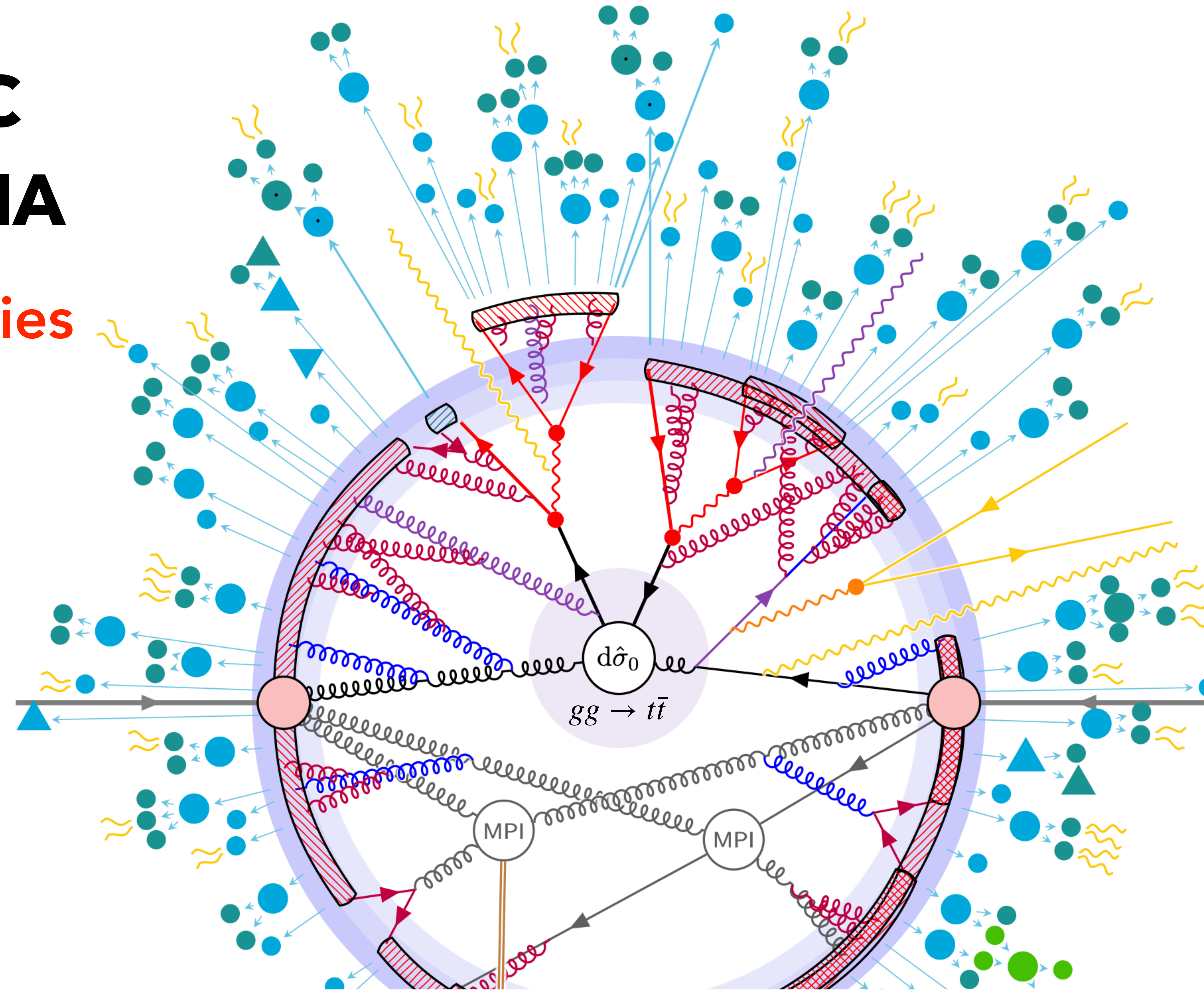
Modelling of LHC Collisions in PYTHIA

— Physics & Uncertainties

Peter Z Skands

University of Oxford
& Monash University

Oxford, April 2024



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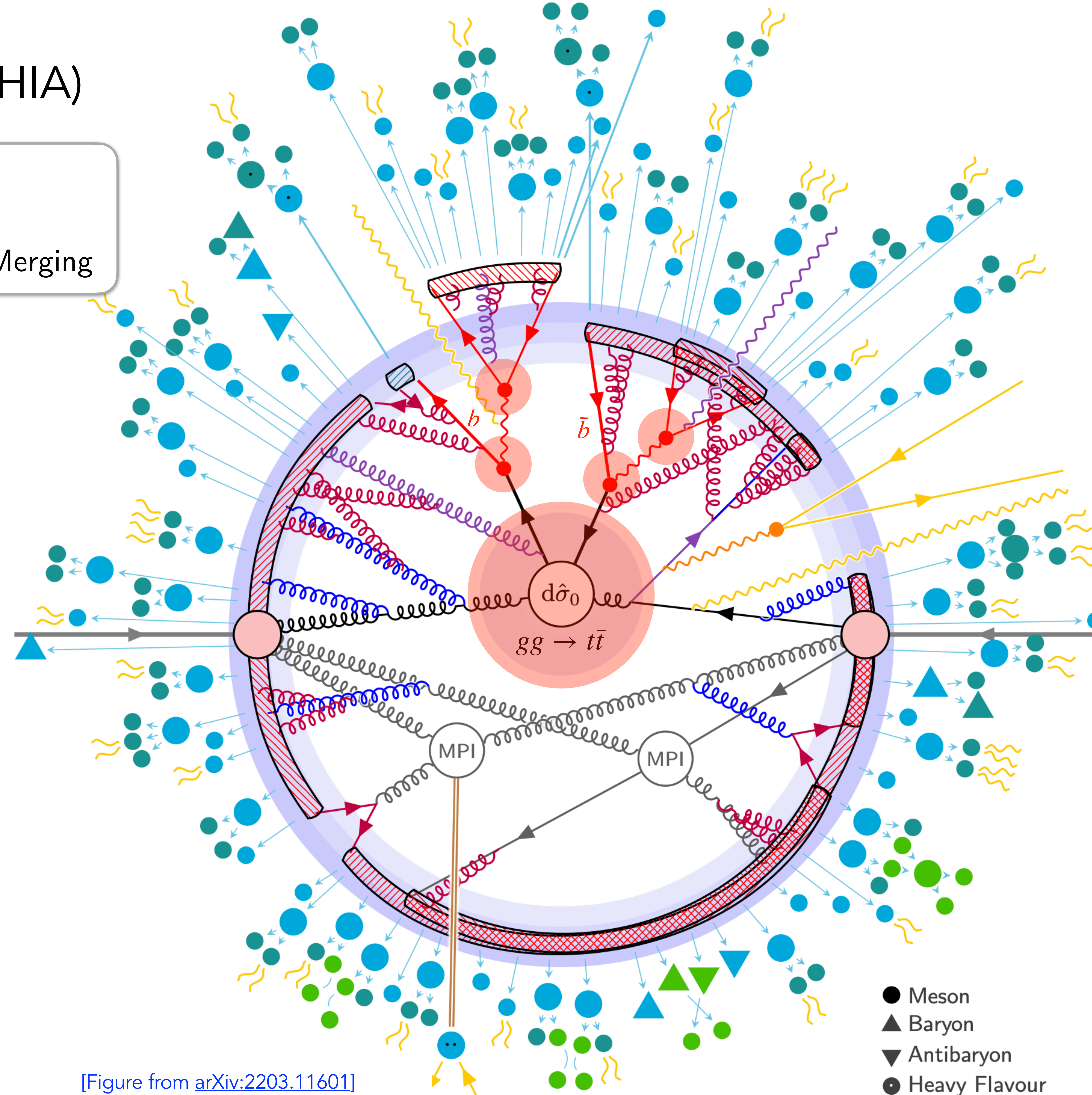
MCnet

An LHC collision (in PYTHIA)

Hard Process	○ Hard Interaction
	● Resonance Decays
	■ MECs, Matching & Merging



[Figure from arXiv:2203.11601]

An LHC collision (in PYTHIA)

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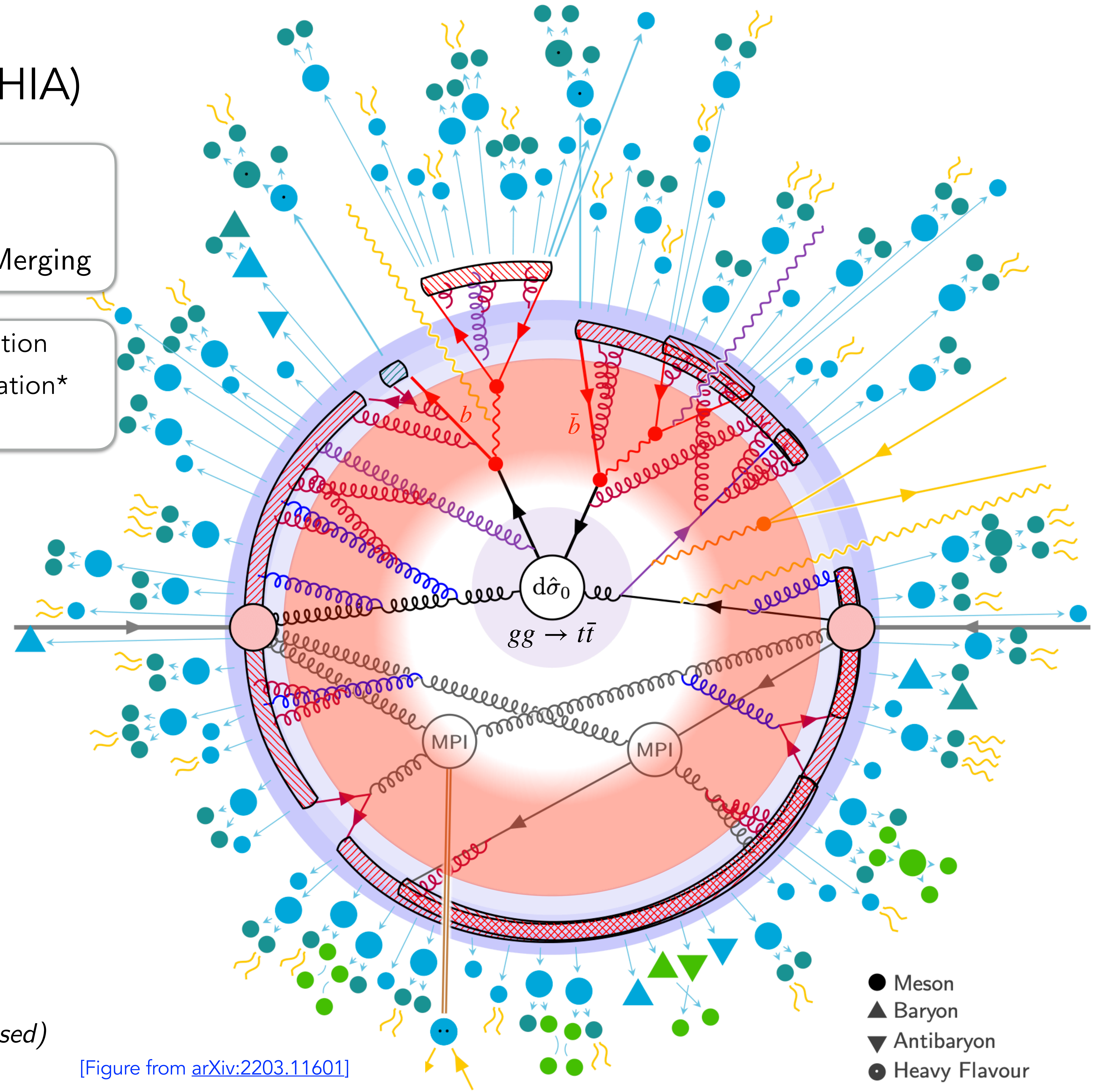
Parton Showers	
	■ QCD Final-State Radiation
	■ QCD Initial-State Radiation*
	■ Electroweak Radiation



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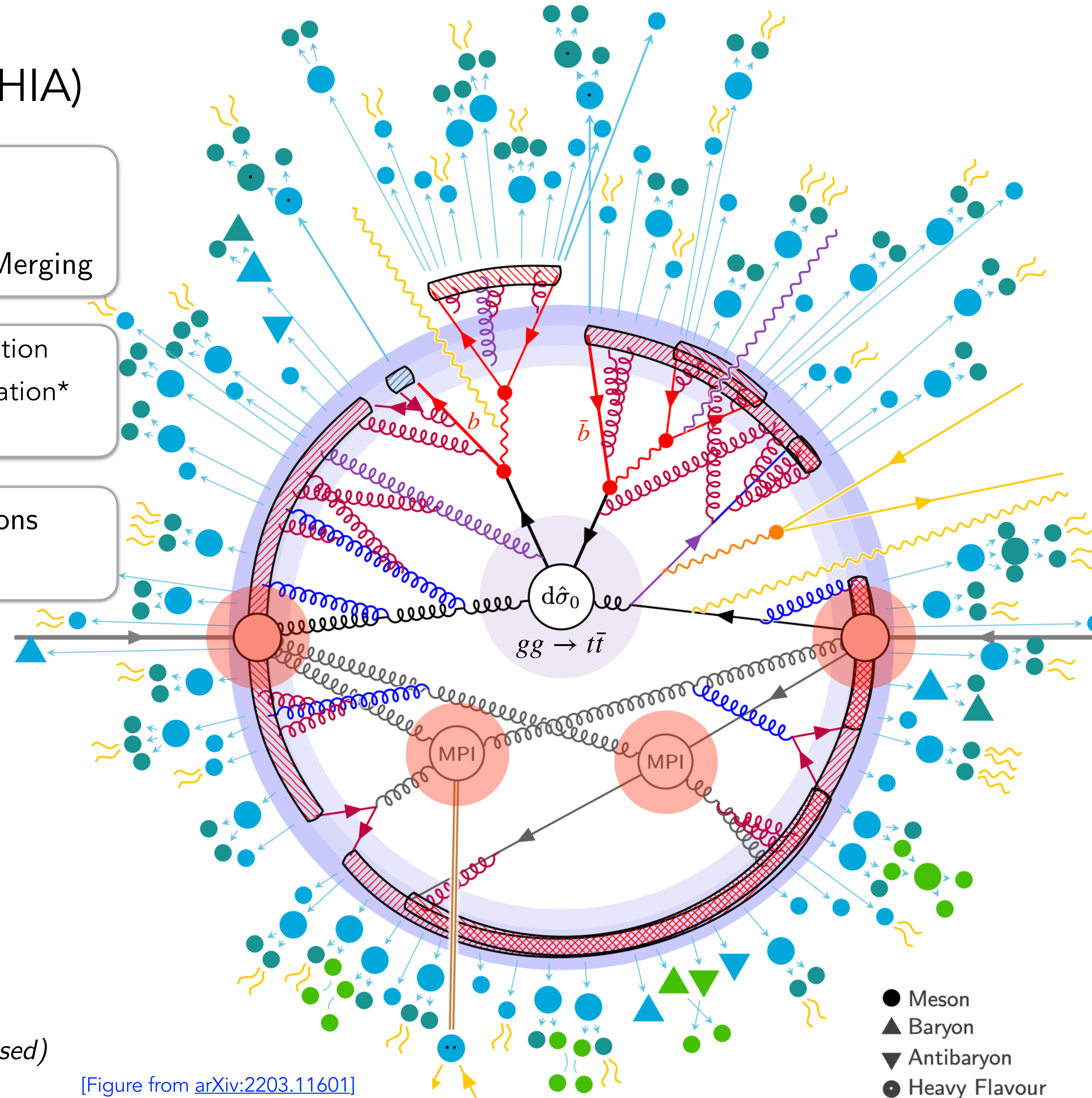
Underlying Event
- ◯ Multiparton Interactions
- □ Beam Remnants*



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An LHC collision (in PYTHIA)

