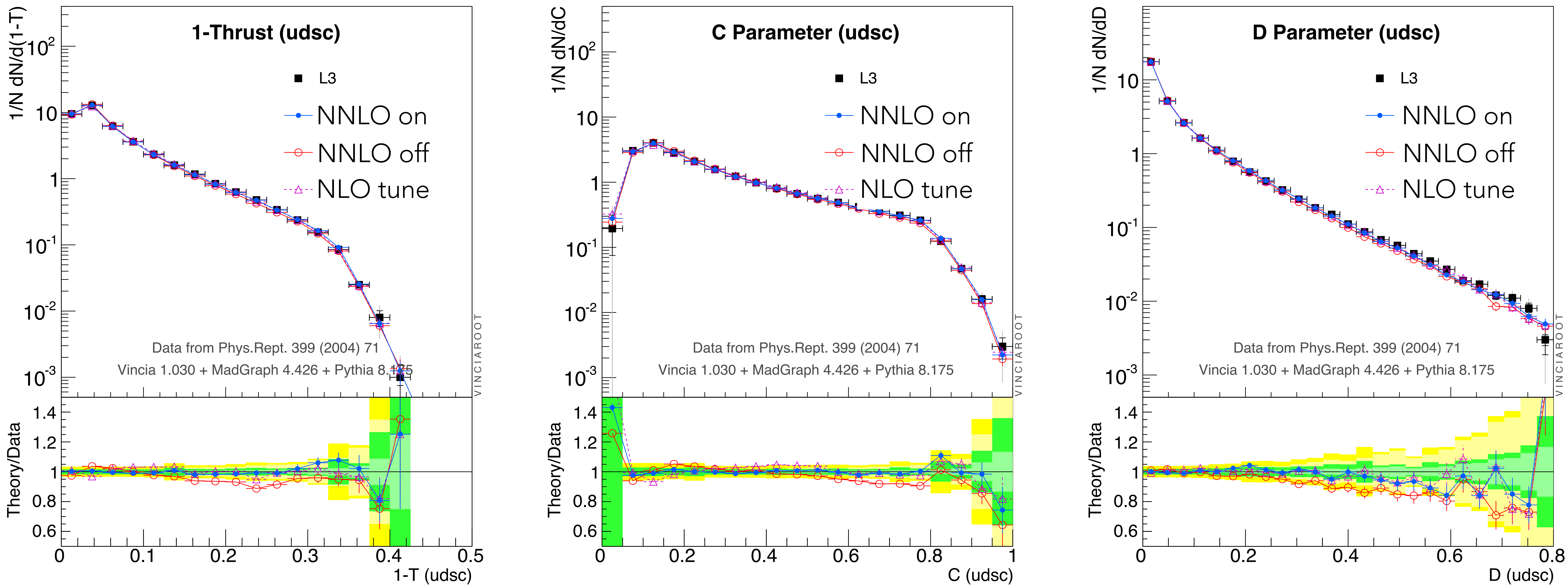
Hartgring, Laenen, **PS**, arXiv:1303.4974

Proof-of-Concept NNLO LEP tune (NNLO Z Decay, ie with NLO 3-jet corrections — using VINCIA)

NNLO tune (3-jet NLO) with $\alpha_s(M_Z) = 0.122$ (2-loop running, CMW)

NLO tune ~ Monash (3-jet LO) with $\alpha_s(M_Z) = 0.139$ (1-loop running, MSbar)

} Comparable values for Λ_{QCD}



Scale variations: How Big?

Poor man's recipe: Use $\sqrt{2}$ instead?

Sure … but still somewhat arbitrary

Instead: add compensation term to preserve soft-gluon limit at $O(\alpha_s^2)$

Still allowing full factor-2 outside that limit.

Pythia includes such a compensation term, at least in context of automated uncertainty bands

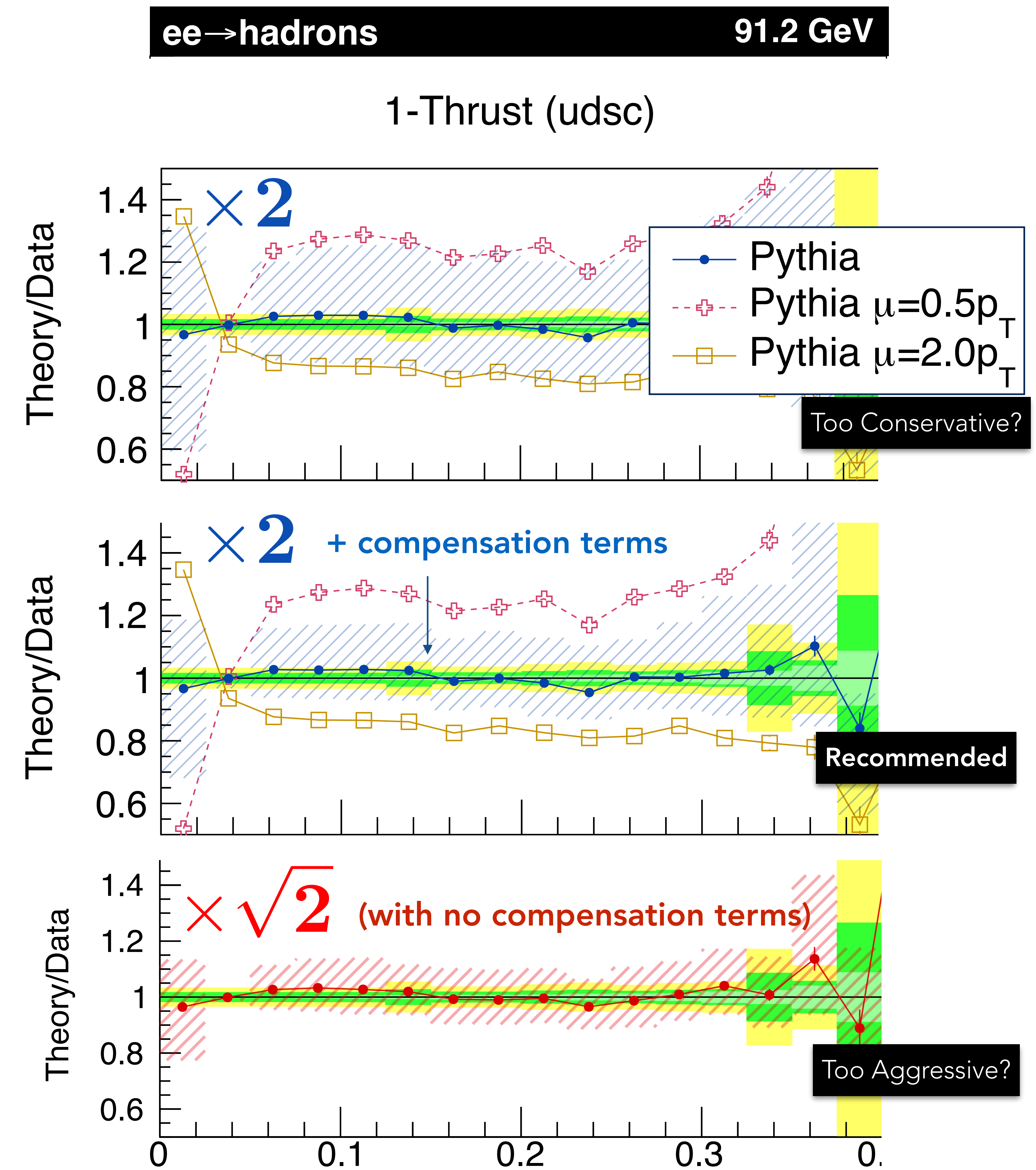
Since aggressive definitions can lead to overcompensation / **extremely** optimistic predictions → *very* small uncertainty bands, we chose a rather conservative definition for PYTHIA → larger bands.

$$P'(t,z) = \frac{\alpha_s(kp_\perp)}{2\pi}\left(1 + (1-\zeta)\frac{\alpha_s(\mu_{\max})}{2\pi}\beta_0 \ln k\right)\frac{P(z)}{t}$$

Kills the compensation outside the soft limit

Small absolute size of compensation

$$\zeta = \begin{cases} z & \text{for splittings with a } 1/z \text{ singularity} \\ 1-z & \text{for splittings with a } 1/(1-z) \text{ singularity} \\ \min(z, 1-z) & \text{for splittings with a } 1/(z(1-z)) \text{ singularity} \end{cases}$$



S. Mrenna & PS: PRD94(2016)074005; arXiv:1605.08352

Matrix-Element Merging — The Complexity Bottleneck

For CKKW-L style merging: (incl UMEPS, NL3, UNLOPS, …)

Need to take **all contributing shower histories into account**.

In conventional parton showers (Pythia, Herwig, Sherpa, …)

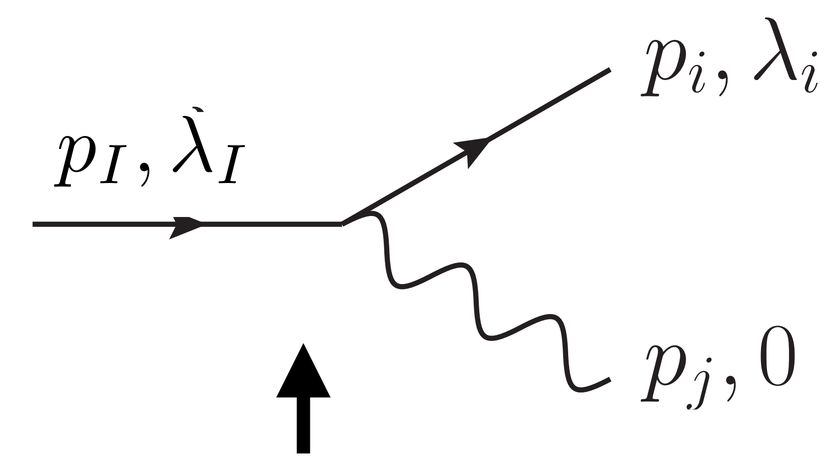
Each phase-space point receives contributions from many possible branching "histories" (aka "clusterings")

# of histories grows ~ # of Feynman Diagrams, **faster than factorial**

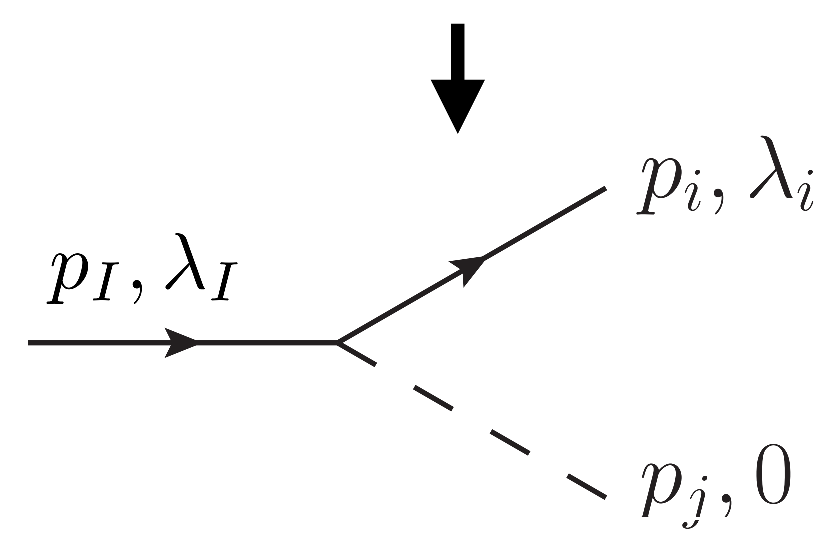
Number of Histories for n Branchings

Starting from a single $q\bar{q}$ pair	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
CS Dipole	2	8	48	384	3840	46080	645120

Bottleneck for merging at high multiplicities (+ high code complexity)



$$\epsilon_0^\mu(p) = \frac{1}{m}\left(p^\mu - \frac{m^2}{p\cdot k}k^\mu\right)$$



Lots of Antenna Functions

$$a^{FF}_{f_\lambda \mapsto f_\lambda V_\lambda} = 2(v-\lambda a)^2 \frac{\tilde{m}^2_{ij}}{(m^2_{ij}-m^2_I)^2}\frac{1}{x_j}$$

$$a^{FF}_{f_\lambda \mapsto f_\lambda V_{-\lambda}} = 2(v-\lambda a)^2 \frac{\tilde{m}^2_{ij}}{(m^2_{ij}-m^2_I)^2}\frac{x_i^2}{x_j}$$

$$a^{FF}_{f_\lambda \mapsto f_{-\lambda} V_\lambda} = 2\frac{1}{(m^2_{ij}-m^2_I)^2}\left((v-\lambda a)m_i\frac{1}{\sqrt{x_i}} - (v+\lambda a)m_I\sqrt{x_i}\right)^2$$

$$a^{FF}_{f_\lambda \mapsto f_\lambda V_0} = \frac{1}{(m^2_{ij}-m^2_I)^2}\left[(v-\lambda a)\left(\frac{m_I^2}{m_j}\sqrt{x_i} - \frac{m_i^2}{m_j}\frac{1}{\sqrt{x_i}} - 2m_j\frac{\sqrt{x_i}}{x_j}\right) + (v+\lambda a)\frac{m_I m_i}{m_j}\frac{x_j}{\sqrt{x_i}}\right]^2$$