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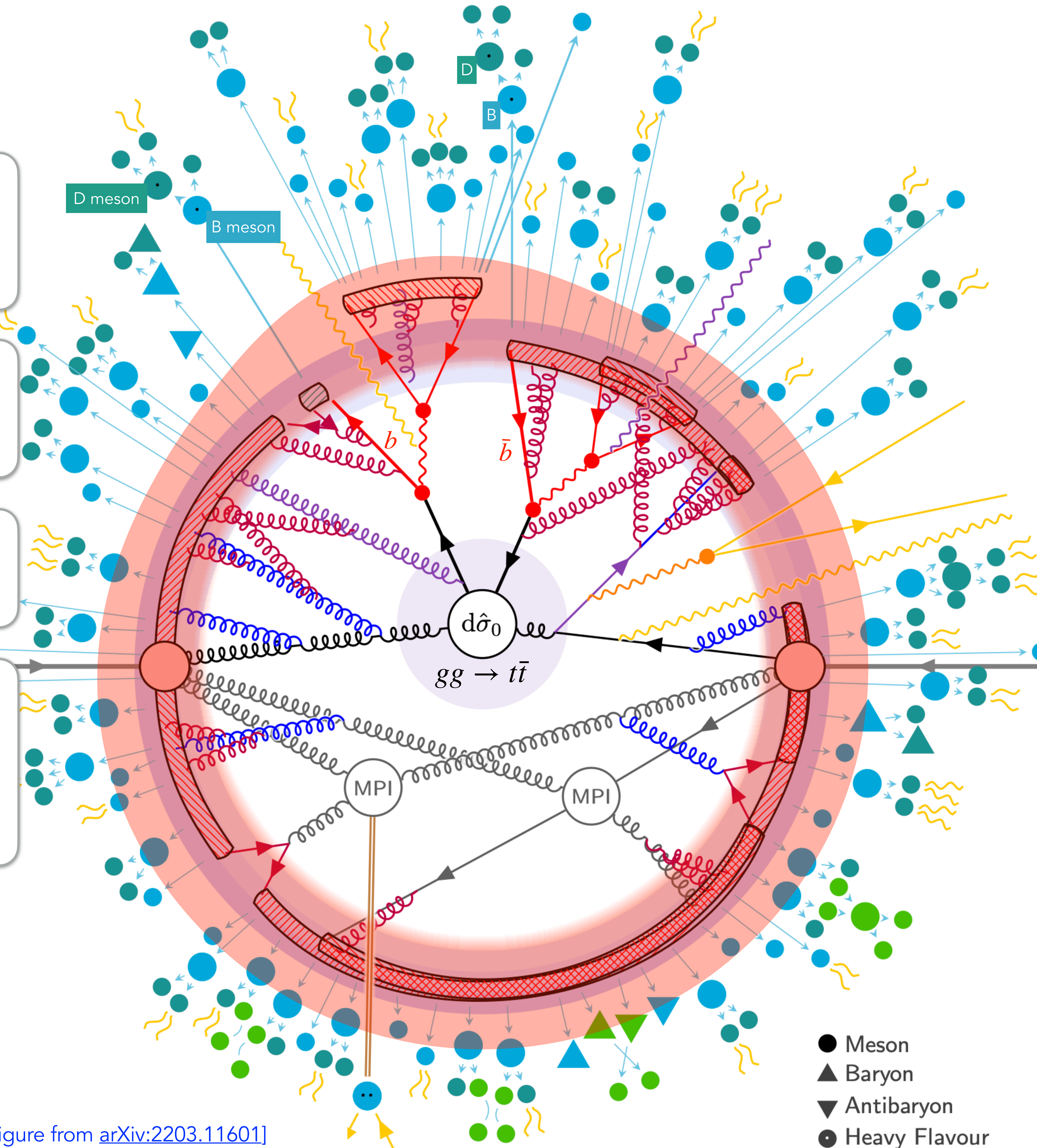
Hadronization	▧ Strings ■ Colour Reconnections ■ String Interactions ■ Bose-Einstein & Fermi-Dirac



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An LHC collision (in PYTHIA)

Hard Process	◯ Hard Interaction ● Resonance Decays ■ MECs, Matching & Merging
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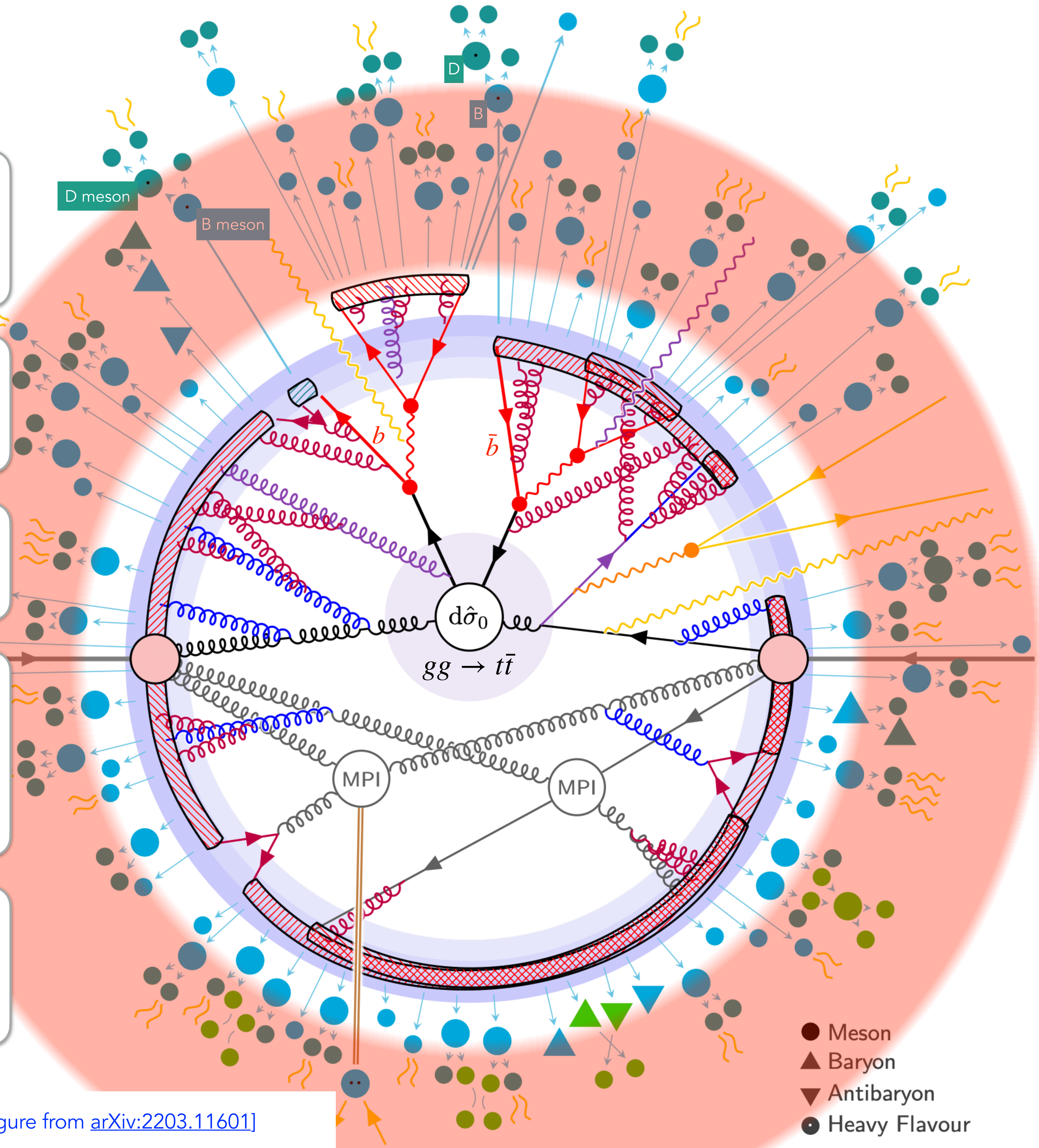
Hadronization	Strings Colour Reconnections String Interactions Bose-Einstein & Fermi-Dirac
Hadron (& τ) Decays	■ Primary Hadrons ■ Secondary Hadrons ■ Hadronic Reinteractions

(: incoming lines are crossed)*



[Figure from arXiv:2203.11601]

Recent Studies

Focus on SM precision environments ↔ BSM backgrounds

1. **NLO** Matching Systematics with POWHEG-Box *(examples: VBF, $t\bar{t}$)*
2. From NLO to **NNLO** *(examples: $t\bar{t}$, V, H, VH, VV, …)*
3. The computational bottleneck in **ME merging** *(example: V+jets)*
4. New Discoveries in **Hadronization** *(examples: HF baryons, JES)*

NB: want to address/explain state of the art & systematics in real contexts → a bit theory heavy

1. NLO + Shower with POWHEG

Nason 2004;
Fixione, Nason, Oleari 2007

(Just focusing on the real-radiation part)

POWHEG generates the first (hardest) emission with $|M_{X+1}|^2$

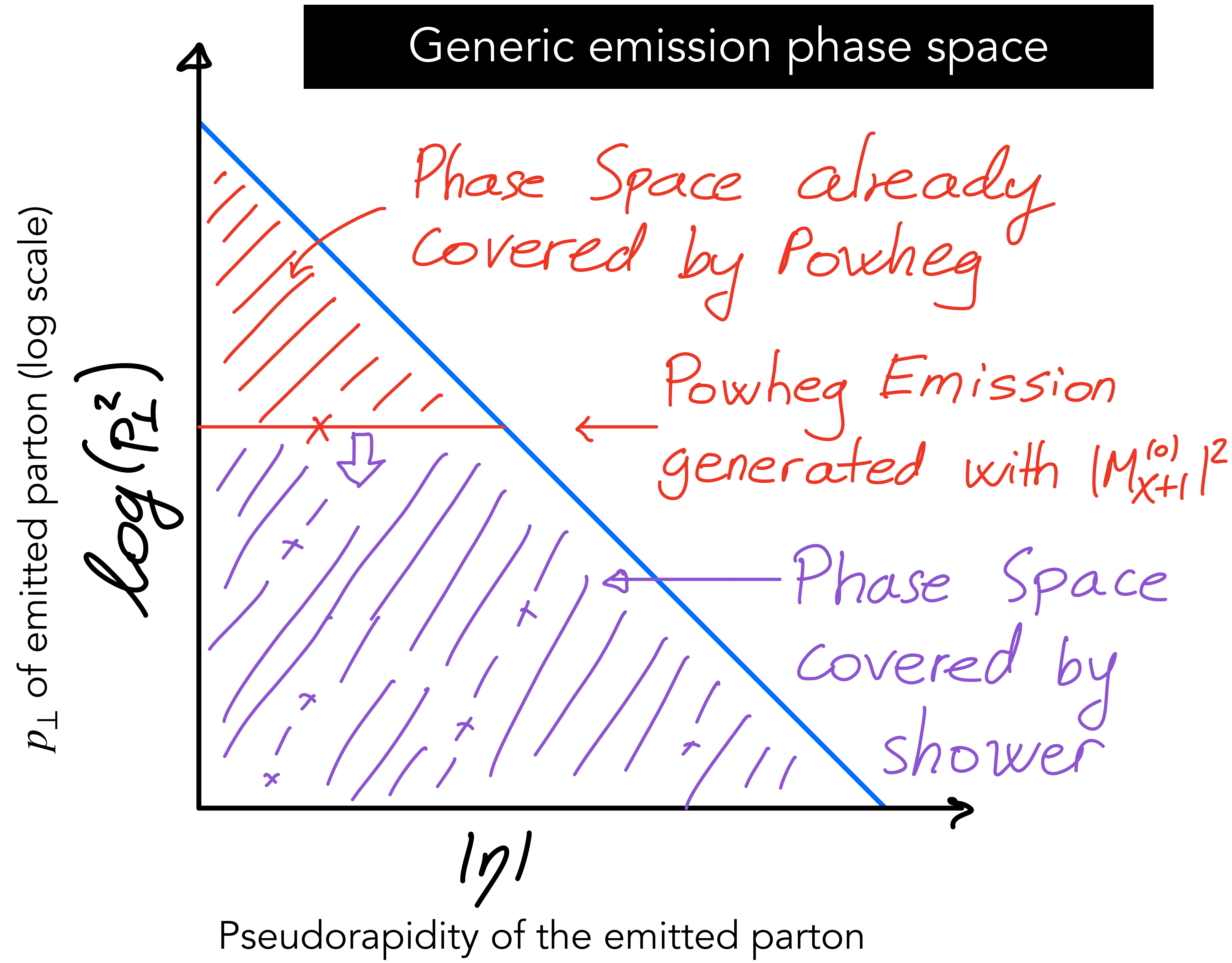
It does so in a shower-like manner: sweeping over phase space, from high to low p_T

Matrix-Element Corrections (MECs)
[Bengtsson & Sjöstrand 1987 + …]

+ NLO Born Normalization
[Nason 2004; Fixione, Nason, Oleari 2007]

Shower then takes over and generates all further emissions

Using soft/collinear approximations



Powheg Box — A Subtlety

[Alioli et al, 2010]

Industry Standard: "Powheg Box"

Exploits having its own definition of "p_T"

$\neq$ shower's definition of p_T

Breaks clean matching

Solution: Vetoed Showers

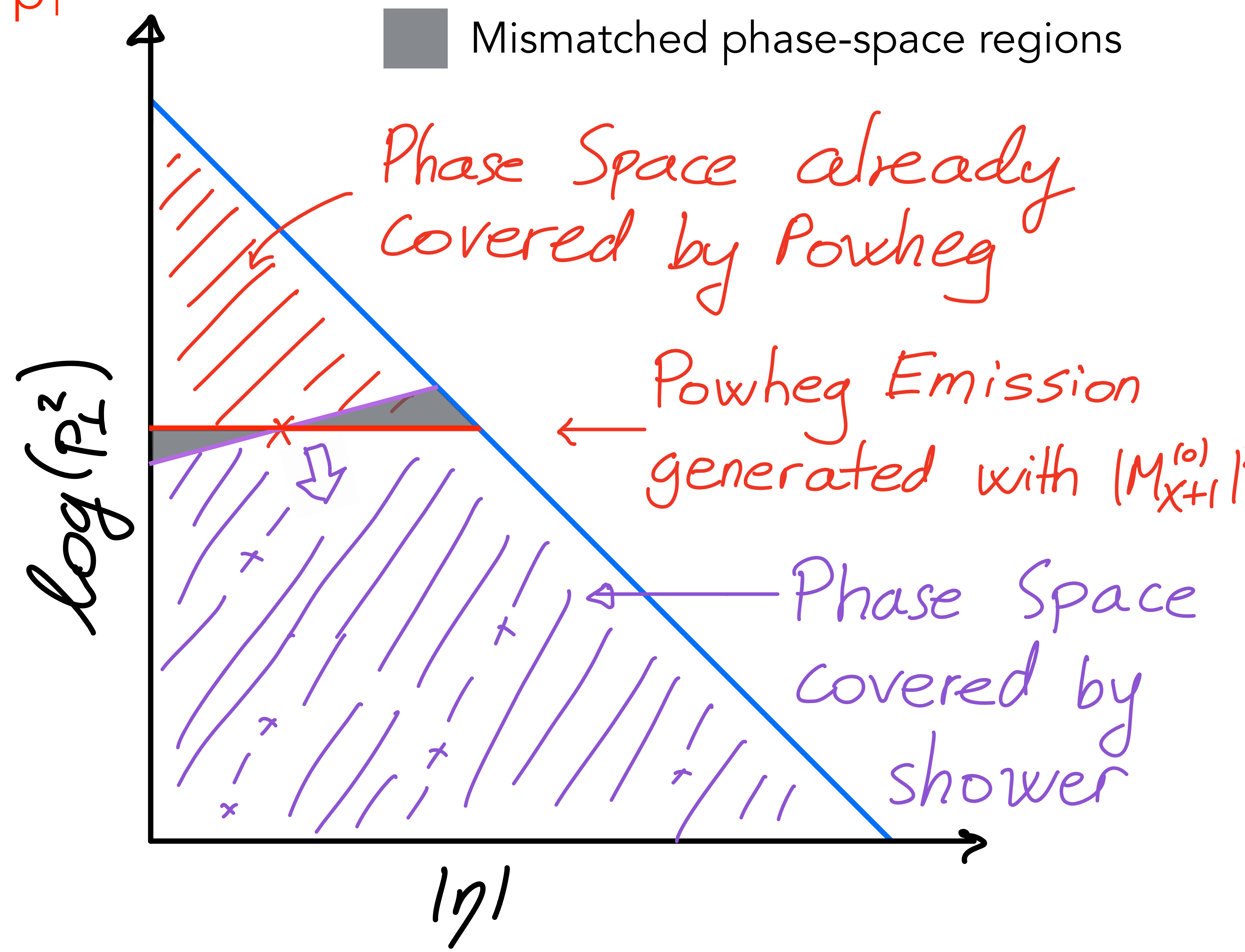
(+ truncated showers)

Works very well for simple cases

Induces an uncertainty/ambiguity

Purely associated with the matching scheme (not physical)

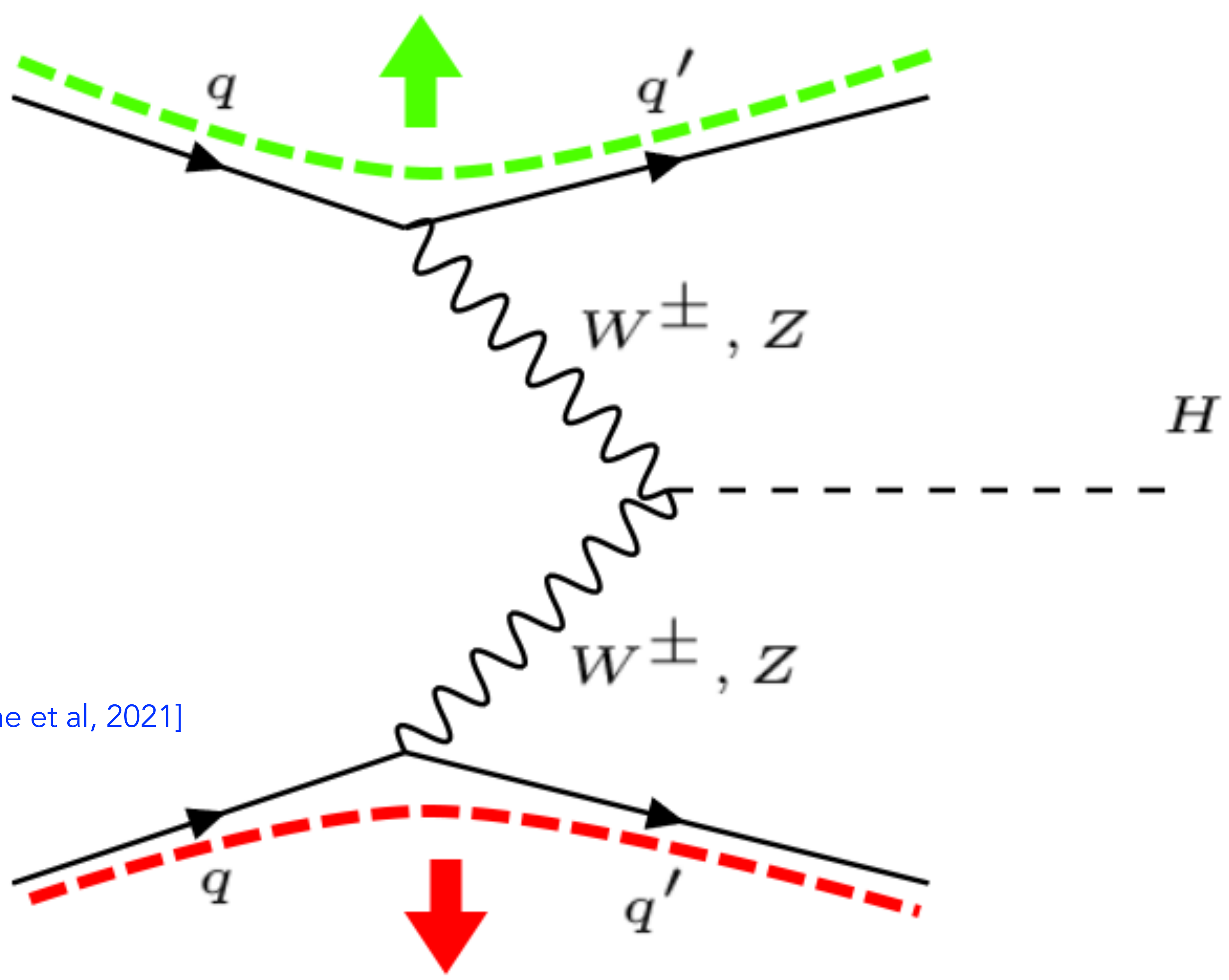
Can be important for complex / multi-scale processes.