$$a^{FF}_{f_\lambda \mapsto f_{-\lambda} V_0} = \frac{(m_I(v+\lambda a) - m_i(v-\lambda a))^2}{m_j^2}\frac{\tilde{m}^2_{ij}}{(m^2_{ij}-m^2_I)^2}x_j.$$

$$a^{FF}_{V_\lambda \mapsto f_\lambda \bar{f}_{-\lambda}} = 2(v-\lambda a)^2 \frac{\tilde{m}^2_{ij}}{(m^2_{ij}-m^2_I)^2}x_j^2$$

$$a^{FF}_{V_\lambda \mapsto f_{-\lambda} \bar{f}_\lambda} = 2(v+\lambda a)^2 \frac{\tilde{m}^2_{ij}}{(m^2_{ij}-m^2_I)^2}x_i^2$$

$$a^{FF}_{V_\lambda \mapsto f_{-\lambda} \bar{f}_{-\lambda}} = 2\frac{1}{(m^2_{ij}-m^2_I)^2}\left((v+\lambda a)m_i\sqrt{\frac{x_j}{x_i}} + (v-\lambda a)m_j\sqrt{\frac{x_i}{x_j}}\right)^2$$

$$a^{FF}_{V_0 \mapsto f_\lambda \bar{f}_\lambda} = \frac{((v+\lambda a)m_i - (v-\lambda a)m_j)^2}{m_I^2}\frac{\tilde{m}^2_{ij}}{(m^2_{ij}-m^2_I)^2}$$

$$a^{FF}_{V_0 \mapsto f_\lambda \bar{f}_{-\lambda}} = \frac{1}{(m^2_{ij}-m^2_I)^2} \times \left[(v-\lambda a)\left(2m_I\sqrt{x_i x_j} - \frac{m_i^2}{m_I}\sqrt{\frac{x_j}{x_i}} - \frac{m_j^2}{m_I}\sqrt{\frac{x_i}{x_j}}\right) + (v+\lambda a)\frac{m_i m_j}{m}\frac{1}{\sqrt{x_i x_j}}\right]^2.$$

$$a^{FF}_{f_\lambda f_\lambda H} = \frac{e^2}{4s_w^2}\frac{m_i^4}{s_w^2}\frac{1}{(m^2_{ij}-m^2_I)^2}\left(\sqrt{x_i} + \frac{1}{\sqrt{x_i}}\right)^2$$

$$a^{FF}_{f_\lambda f_{-\lambda} H} = \frac{e^2}{4s_w^2}\frac{m_i^2}{s_w^2}\frac{\tilde{m}^2_{ij}}{(m^2_{ij}-m^2_I)^2}x_j.$$

$$a^{FF}_{V_\lambda \mapsto V_\lambda V_\lambda} = 2g_v^2\frac{\tilde{m}^2_{ij}}{(m^2_{ij}-m^2_I)^2}\frac{1}{x_i x_j}$$

$$a^{FF}_{V_\lambda \mapsto V_\lambda V_{-\lambda}} = 2g_v^2\frac{\tilde{m}^2_{ij}}{(m^2_{ij}-m^2_I)^2}\frac{x_i^3}{x_j}$$

$$a^{FF}_{V_\lambda \mapsto V_{-\lambda} V_\lambda} = 2g_v^2\frac{\tilde{m}^2_{ij}}{(m^2_{ij}-m^2_I)^2}\frac{x_j^3}{x_i}$$

$$a^{FF}_{V_\lambda \mapsto V_\lambda V_0} = g_v^2\frac{1}{(m^2_{ij}-m^2_I)^2}\frac{(m_I^2 - m_i^2 - \frac{1+x_i}{x_j}m_j^2)^2}{m_j^2}$$

$$a^{FF}_{V_\lambda \mapsto V_0 V_\lambda} = g_v^2\frac{1}{(m^2_{ij}-m^2_I)^2}\frac{(m_I^2 - m_j^2 - \frac{1+x_j}{x_i}m_i^2)^2}{m_i^2}$$

$$a^{FF}_{V_\lambda \mapsto V_0 V_0} = \frac{g_v^2}{2}\frac{(m_I^2 - m_i^2 - m_j^2)^2}{m_i^2 m_j^2}\frac{\tilde{m}^2_{ij}}{(m^2_{ij}-m^2_I)^2}x_i x_j.$$

$$a^{FF}_{V_\lambda \mapsto V_\lambda H} = \frac{e^2}{s_w^2}\frac{m_v^4}{m_w^2}\frac{1}{(m^2_{ij}-m^2_I)^2}$$

$$a^{FF}_{V_\lambda \mapsto V_0 H} = \frac{e^2}{2s_w^2}\frac{m_v^2}{m_w^2}\frac{\tilde{m}^2_{ij}}{(m^2_{ij}-m^2_I)^2}x_i x_j$$

$$a^{FF}_{V_0 \mapsto V_\lambda H} = \frac{e^2}{2s_w^2}\frac{m_v^2}{m_w^2}\frac{\tilde{m}^2_{ij}}{(m^2_{ij}-m^2_I)^2}\frac{x_j}{x_i}$$

$$a^{FF}_{V_0 \mapsto V_0 H} = \frac{e^2}{4s_w^2}\frac{1}{m_w^2}\frac{1}{(m^2_{ij}-m^2_I)^2}\left(m_I^2 - 2m_i^2\left(x_i + \frac{1}{x_i}\right)\right)^2.$$

$$a^{FF}_{H \mapsto f_\lambda \bar{f}_\lambda} = \frac{e^2}{4s_w^2}\frac{m_i^2}{s_w^2}\frac{\tilde{m}^2_{ij}}{(m^2_{ij}-m^2_I)^2}$$

$$a^{FF}_{H \mapsto f_\lambda \bar{f}_{-\lambda}} = \frac{e^2}{4s_w^2}\frac{m_i^4}{s_w^2}\frac{1}{(m^2_{ij}-m^2_I)^2}\left(\sqrt{\frac{x_i}{x_j}} - \sqrt{\frac{x_j}{x_i}}\right)^2.$$