E.g., Nason, Oleari arXiv:1303.3922
VBF: Höche et al., SciPost Phys. 12 (2022) 1

A More Complex Process

Vector boson fusion, $qq \to q'q'H$



[Höche et al, 2021]

Multiple emitters

⇝ several overlapping phase spaces

And many possible p_T definitions:

$p_\perp$ with respect to the beam

$p_\perp$ with respect to the final-state q' partons

$p_\perp$ with respect to either of the $(q_* q')$ dipoles (crossed)

$p_\perp$ with respect to the H?

(+ PYTHIA defines a problematic $(q'q')$ dipole)

+ Interpolations/combinations of the above …

Again, POWHEG-Box generates the first emission, which it judges to be the "hardest" according to its own p_T definition

Note: similar concerns for any process with coloured partons in the final state at Born level

$t\bar{t}$ (& $t \to bW$), V/H + jet(s), *dijets, trijets, …*

POWHEG-Box Matching Systematics

Varying the POWHEG-Box ↔ PYTHIA hardness-scale ambiguity

`POWHEG:pThard = 0` # Veto at $p_{\perp j;i}^{\mathrm{POWHEG}}$ = **SCALUP** = scale at which POWHEG says it emitted this parton

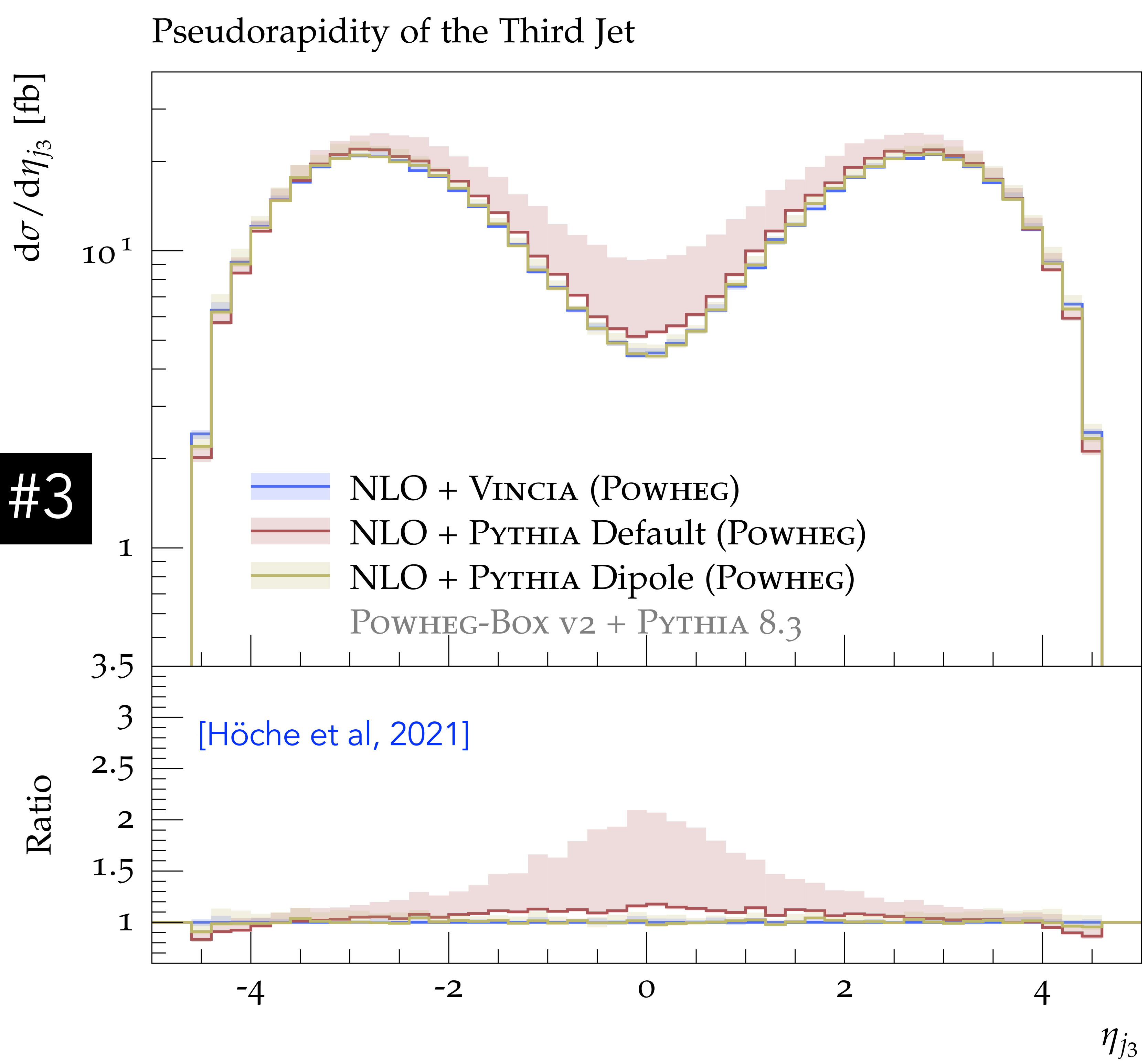
`POWHEG:pThard = 1` # Veto at $\min_i(p_{\perp j;i}^{\mathrm{POWHEG}})$ = smallest scale at which POWHEG **could** have emitted this **parton**

`POWHEG:pThard = 2` # Veto at $\min_{i,j}(p_{\perp j;i}^{\mathrm{POWHEG}})$ = smallest scale at which POWHEG **could** have produced this **event**

[Nason, Oleari 2013]

Less radiation ↓

Jet #3



Powheg + Pythia Default

Big variation with pThard choice ☹️

Tends to fill in the rapidity gap **even for the 3rd jet** (which **should** be under control in POWHEG VBF)

Powheg + Pythia Dipole

Powheg + Vincia

Very little dependence on pThard 😊

Born-Level **NLO accuracy preserved** ✅

VBF: 4th Jet = First Pure-Shower Emission

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[Nason, Oleari 2013]

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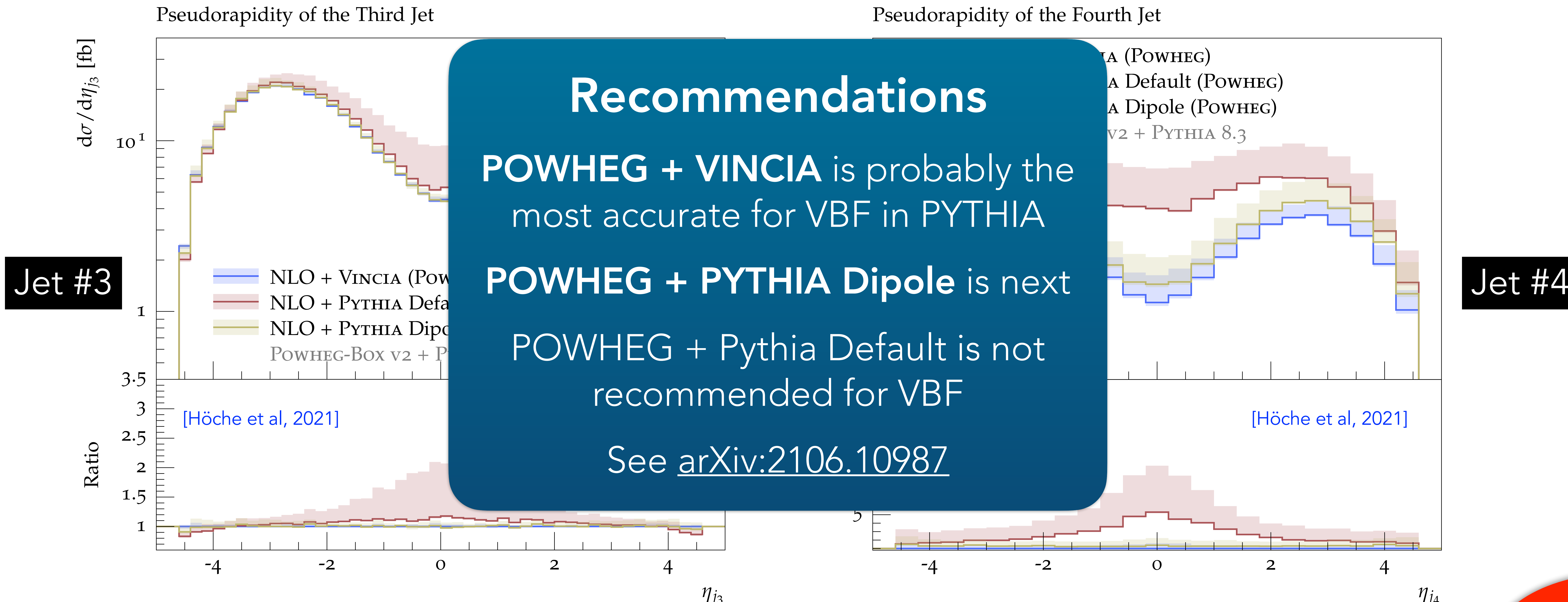


Recommendations

POWHEG + VINCIA is probably the most accurate for VBF in PYTHIA

POWHEG + PYTHIA Dipole is next

POWHEG + Pythia Default is not recommended for VBF

See arXiv:2106.10987

2. From NLO to NNLO

MiNNLO$_{PS}$ builds on (extends) POWHEG NLO for X + jet

[Hamilton et al. 1212.4504, Monni et al. 1908.06987]

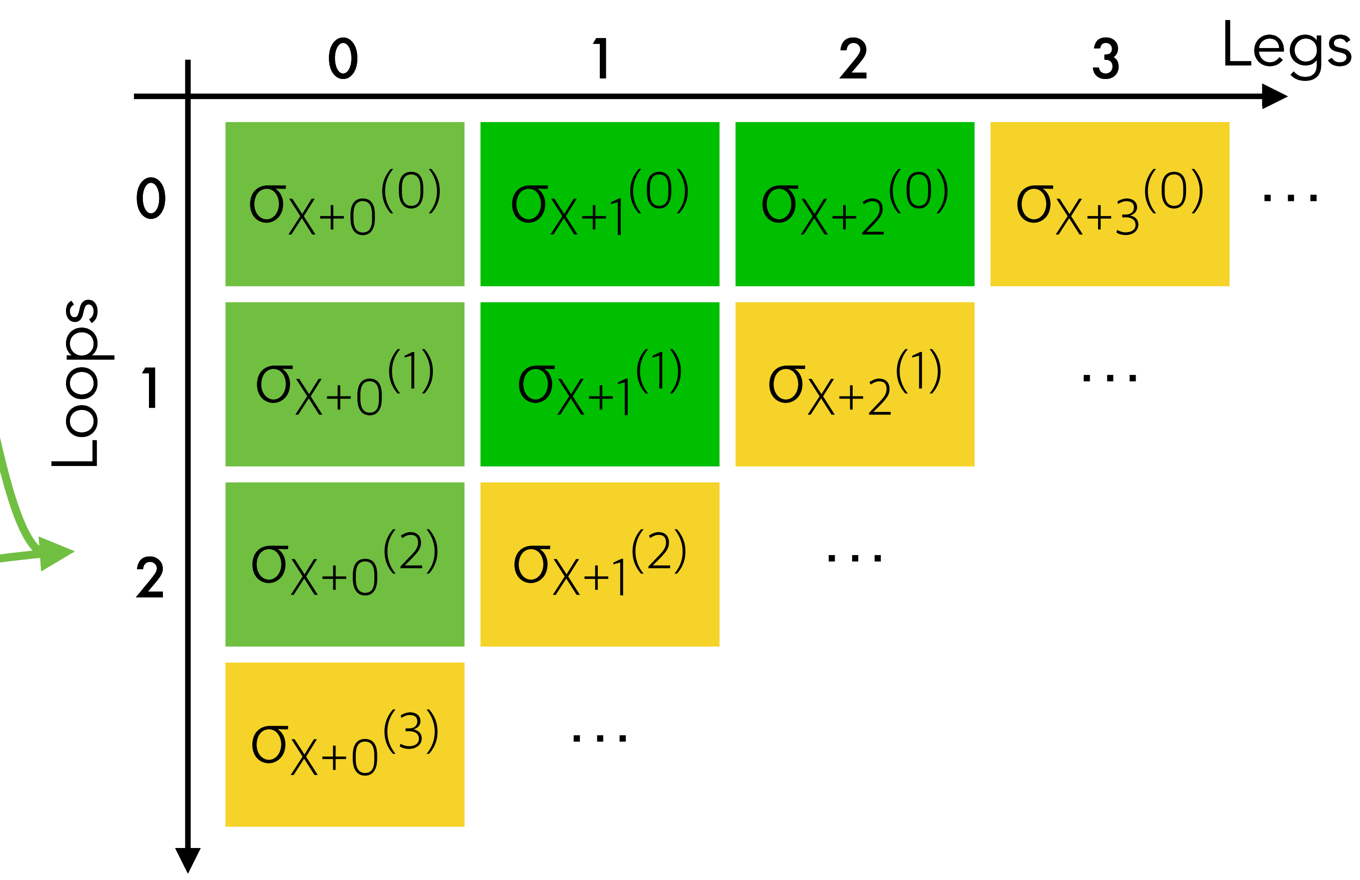
Allow the first jet to approach $p_\perp \to 0$ ~ X + 0

Tame divergence with analytic (NNLL) Sudakov

(introduces additional hardness scale = resummation scale)

Normalize inclusive $d\sigma_X$ to NNLO

($\mathcal{O}(\alpha_s^3)$ ambiguity on how to "spread" the additional contributions in phase space.)



~ Best you can do with current off-the-shelf parton showers

Is approximate; introduces some ambiguities:

$p_\perp^{\text{Shower}}$ vs $p_\perp^{\text{Powheg}}$ vs $Q_{NNLL}^{\text{resummation}}$ & differential NNLO spreading

(+ possible efficiency bottleneck: $p_\perp \to 0$ singularity × Sudakov veto)

What if we could lift that restriction?

MiNNLOPS inherits some issues from POWHEG-Box

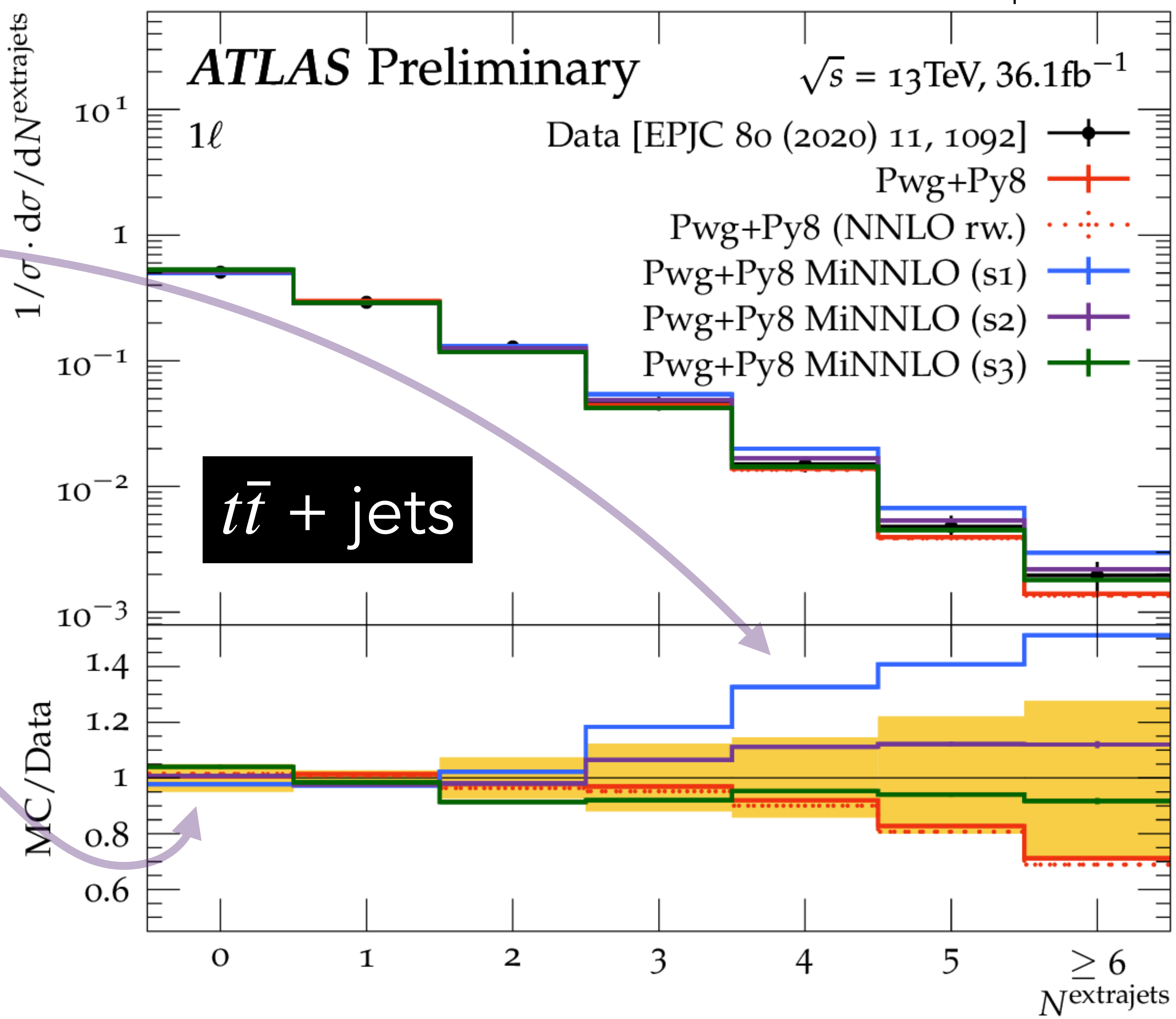
Large dependence on pThard scale

Big variations in predictions for further jets

Calculation "anchored" in NLO for X+jet

$\Longrightarrow$ Fairly big variations for Born-level (0-jet) observable.

ATL-PHYS-PUB-2023-029. September 2023.



Recommendations to Users of these Calculations

- **MiNNLO$_{PS}$ is an *approximate* matching scheme**

  ~ Best you can do with current off-the-shelf parton showers!

  But: does not "match" shower to NNLO point by point in phase space

  (Impossible to do so with LL showers.)

  Does not (always) do vetoed showers

  (This can in principle be done.)

  Depends on several auxiliary scales

  (Intrinsic to scheme. Physical observables should not depend on them → *vary!*)

→ **Do comprehensive variations to estimate scheme uncertainties**

  Subsequent shower not fully *guaranteed* to preserve accuracy

  (Also applies to POWHEG + showers)

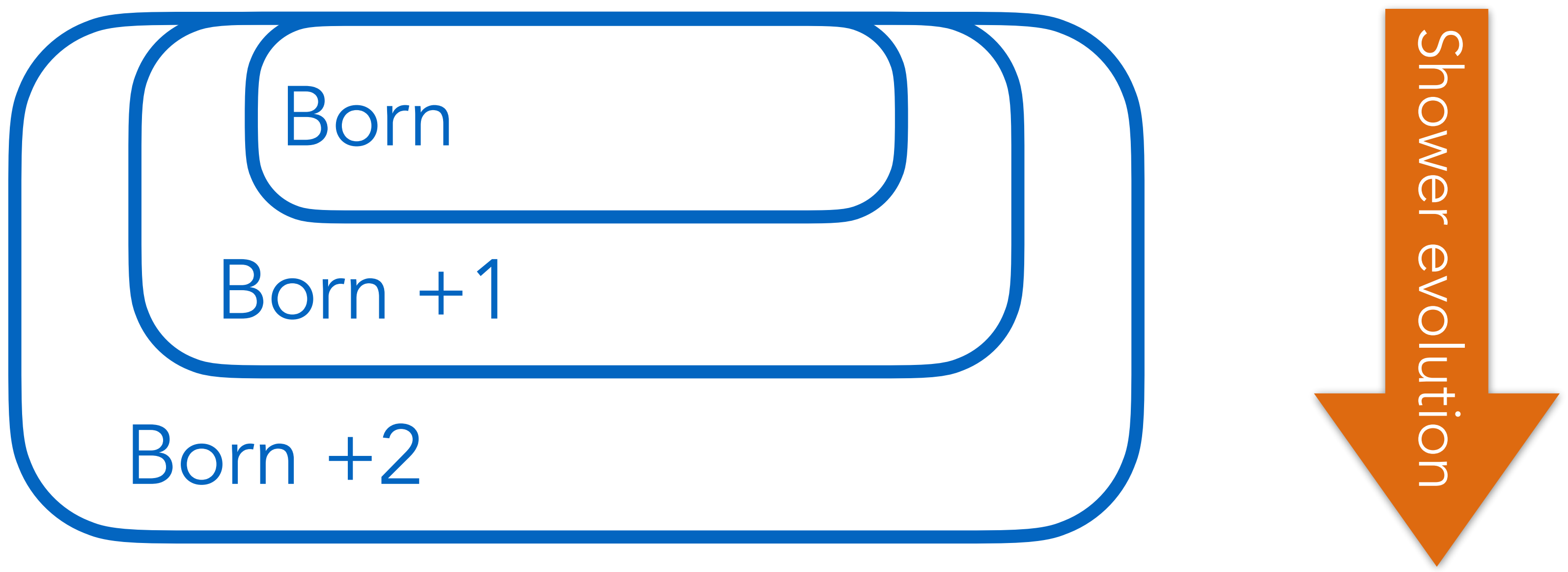
Towards True* NNLO Matching

*In the sense of the fixed-order and shower calculations matching each other point by point in each phase space

Idea: Use (nested) Shower Markov Chain as NNLO Phase-Space Generator

Harnesses the power of showers as efficient phase-space generators for QCD

Pre-weighted with the (leading) QCD singular structures = soft/collinear poles



Different from conventional Fixed-Order phase-space generation (eg VEGAS)

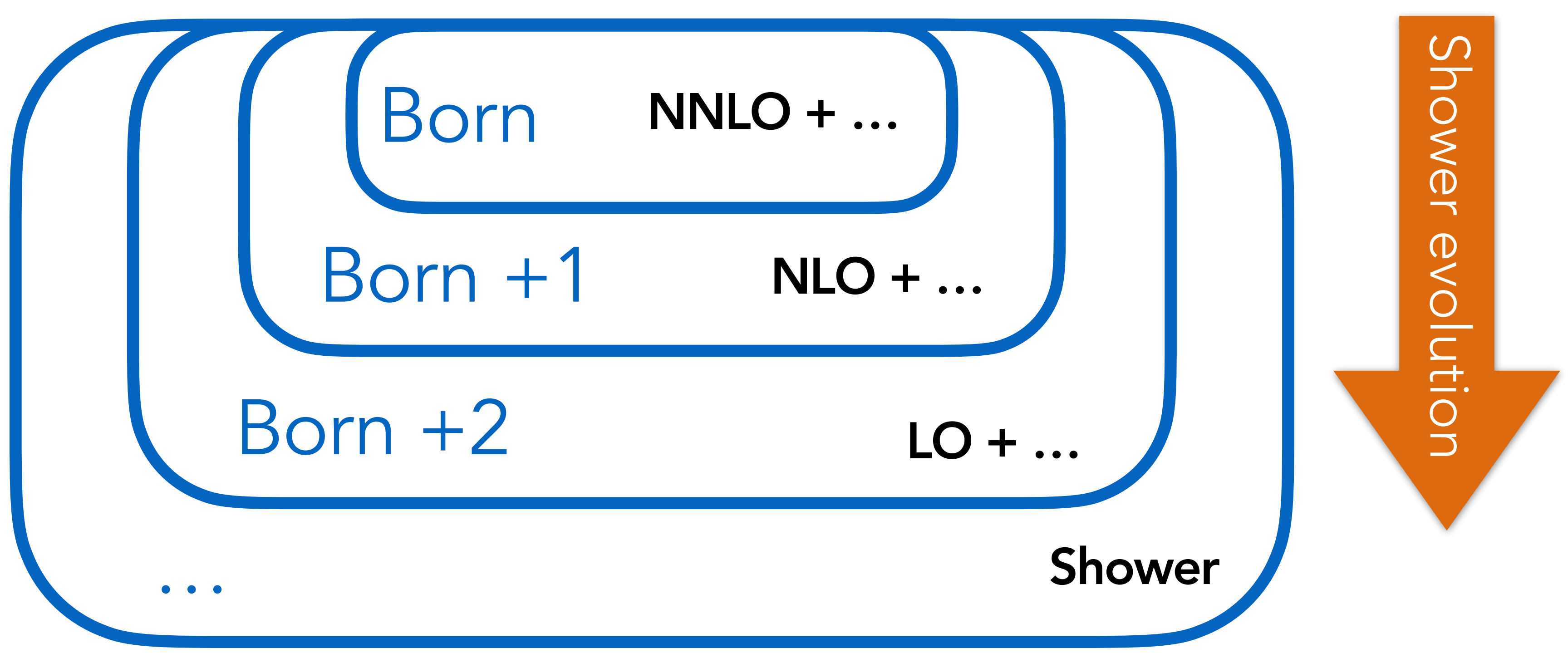
Towards True* NNLO Matching

*In the sense of the fixed-order and shower calculations matching each other point by point in each phase space

Continue shower afterwards

No auxiliary / unphysical scales

⇒ expect small matching systematics



arXiv:2108.07133
& arXiv:2310.18671

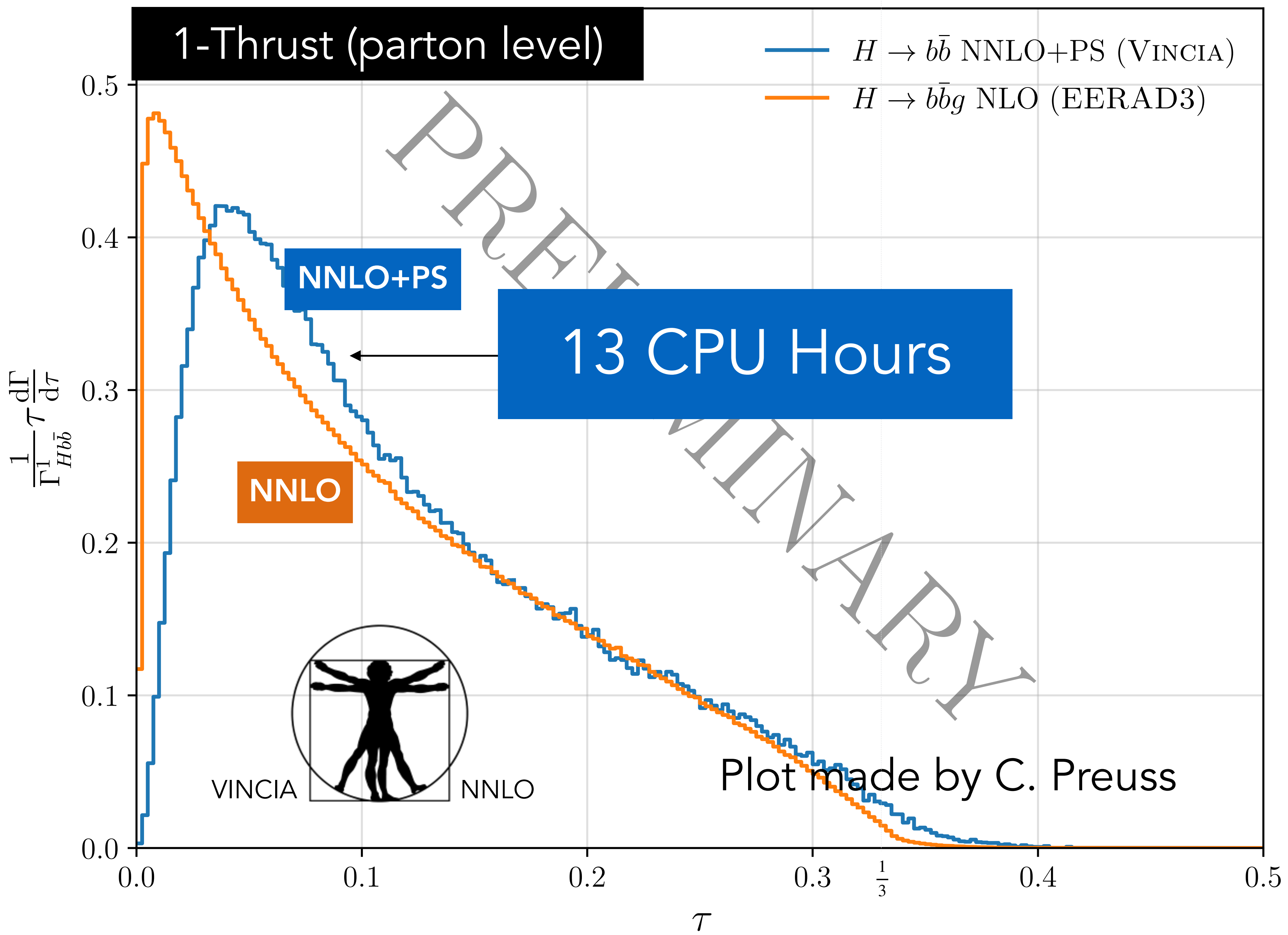
Need:

1. Born-Local NNLO ($\mathcal{O}(\alpha_s^2)$) K-factors: $k_{\mathrm{NNLO}}(\Phi_2)$
2. NLO ($\mathcal{O}(\alpha_s^2)$) MECs in the first $2 \to 3$ shower emission: $k_{\mathrm{NLO}}^{2\to3}(\Phi_3)$
3. LO ($\mathcal{O}(\alpha_s^2)$) MECs for next (iterated) $2 \to 3$ shower emission: $k_{\mathrm{LO}}^{3\to4}(\Phi_4)$
4. Direct $2 \to 4$ branchings for unordered sector, with LO ($\mathcal{O}(\alpha_s^2)$) MECs: $k_{\mathrm{LO}}^{2\to4}(\Phi_4)$

NEW

VINCIA NNLO

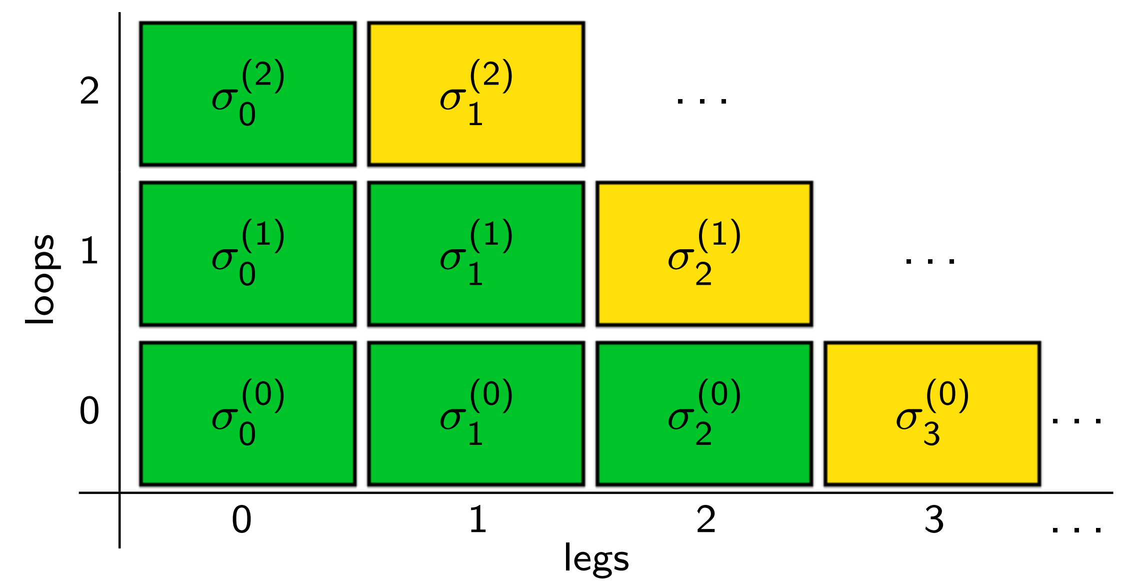
Preview: VINCIA NNLO+PS for $H \to b\bar{b}$



"NNLO" Reference = **EERAD3** NLO $H \to b\bar{b}g$

Coloretti, Gehrmann-de Ridder, Preuss, JHEP 06 (2022) 009

NNLO accuracy in $H \to 2j$ implies **NLO correction in first emission** and **LO correction in second emission.**



So for Thrust, NNLO $H \to b\bar{b}$ is effectively

NLO for $\tau < 1/3$

LO for $\tau > 1/3$

VINCIA NNLO+PS: shower as phase-space generator: efficient & no negative weights!