Collinear Limits

$$\tilde{m}_{ij}^2 = m_{ij}^2 - \frac{m_i^2}{z^2} - \frac{m_j^2}{(1-z)^2}$$

λ_I	λ_i	λ_j	$f \to f'V$
λ	λ	λ	$2(v-\lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}\frac{1}{1-z}$
λ	λ	$-\lambda$	$2(v-\lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}\frac{z^2}{1-z}$
λ	$-\lambda$	λ	$2\frac{1}{(m_{ij}^2-m_I^2)^2}\left(m_I(v-\lambda a)\sqrt{z} - m_i(v+\lambda a)\frac{1}{\sqrt{z}}\right)^2$
λ	$-\lambda$	$-\lambda$	0
λ	λ	0	$\frac{1}{(m_{ij}^2-m_I^2)^2}\left[(v-\lambda a)\left(\frac{m_I^2}{m_j}\sqrt{z} - \frac{m_i^2}{m_j}\frac{1}{\sqrt{z}} - 2m_j\frac{\sqrt{z}}{1-z}\right) + (v+\lambda a)\frac{m_i m_I}{m_j}\frac{1-z}{\sqrt{z}}\right]^2$
λ	$-\lambda$	0	$\frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}(1-z)\left(\frac{m_i}{m_j}(v-\lambda a) - \frac{m_I}{m_j}(v+\lambda a)\right)^2$

Rows 1–2 → $P(z) \propto \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2}\frac{1+z^2}{1-z}$ — Pure vector

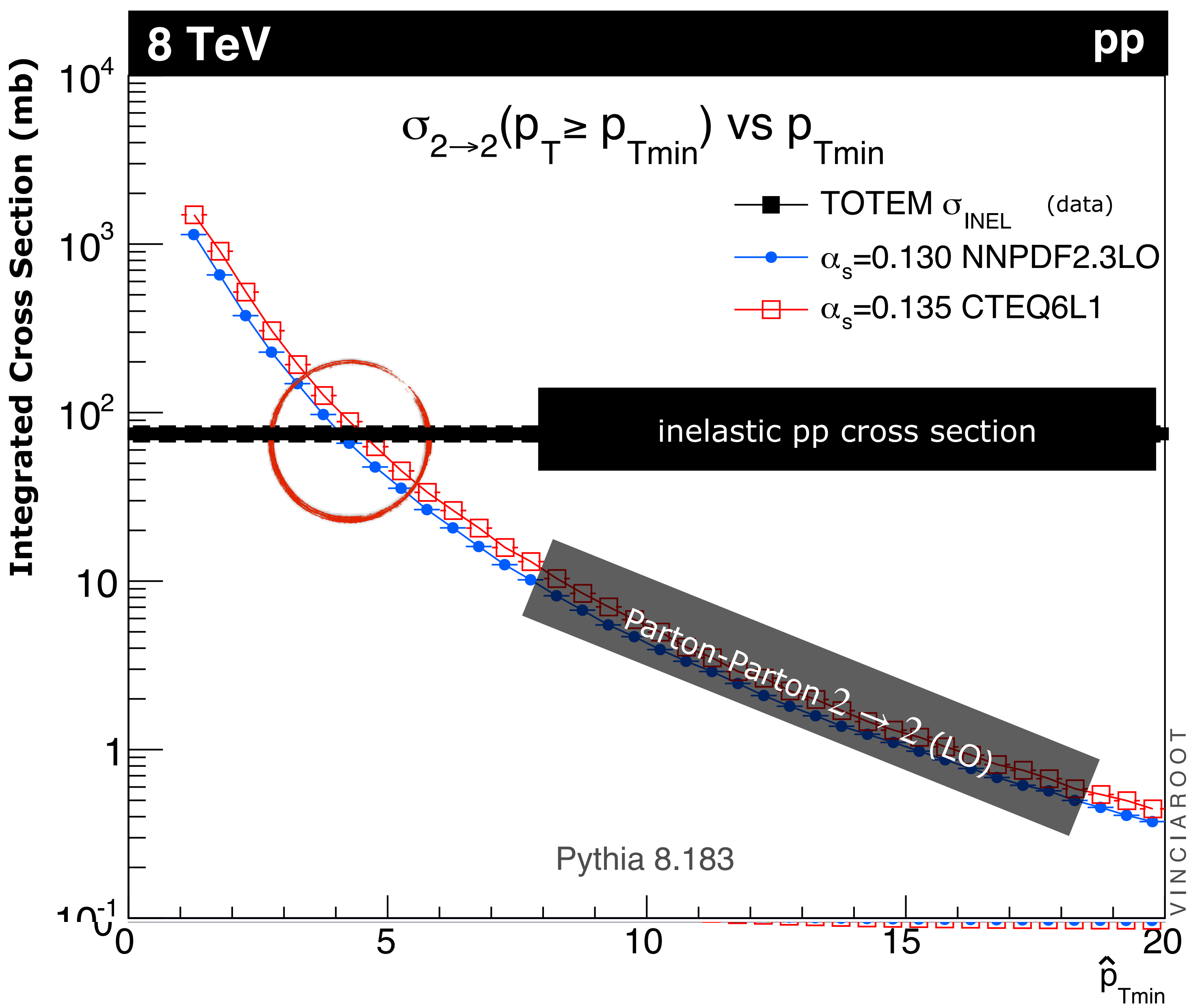
Row 3 — Pure vector; rows 3 and 5 → $P(z) \propto \frac{m^2}{(m_{ij}^2 - m_I^2)^2}$