➤ Looks ~ 5 x ***faster*** than **EERAD3** (for equivalent unweighted stats)

+ is **matched to shower** (add shower resummation without auxiliary input/scales) + can be **hadronized**

Proof of concepts now done for $Z/H \to q\bar{q}$; work remains for pp (& for N^{n}LL accuracy)

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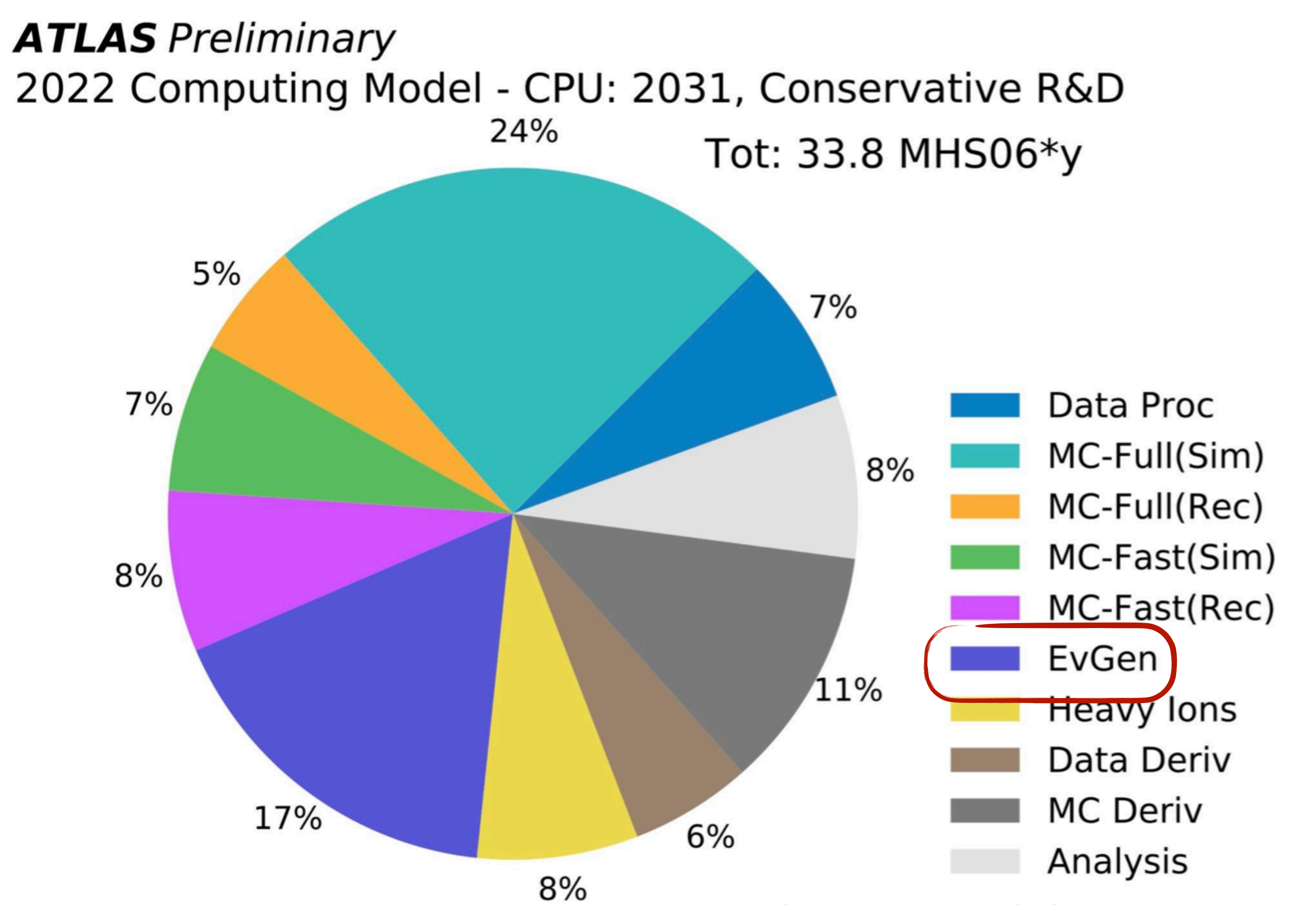
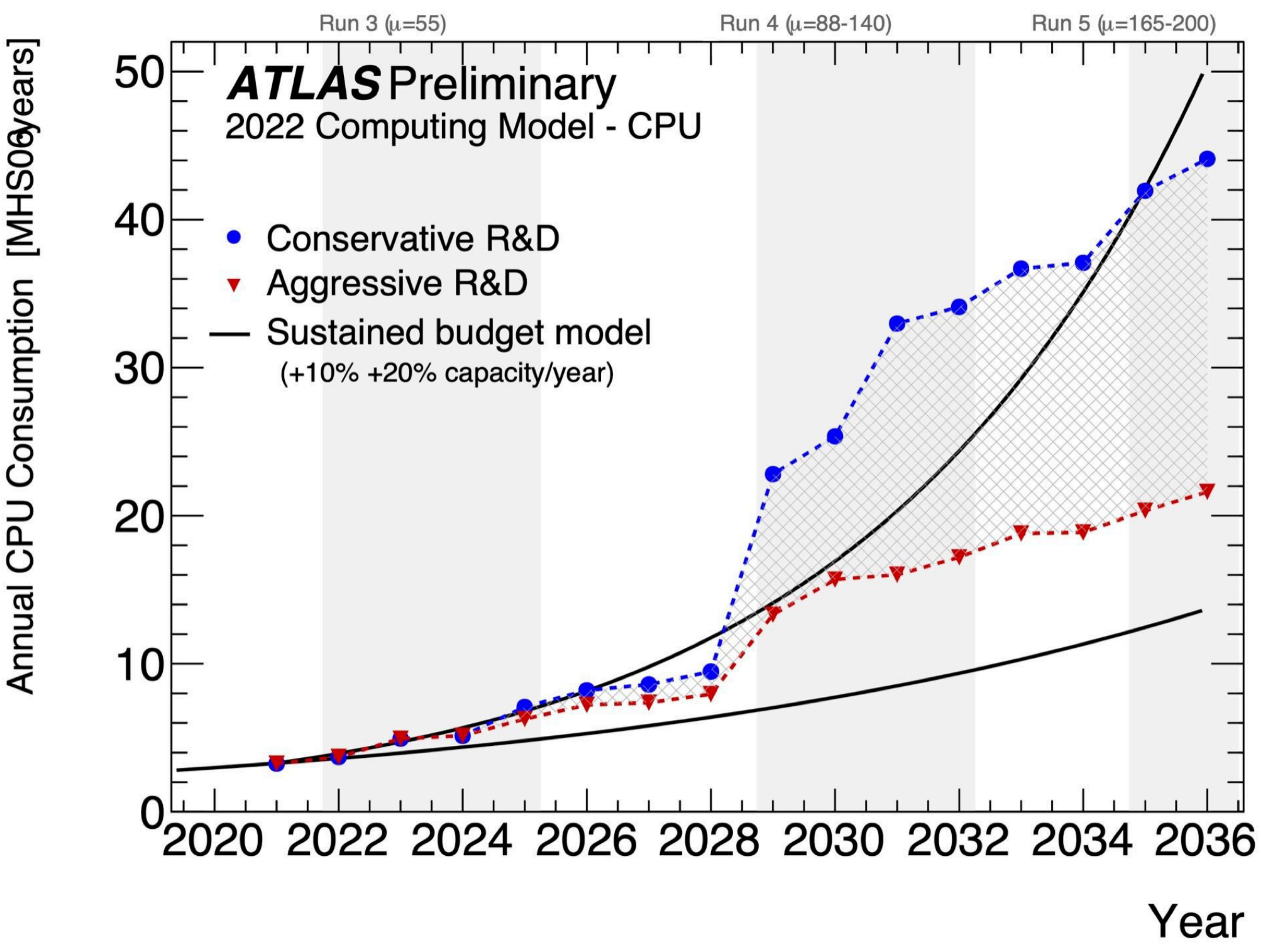
The Computational Bottleneck in ME Merging

Condensed remarks from talk by T. Moskalets (ATLAS) at CERN Workshop Nov 2023

Something about CPU resources



CERN-LHCC-2022-005



- **Largest fraction of EvGen CPU time is taken by generation of multi-leg MC predictions**
  - namely, multijet merged Sherpa V+jets

Matrix-Element Merging — The Complexity Bottleneck

For CKKW-L style merging: (incl UMEPS, NL3, UNLOPS, …)

Need to take **all contributing "shower histories" into account**.

In conventional parton showers (Pythia, Herwig, Sherpa, …)

Each phase-space point receives contributions from many possible branching "histories" (aka "clusterings")

# of histories grows ~ # of Feynman Diagrams, **faster than factorial**

Number of Histories for n Branchings

Starting from a single $q\bar{q}$ pair	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
CS Dipole	2	8	48	384	3840	46080	645120

Bottleneck for merging at high multiplicities (+ high code complexity)

Sector Showers (without maths)

NEW

VINCIA's shower is unique in being a "Sector Shower"

PS & Villarejo JHEP 11 (2011) 150
Brooks, Preuss, **PS** JHEP 07 (2020) 032

Partition N-gluon Phase Space into N "sectors" (using step functions).

Each sector ↔ one specific gluon being the "softest" in the event

Inside each sector, only one kernel contributes (the most singular one)!

Sector Kernel = the eikonal for the soft gluon and its collinear DGLAP limits for $z > 0.5$.

➜ Unique properties: shower operator is **bijective** and is a **true Markov chain**

The crucial aspect:

Only a **single history** contributes to each phase-space point !

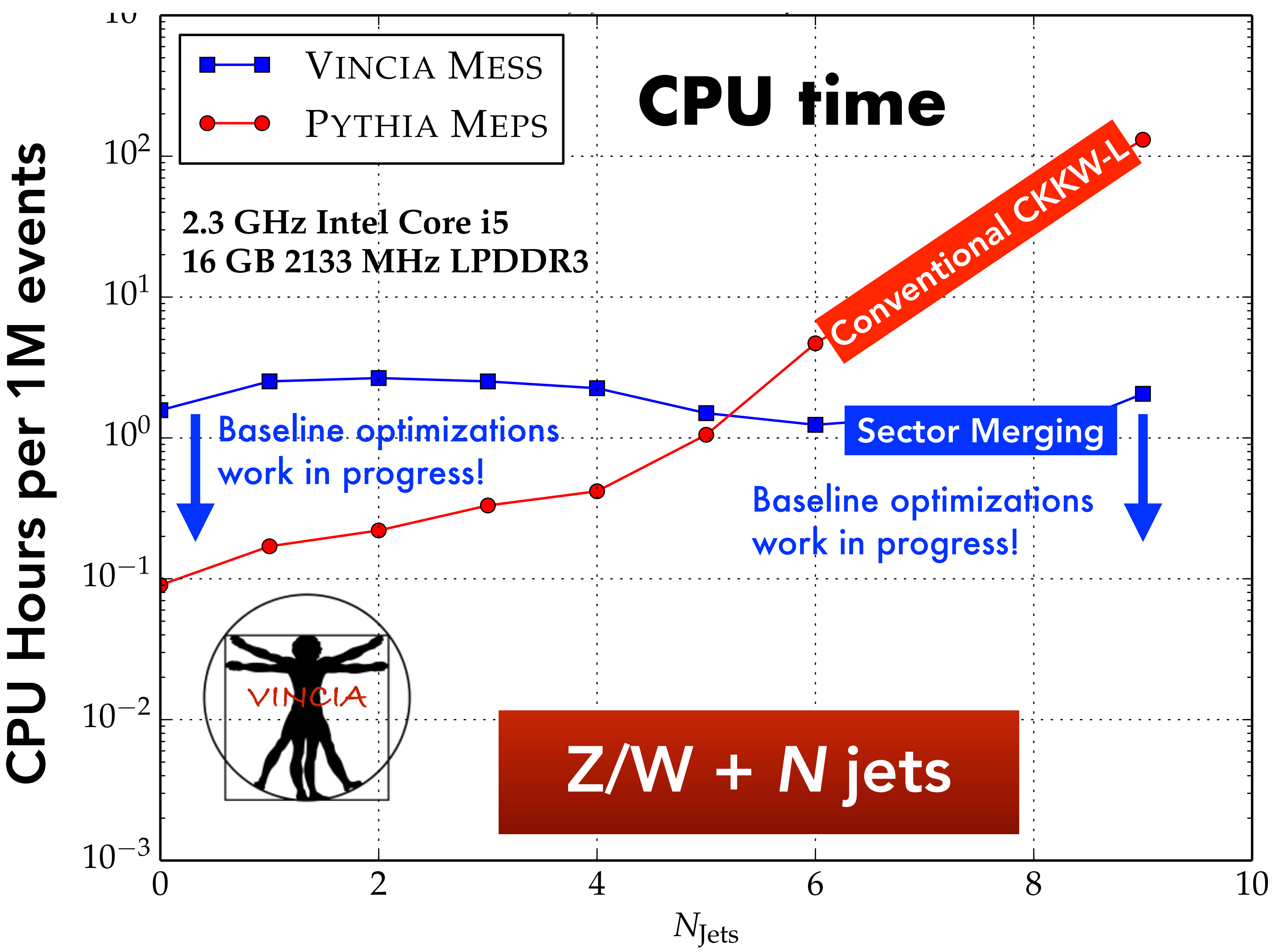
$\Longrightarrow$ **Factorial growth of number of histories reduced to constant!**

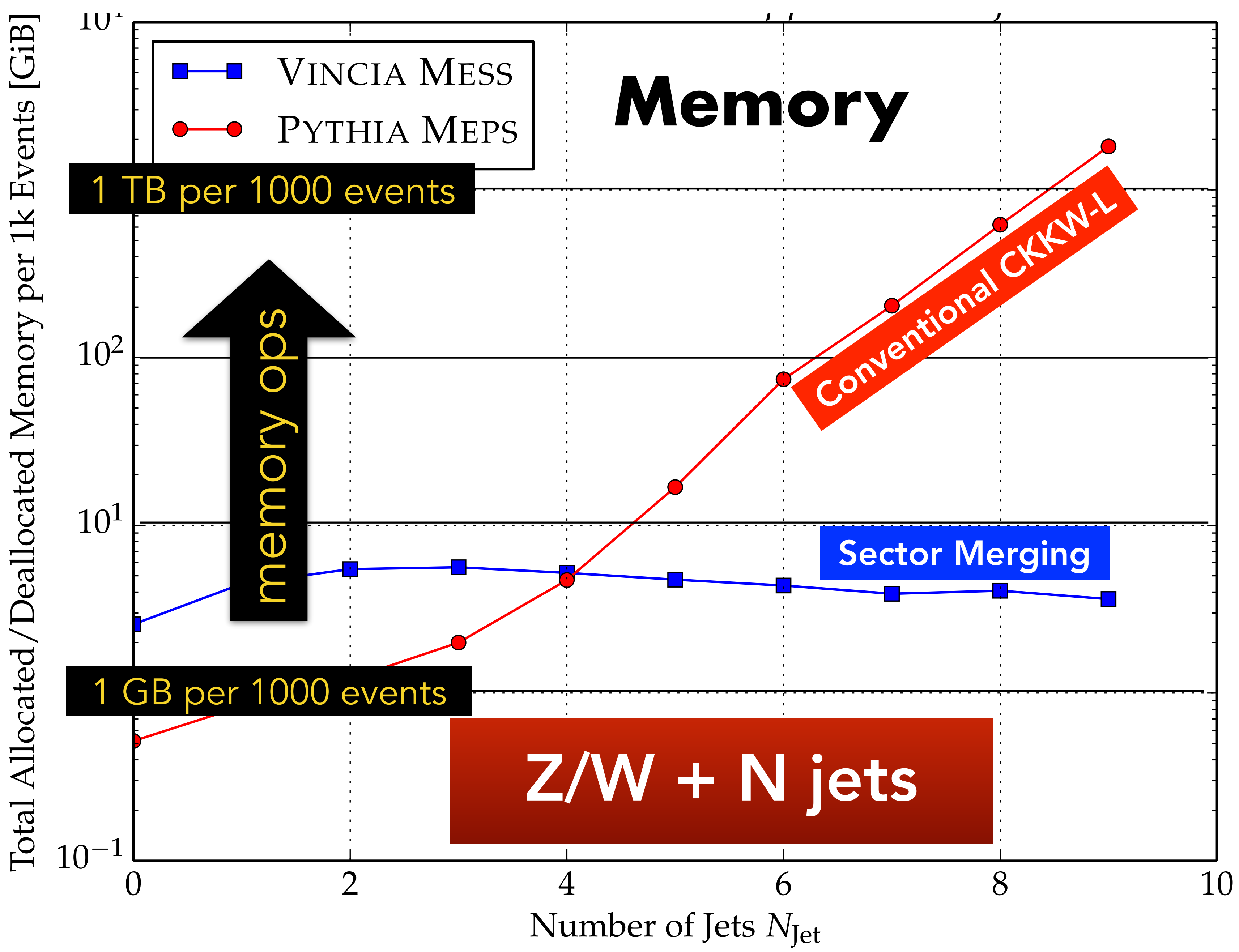
(And the number of sectors only grows linearly with the number of gluons)

($g \to q\bar{q}$ ➜ leftover factorial in number of same-flavour quarks; not a big problem)

Sectorized CKKW-L Merging publicly available from Pythia 8.306

Brooks & Preuss, "Efficient multi-jet merging with the VINCIA sector shower", arXiv:2008.09468





Demonstrated constant scaling with multiplicity. Extensions now pursued:

Optimisations of baseline algorithm

Sectorized **iterated tree-level ME corrections** (demonstrated in PS & Villarejo arXiv:1109.3608)

Sectorized **multi-leg merging at NLO** (active research grants, with **C. Preuss, Wuppertal**)

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New Discoveries in Hadronization

NEW

LHC experiments report very large **(factor-10)** enhancements in **heavy-flavour** baryon-to-meson ratios **at low p_T!**

Not predicted by default Pythia (Monash)

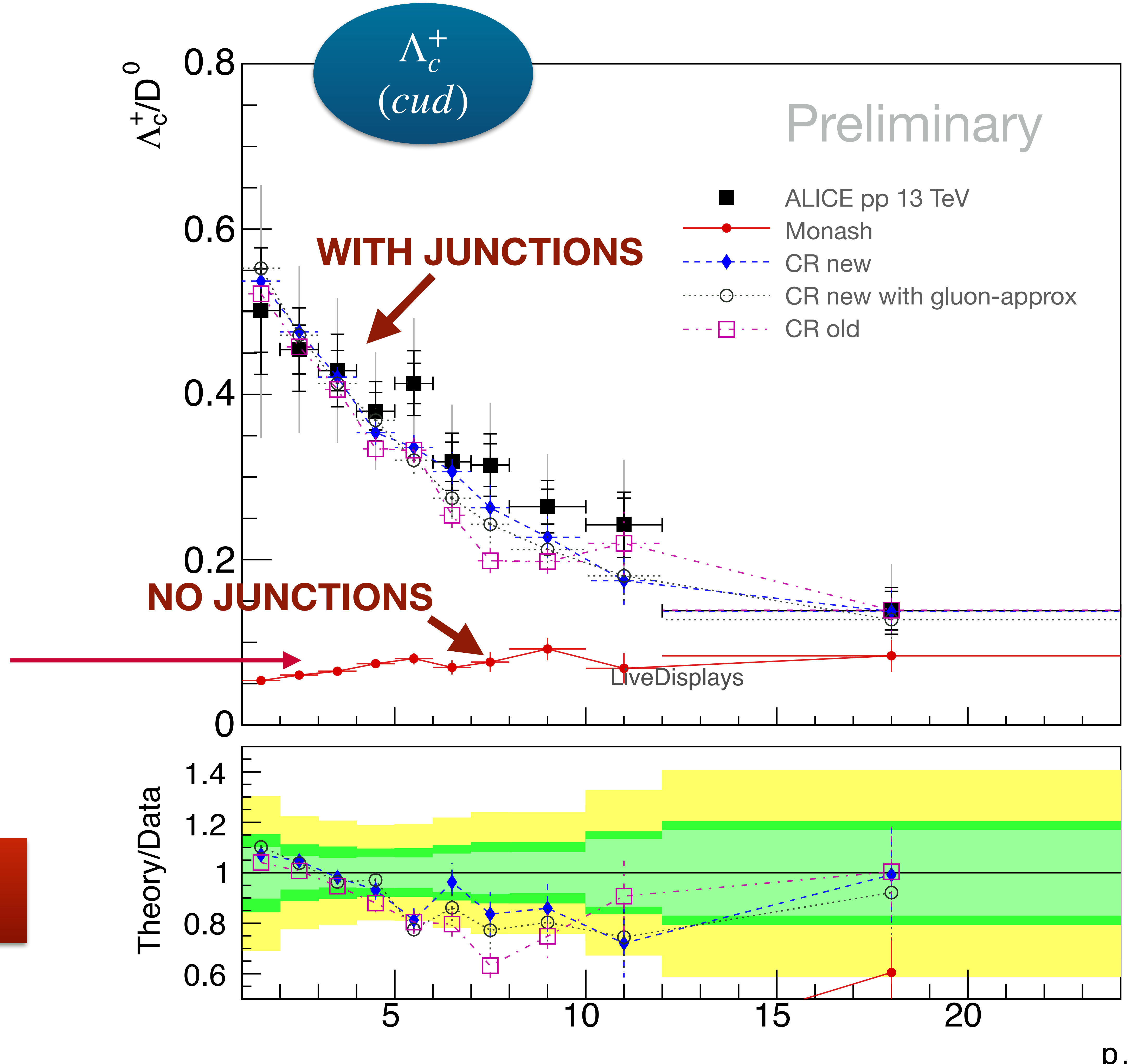


Figure from [Altmann & Skands, in progress]

Very exciting!

LHCb: also in Bottom

Λ_b asymmetry

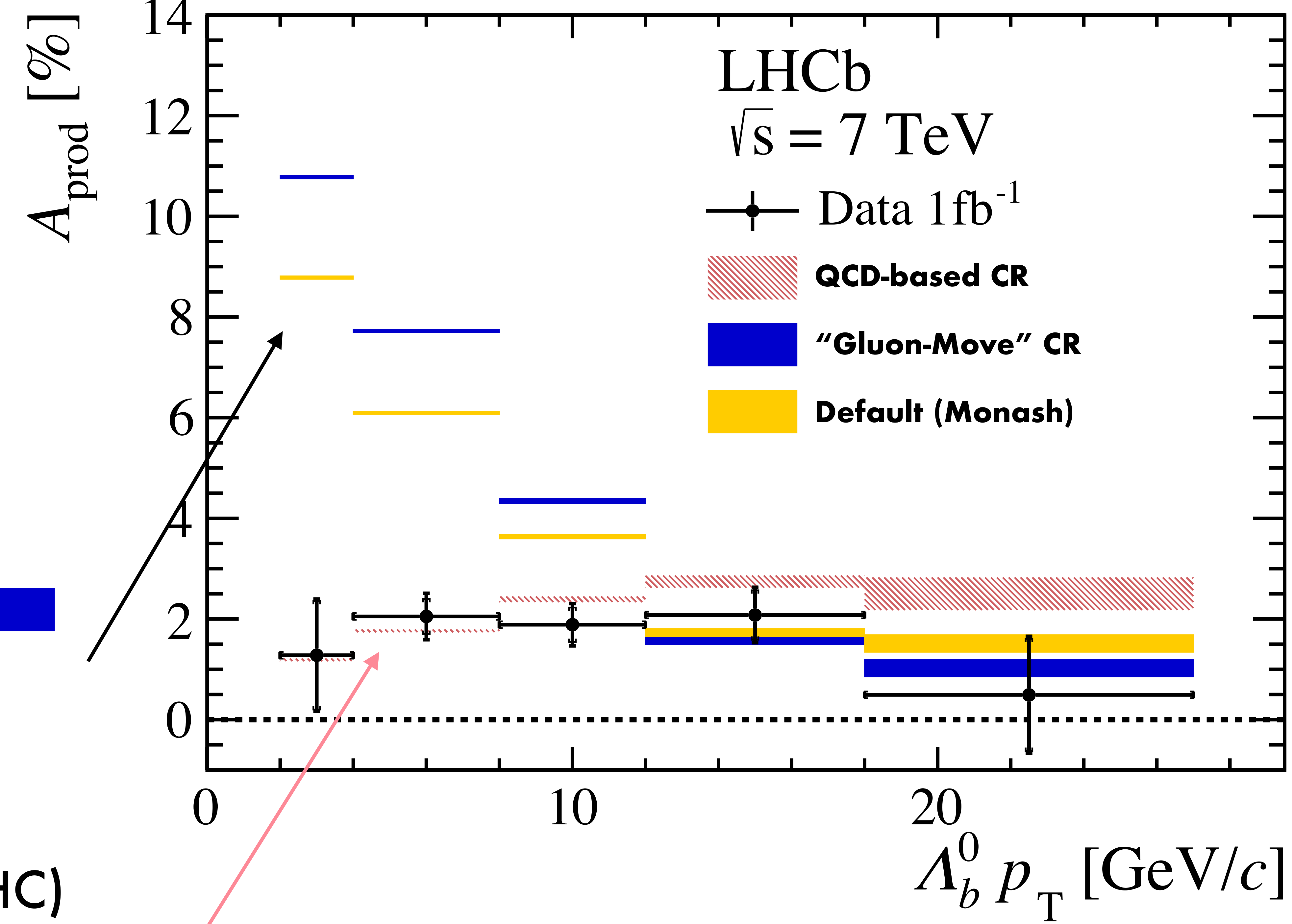
$$A = \frac{\sigma(\Lambda_b^0) - \sigma(\overline{\Lambda}_b^0)}{\sigma(\Lambda_b^0) + \sigma(\overline{\Lambda}_b^0)}$$

Baseline Expectations: &

b quark combines with the proton beam remnant $\Longrightarrow \Lambda_b$ production

Not possible for $\bar{\Lambda}_b$ (no $\bar{p}$ remnant at LHC)

LHCb, JHEP 10 (2021) 060 • arXiv: 2107.09593

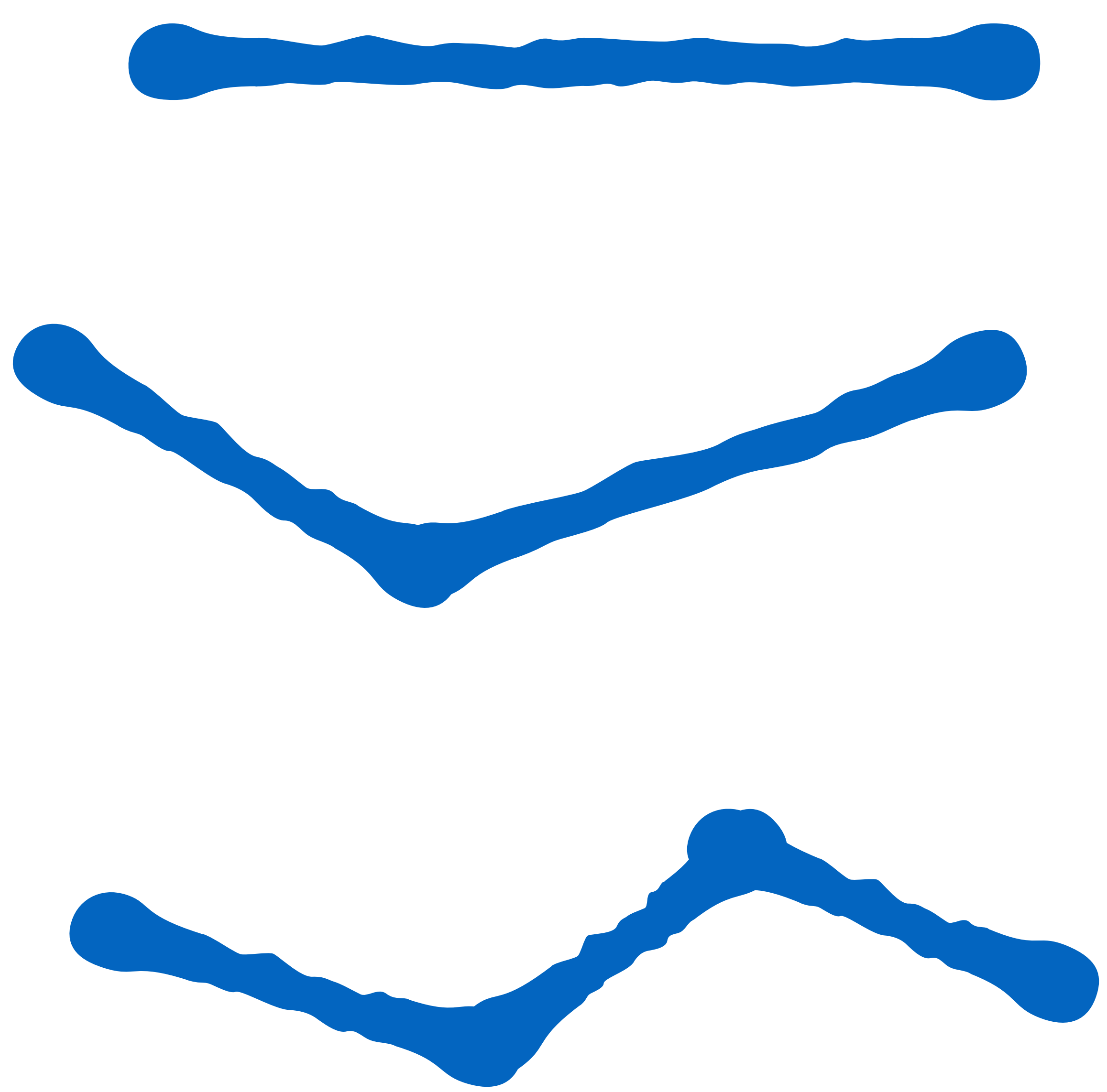


QCD CR with "string junctions" [Christiansen & Skands JHEP 08 (2015) 003]

Adds large amount of low-p$_T$ Λ_b and $\bar{\Lambda}_b$, in equal amounts. Dilutes asymmetry!

What are "String Junctions"?

Open Strings

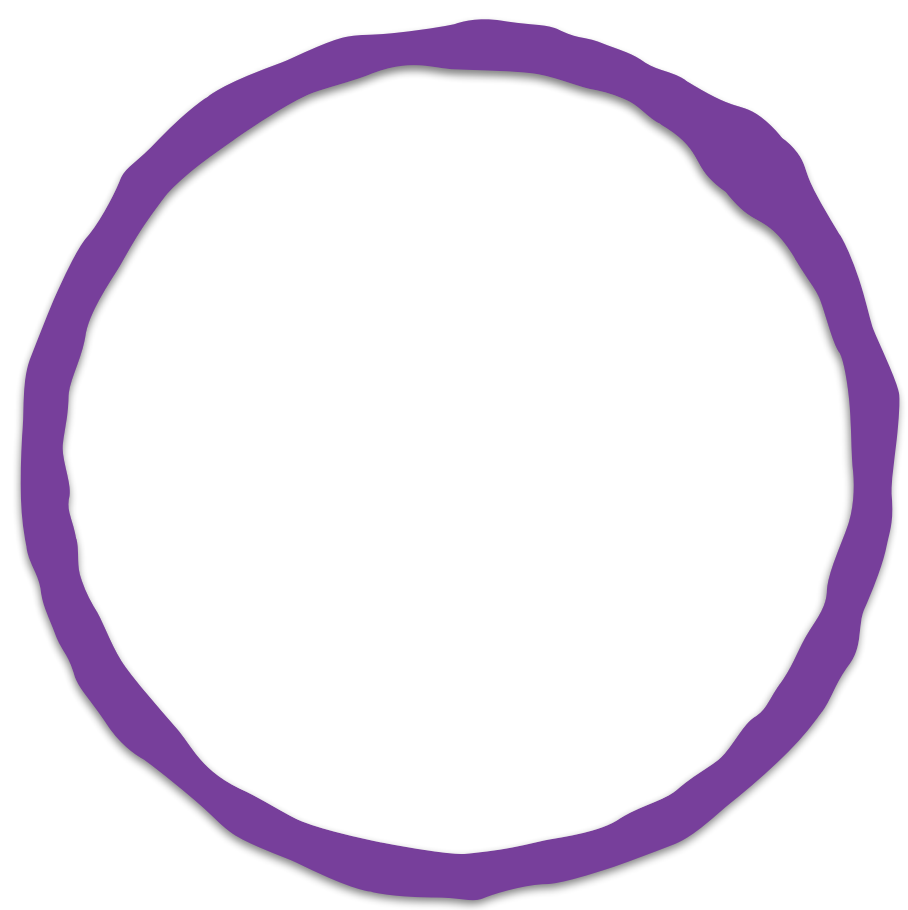


$q\bar{q}$ strings (with gluon kinks)