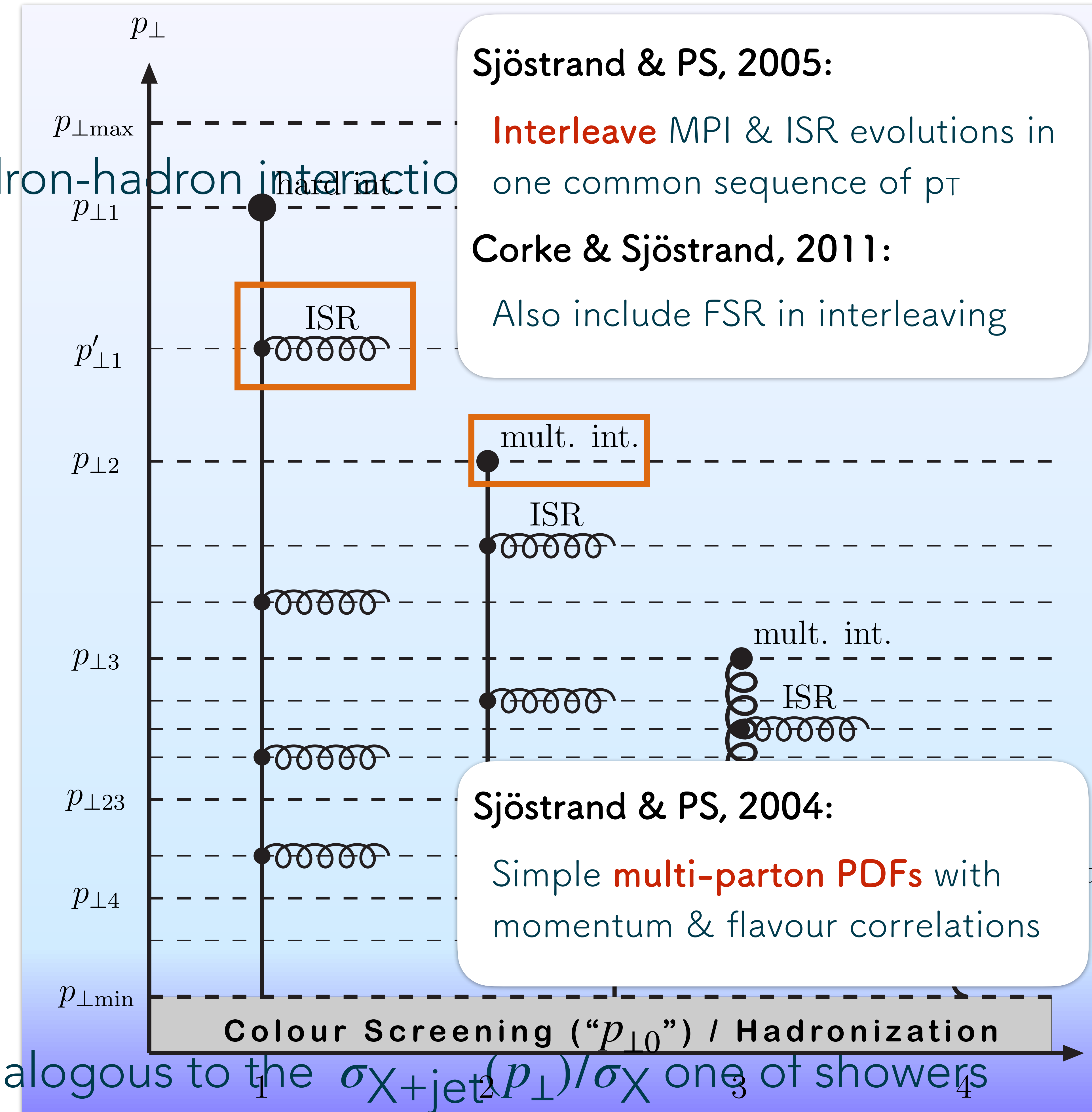
Row 5 — Vector + Scalar

Row 6 → $P(z) \propto \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2}(1-z)$ — Pure scalar

$$\frac{\sigma_{\text{parton-parton}}(\hat{p}_\perp)}{\sigma_{\text{hadron-hadron}}} > 1$$

$\Longrightarrow$ several parton-parton interactions *per* hadron-hadron interactio





Sjöstrand & PS, 2005:

Interleave MPI & ISR evolutions in one common sequence of pT

Corke & Sjöstrand, 2011:

Also include FSR in interleaving

Sjöstrand & PS, 2004:

Simple **multi-parton PDFs** with momentum & flavour correlations

Figure from Sjöstrand & PS, 2005

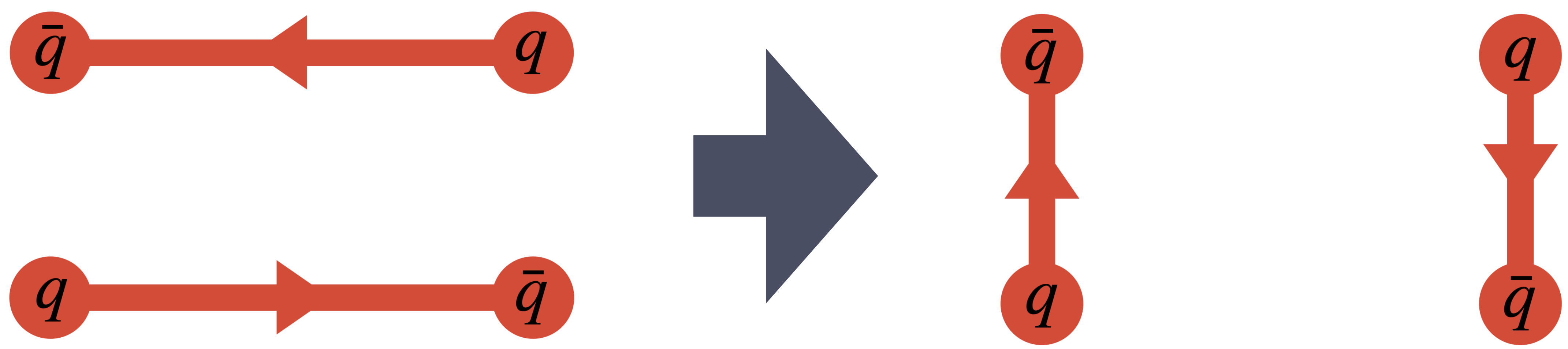
Sjöstrand & van Zijl, 1985:

Cast as **Sudakov-style evolution equation**, analogous to the $\sigma_{X+jet}(p_\perp)/\sigma_X$ one of showers

Stochastically restores colour-space ambiguities according to **SU(3) algebra**

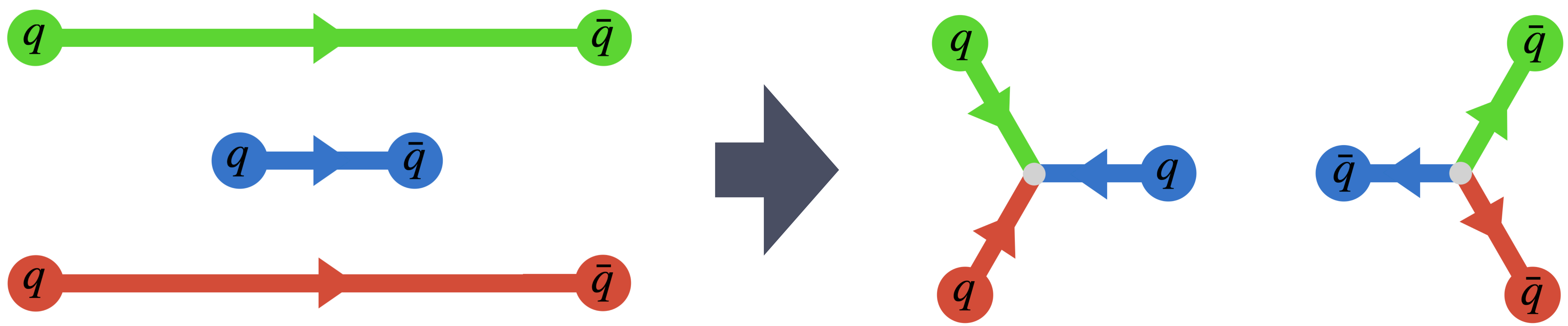
[Christiansen & **PS** JHEP 08 (2015) 003]

➢ Allows for reconnections to minimise string lengths



Dipole-type reconnection

What about the red-green-blue colour singlet state?



Junctions!

In Progress: Strangeness Enhancement from Close-Packing

Idea: each string exists in an effective background produced by the

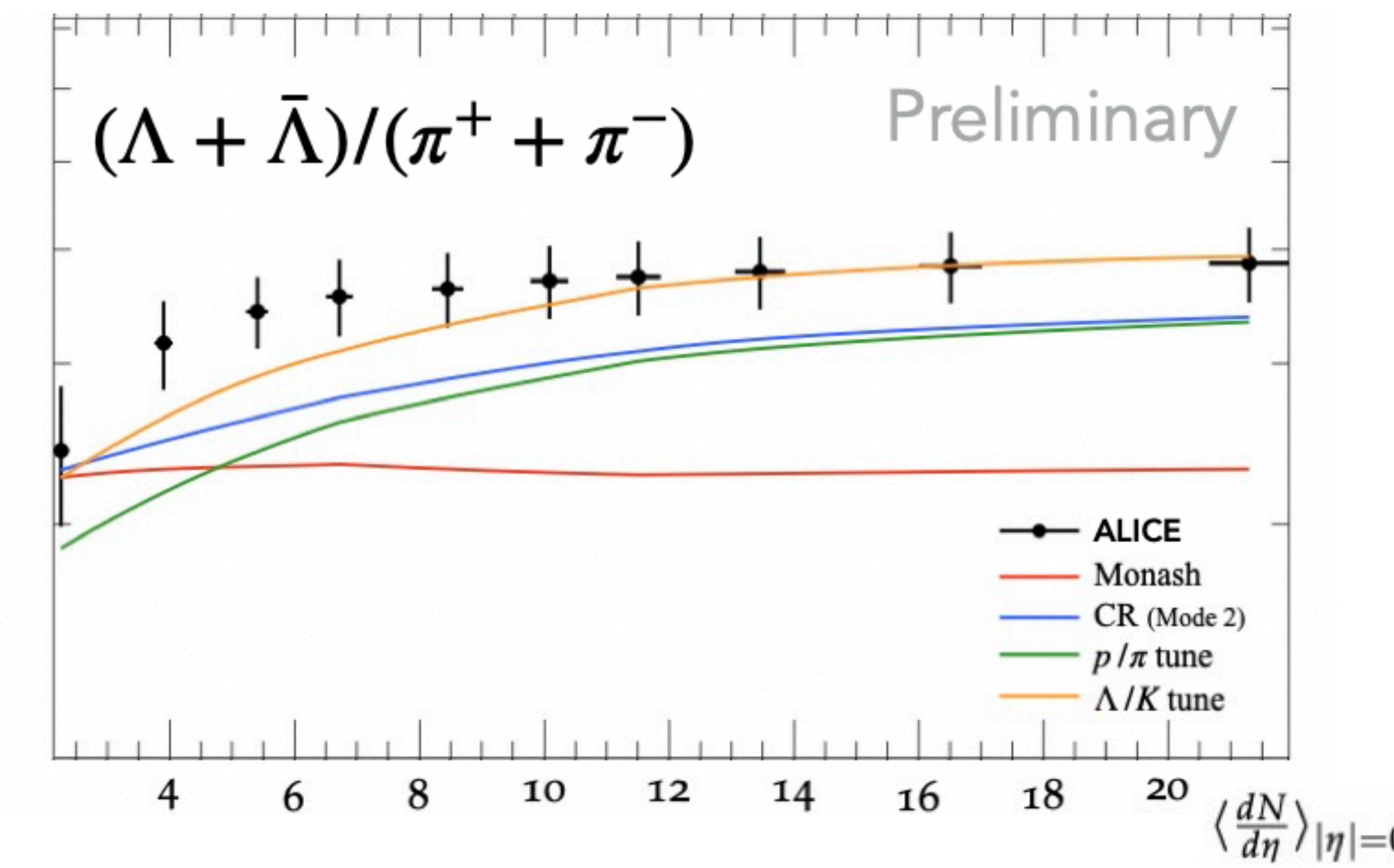
Close-packing

$C_8 = 2.25 C_F$ $C_6 = 2.5 C_F$

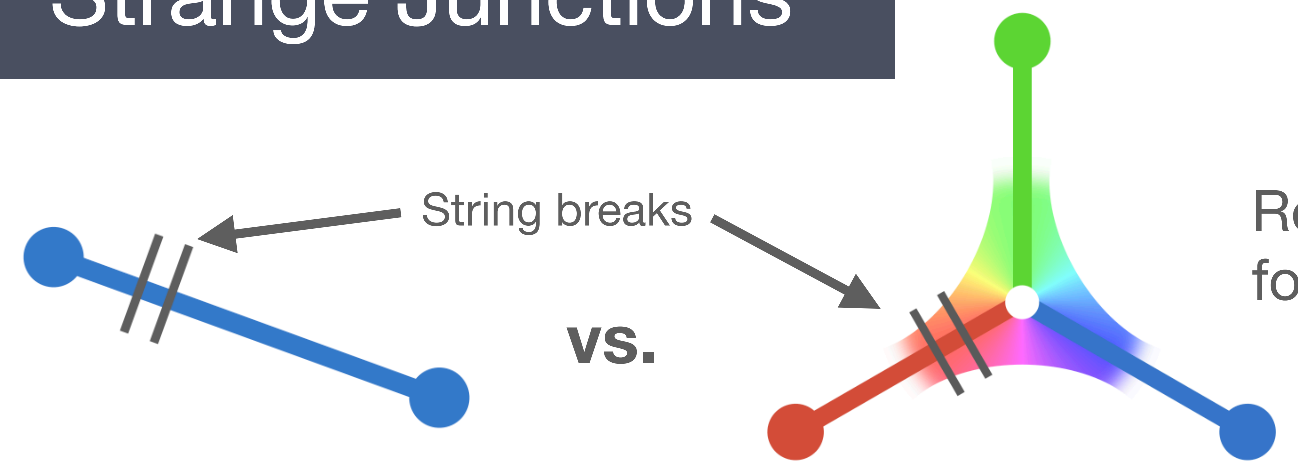
Dense string environments