E.g., $Z \to q\bar{q}$ + shower

$H \to b\bar{b}$ + shower

Closed Strings

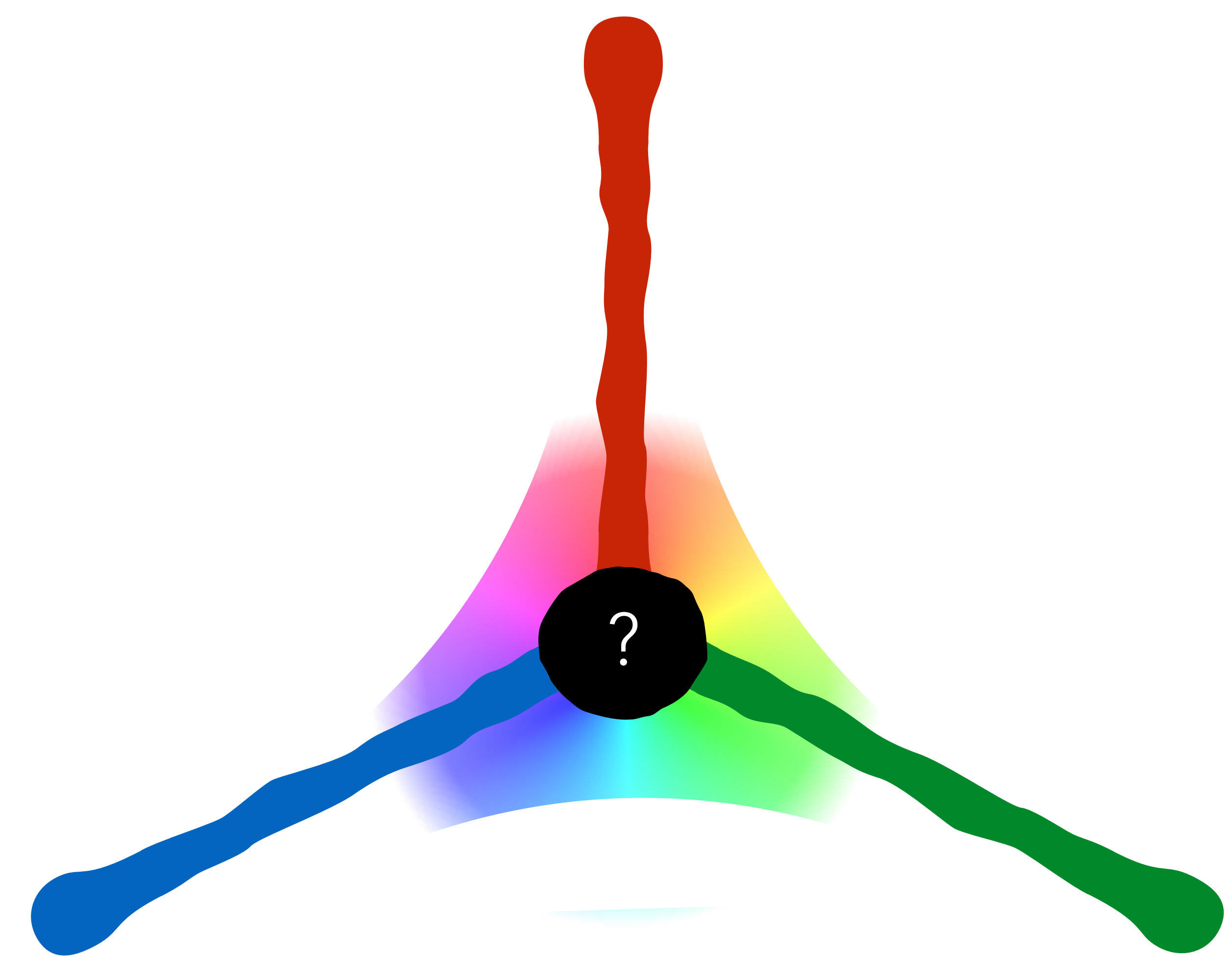


Gluon rings

E.g., $H \to gg$ + shower

$\Upsilon \to ggg$ + shower

SU(3) String Junction



Open strings with $N_C = 3$ endpoints

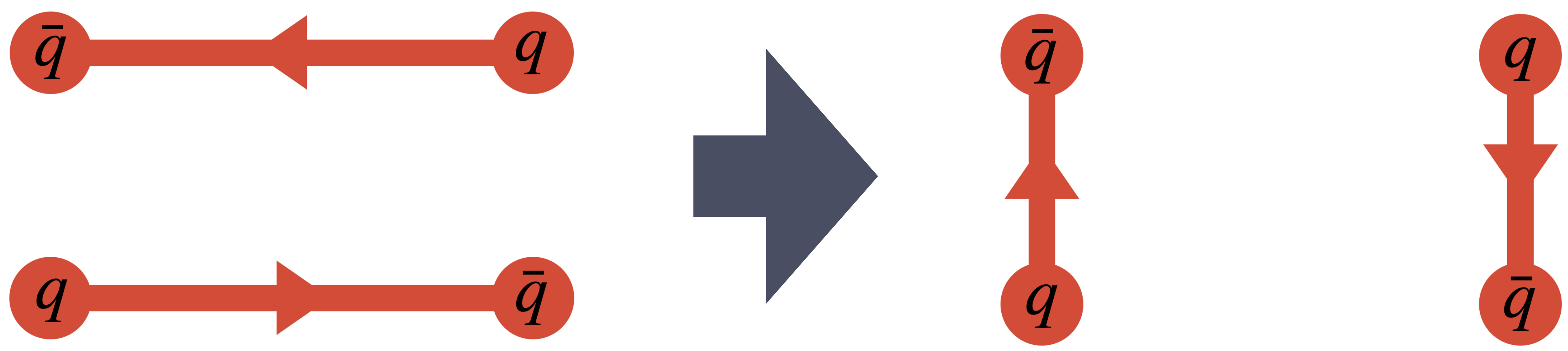
E.g., Baryon-Number violating neutralino decay $\tilde{\chi}^0 \to qqq$ + shower

How do Colour Reconnections Create String Junctions?

Stochastically restores colour-space ambiguities according to **SU(3) algebra**

[Christiansen & **PS** JHEP 08 (2015) 003]

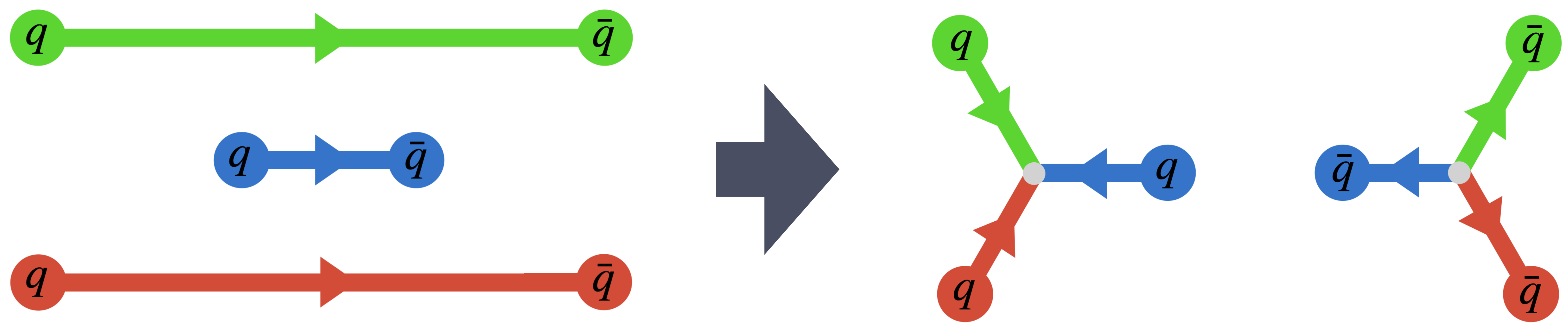
➢ Allows for reconnections to minimise string lengths



Dipole-type reconnection

This is what most CR models do

What about the red-green-blue colour singlet state?

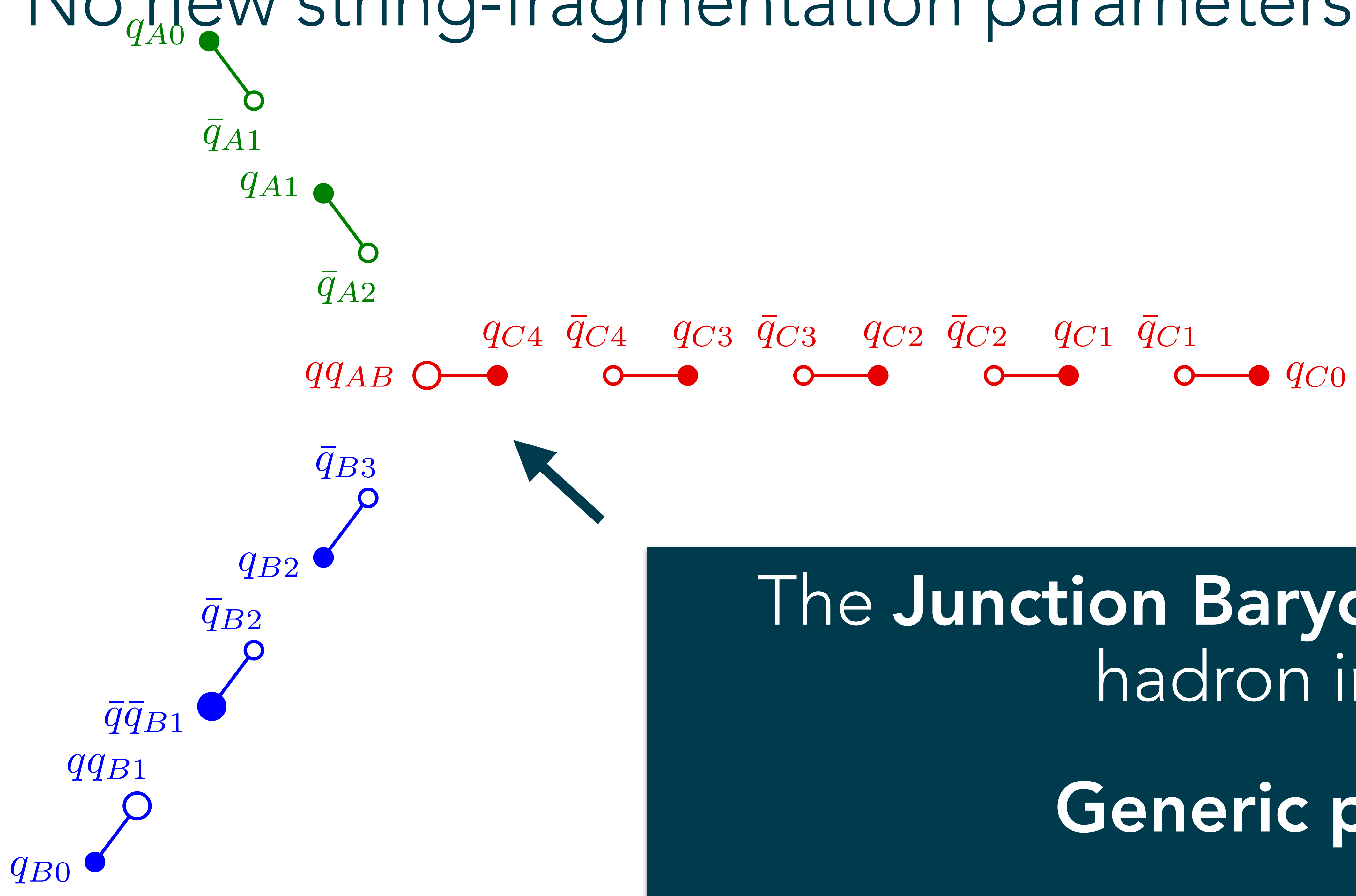


Junctions!

What do String Junctions do?

Assume Junction Strings have same properties as ordinary ones
(u:d:s, Schwinger p_T, etc)

➤ No new string-fragmentation parameters



[Sjöstrand & **PS**, NPB 659 (2003) 243]
[+ J. Altmann & **PS**, in progress]

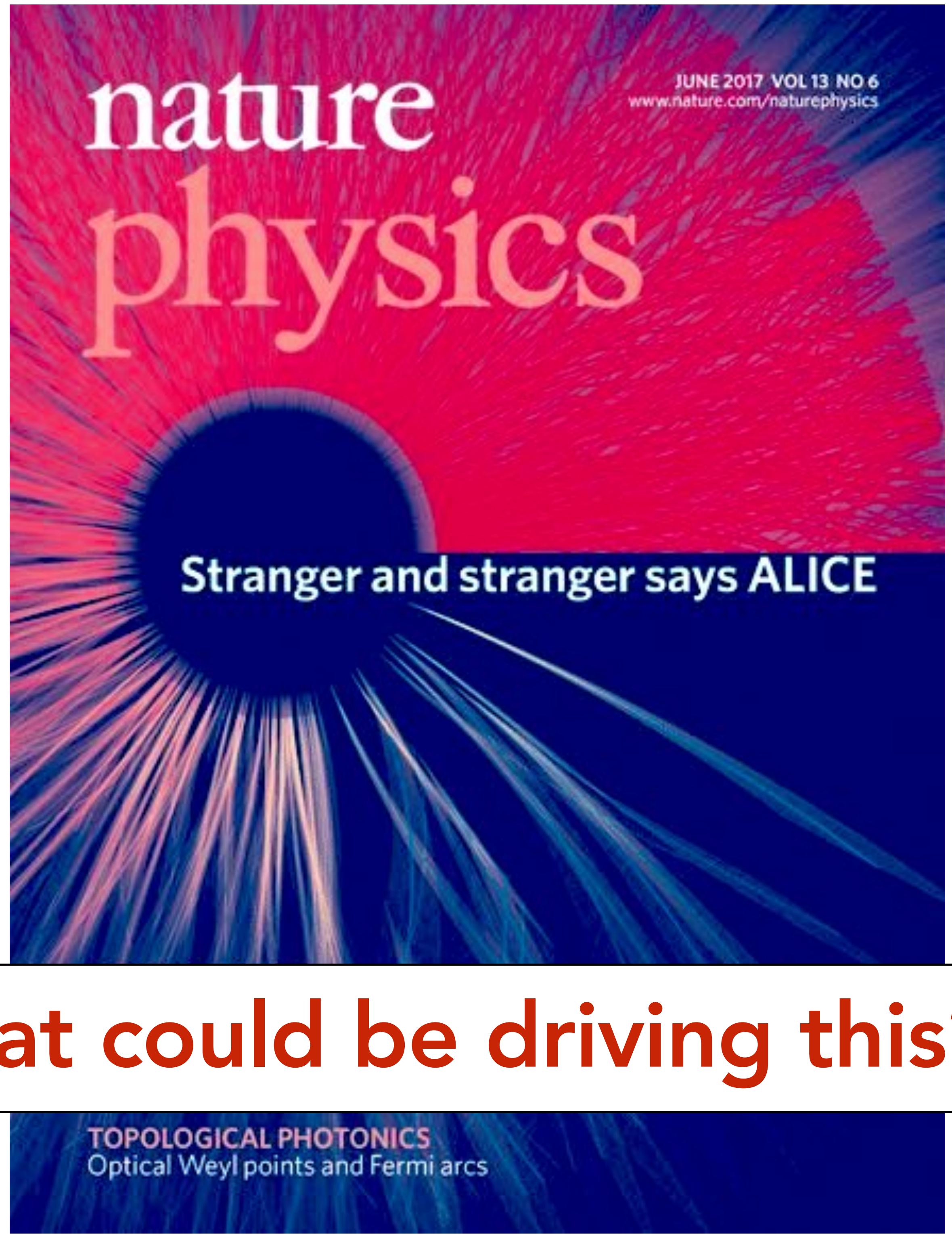
The **Junction Baryon** is the most "subleading" hadron in all three "jets".

Generic prediction: low p_T

A Smoking Gun for String Junctions: **Baryon enhancements at low p_T**

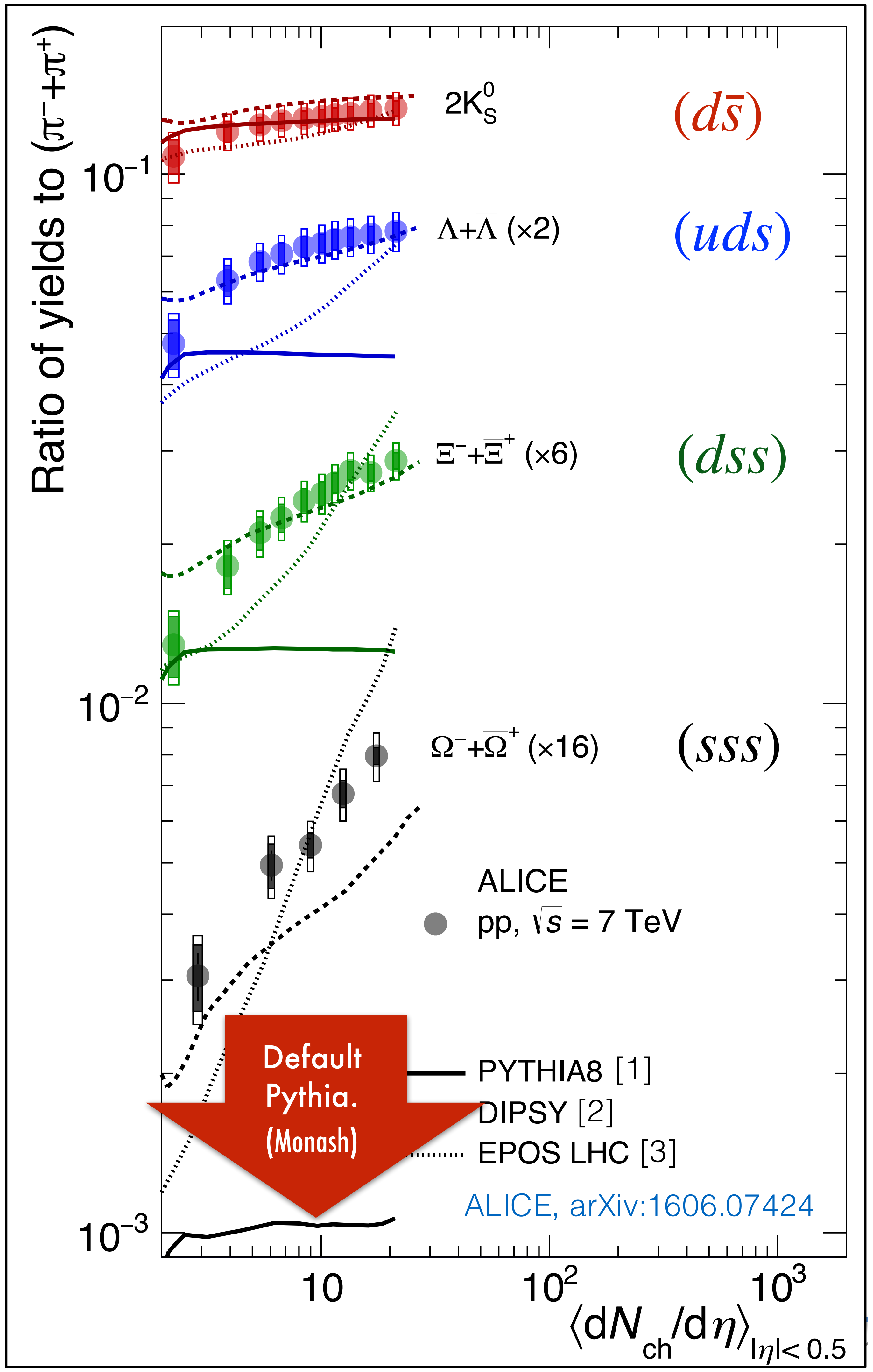
What a **strange** world we live in, said Alice

We also know ratios of strange hadrons to pions strongly increase with event activity



June 2017

What could be driving this?