→ Casimir scaling of **effective string tension**

→ Higher probability of strange quarks

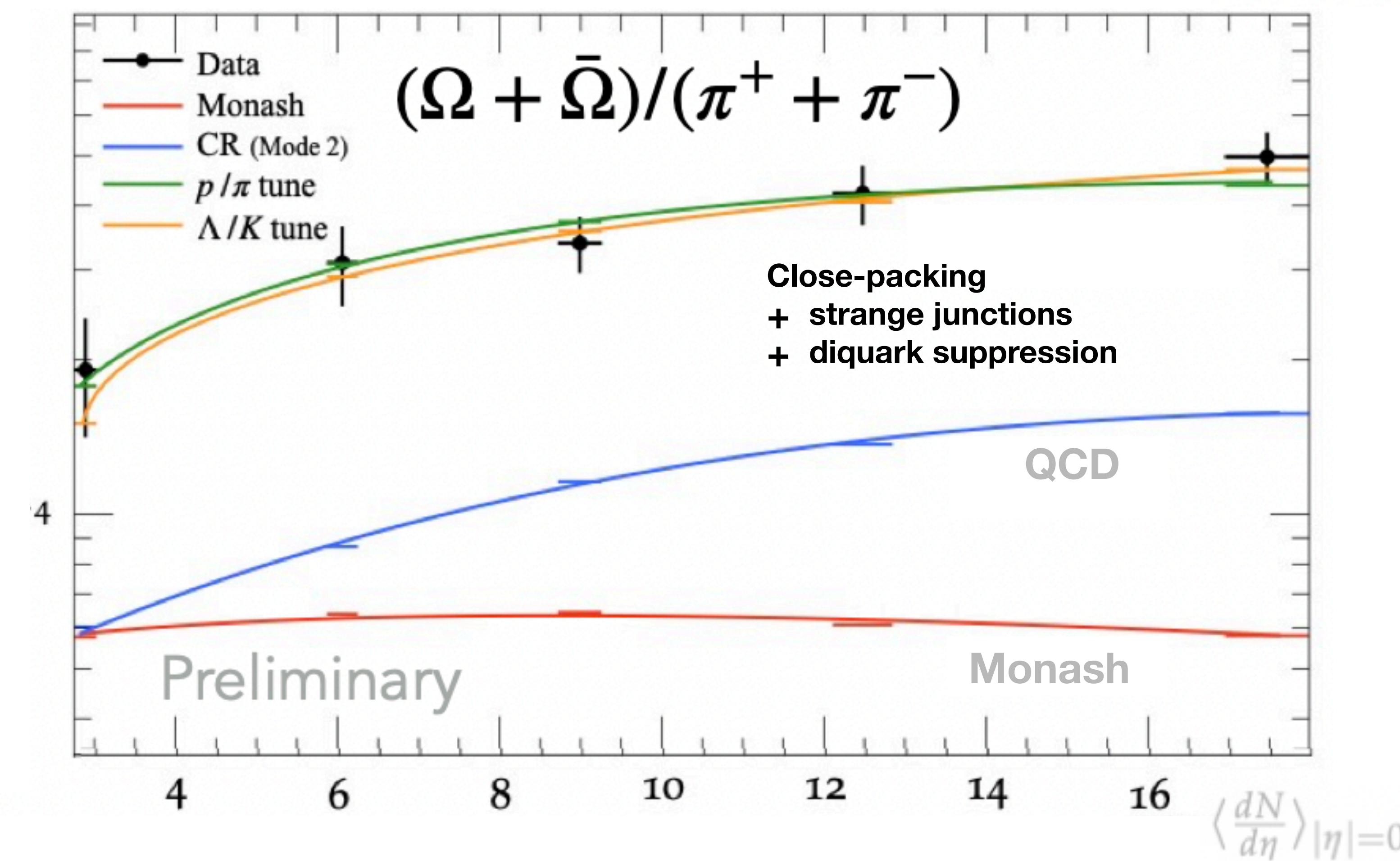


Strange Junctions



Results in strangeness enhancement focused in baryon sector

String tension could be different from the vacuum case compared to near a junction

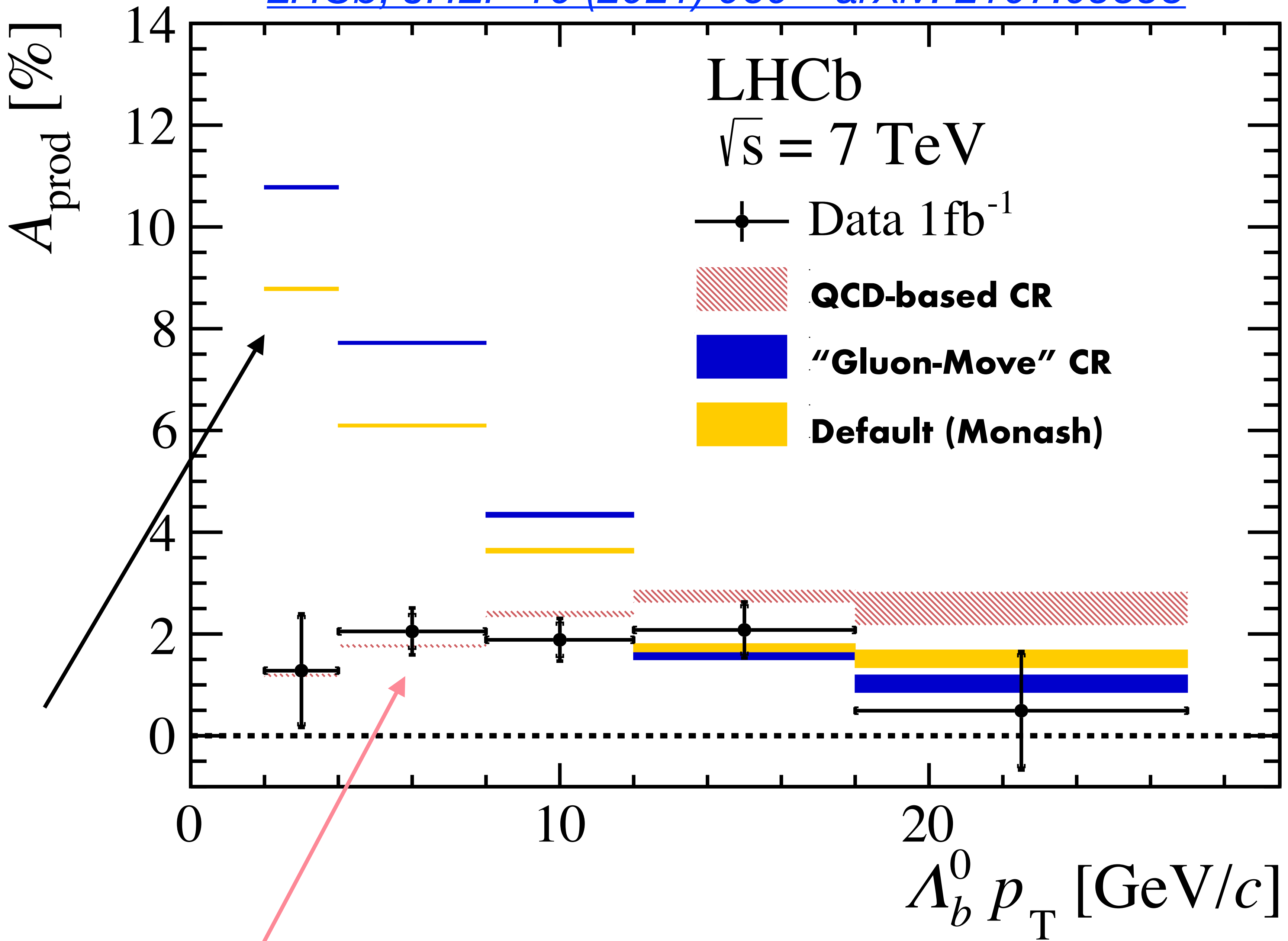


Slide adapted from J. Altmann

Λ_b asymmetry

$$A = \frac{\sigma(\Lambda_b^0) - \sigma(\overline{\Lambda}_b^0)}{\sigma(\Lambda_b^0) + \sigma(\overline{\Lambda}_b^0)}$$

LHCb, JHEP 10 (2021) 060 • arXiv: 2107.09593



Without junction CR, an important source of low-pT Λ_b production is when a b quark combines with the proton beam remnant.

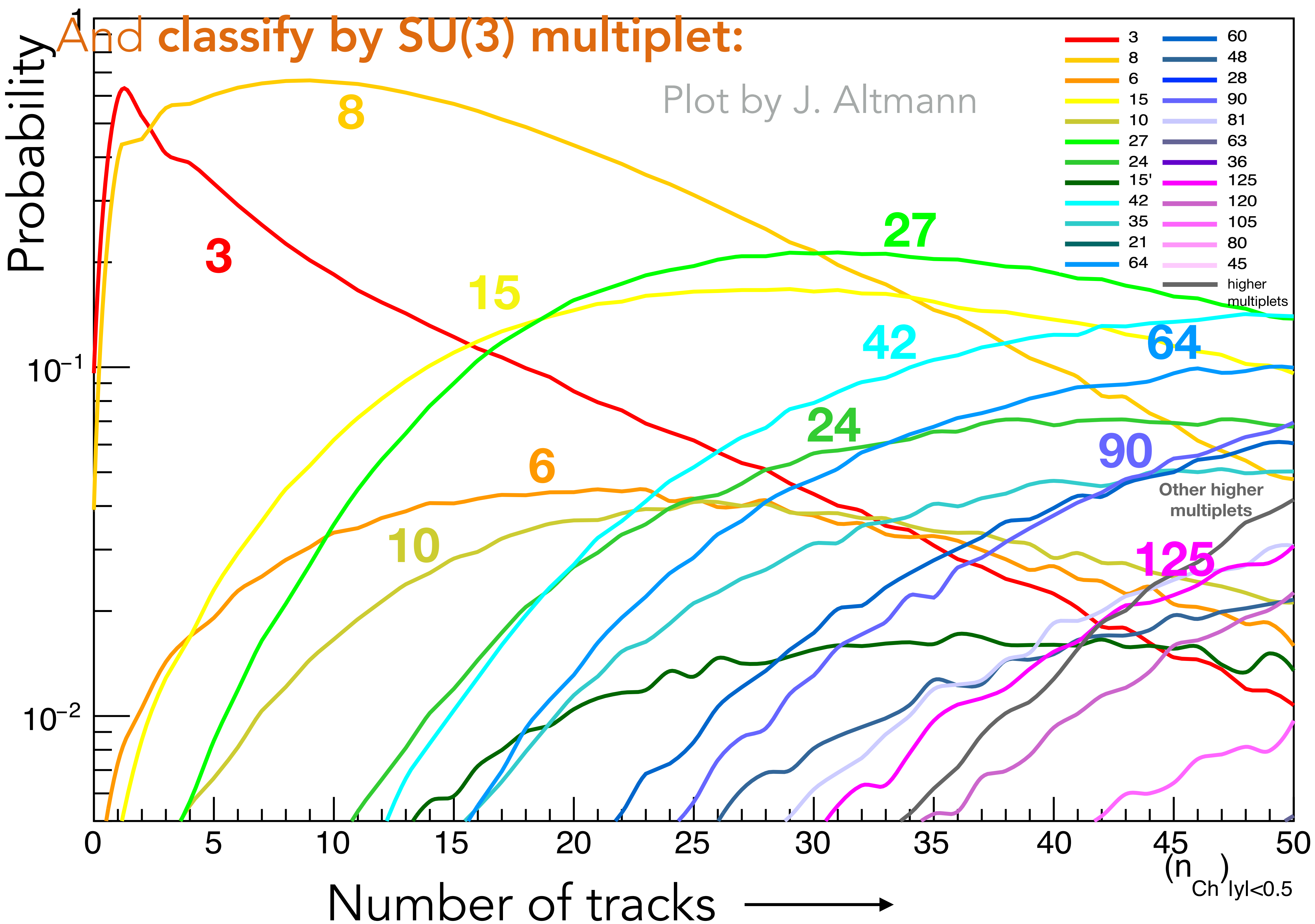
Not possible for $\bar{\Lambda}_b$ (no $\bar{p}$ remnant at LHC)

QCD CR adds large amount of low-pT junction Λ_b and $\bar{\Lambda}_b$, in equal amounts. Dilutes asymmetry!

MPI $\Longrightarrow$ lots of coloured partons scattered into the final states

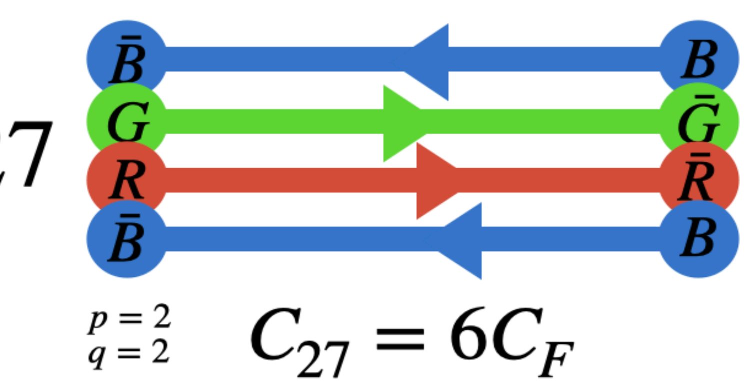
Count **# of (oriented) flux lines** crossing $y = 0$ in pp collisions (according to PYTHIA)

And **classify by SU(3) multiplet:**



Confining fields may be reaching **higher effective representations** than simple quark-antiquark (3) ones.

E.g.: 27



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Two approaches in PYTHIA:

1) Colour Ropes (Lund)

2) Close-Packing (Monash)