In Progress: Strangeness Enhancement from Close-Packing

NEW

Idea: each string exists in an effective background produced by the others

Close-packing

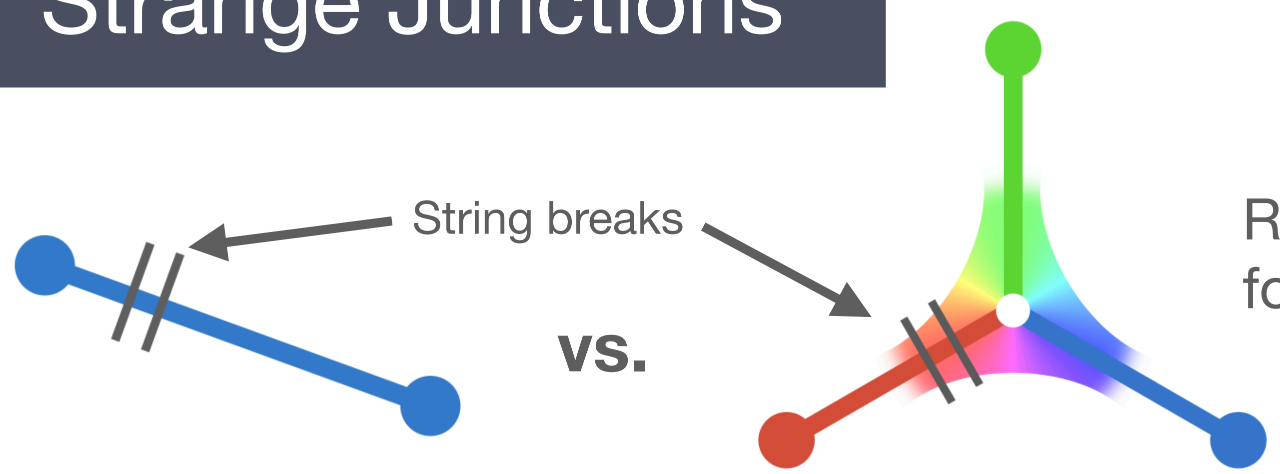
$C_8 = 2.25 C_F$ $\qquad$ $C_6 = 2.5 C_F$

Dense string environments

→ Casimir scaling of **effective string tension**

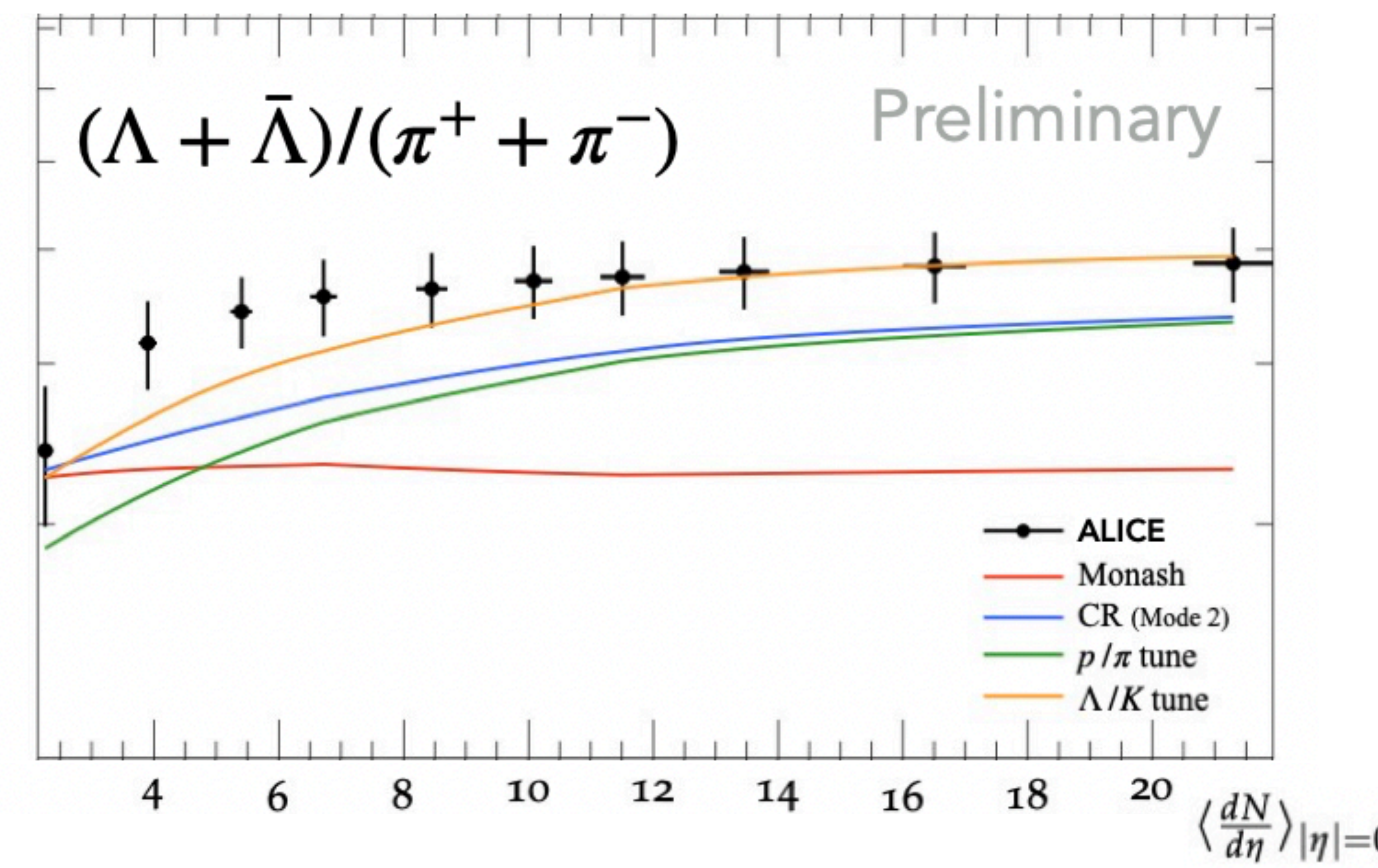
→ Higher probability of strange quarks

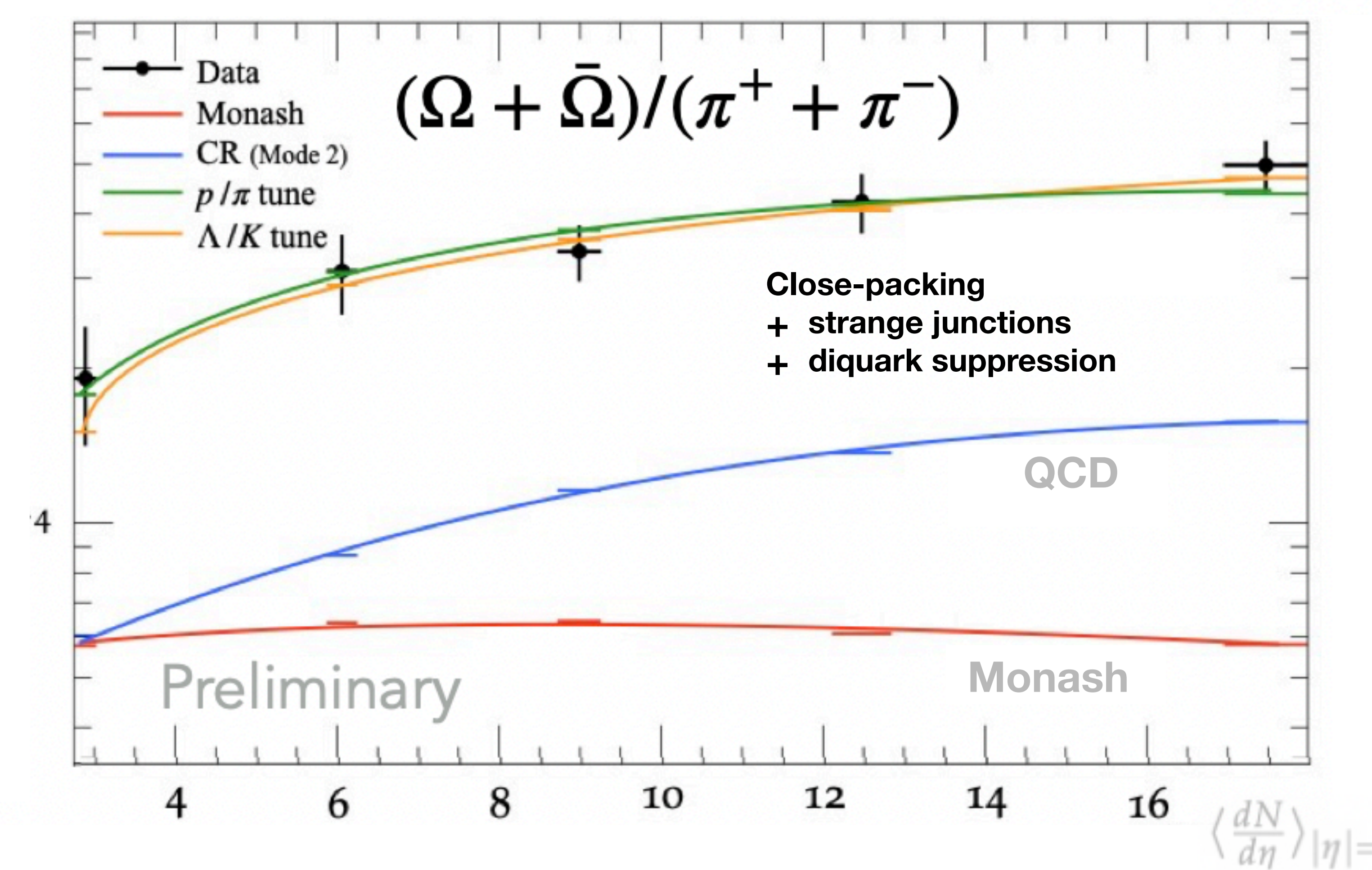
Strange Junctions



Results in strangeness enhancement focused in baryon sector

String tension could be different from the vacuum case compared to near a junction





Slide adapted from J. Altmann

Particle Composition: Impact on Jet Energy Scale

ATLAS PUB Note

ATL-PHYS-PUB-2022-021

29th April 2022

Dependence of the Jet Energy Scale on the Particle Content of Hadronic Jets in the ATLAS Detector Simulation

The dependence of the ATLAS jet energy measurement on the modelling in Monte Carlo simulations of the particle types and spectra within jets is investigated. **It is found that the hadronic jet response, i.e. the ratio of the reconstructed jet energy to the true jet energy, varies by ~ 1–2% depending on the hadronisation model used in the simulation. This effect is mainly due to differences in the average energy carried by kaons and baryons in the jet.** Model differences observed for jets initiated by *quarks* or *gluons* produced in the hard scattering process are dominated by the differences in these hadron energy fractions indicating that **measurements of the hadron content of jets and improved tuning of hadronization models can result in an improvement in the precision of the knowledge of the ATLAS jet energy scale.**

Variation largest for gluon jets

- For E_T = **[30, 100, 200] GeV**
- Max JES variation = **[3%, 2%, 1.2%]**

Fraction of jet E_T carried by baryons (and kaons) varies significantly

- Reweighting to force similar baryon and kaon fractions
- Max variation ➜ **[1.2%, 0.8%, 0.5%]**
- Significant potential for improved Jet Energy Scale uncertainties!

Motivates Careful Models & Careful Constraints

- Interplay with advanced UE models
- In-situ constraints from LHC data
- Revisit comparisons to LEP data

Summary & Outlook

State of the art for perturbation theory: NNLO (→ N3LO)

Showers + hadronization mandatory for collider studies (+ resummation extends range)

Now: can use off-the-shelf showers with MiNNLO$_{PS}$

Based on POWHEG-Box + Analytical Resummation + NNLO normalisation

Approximate method; depends on several auxiliary unphysical scales → can exhibit large variations

Work in progress: VinciaNNLO

Based on nested shower-like phase-space generation with second-order MECs

True NNLO matching → Expect small matching systematics

So far only worked out for colour-singlet decays.

(Also developing extensions towards NLL, NNLL showers …)

Beautiful Strings

New discoveries at LHC on particle composition, esp. **baryons and strangeness**

New research grant with LHCb (Warwick) focusing on strings with b-quark endpoints

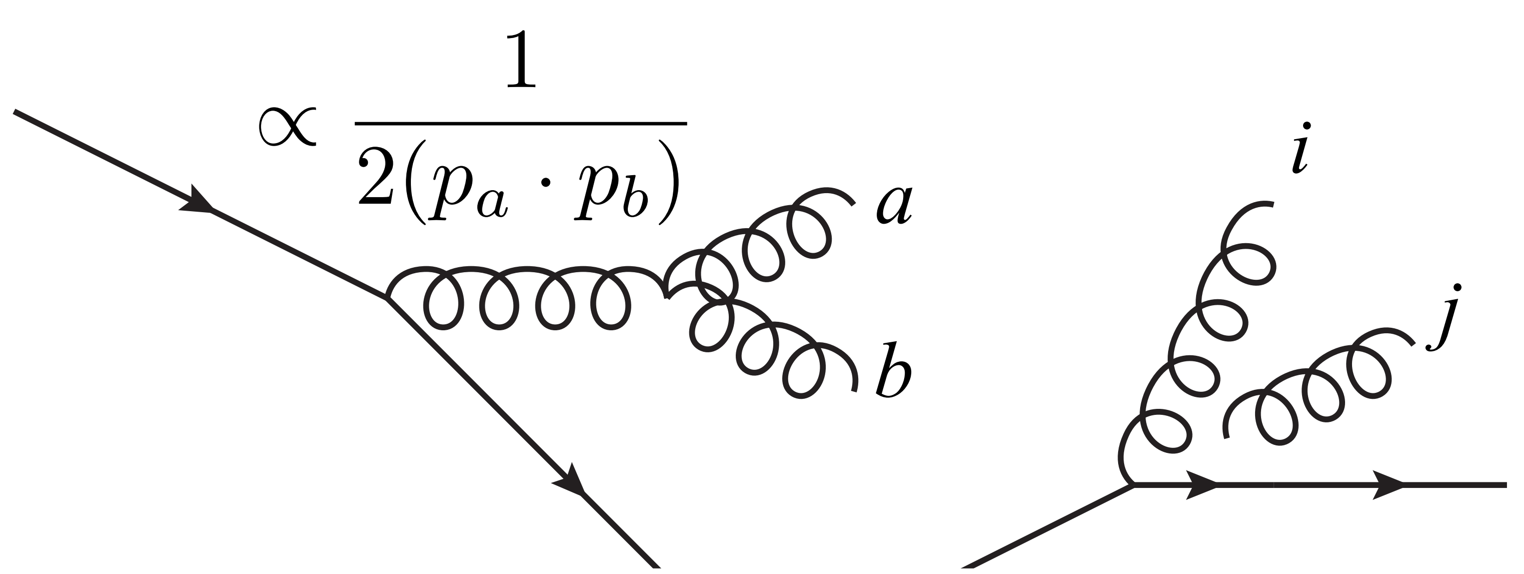
And QED corrections in B decays

Extra Slides

Parton Showers: Theory

see e.g PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389

Most bremsstrahlung is driven by divergent propagators → simple structure



Mathematically, **gauge amplitudes factorize** in **singular limits**

Partons ab → **collinear**:

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \overset{a||b}{\to} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

$P(z)$ = **DGLAP splitting kernels**", with $z = E_a/(E_a + E_b)$

Gluon j → **soft**:

$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \overset{j_g \to 0}{\to} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

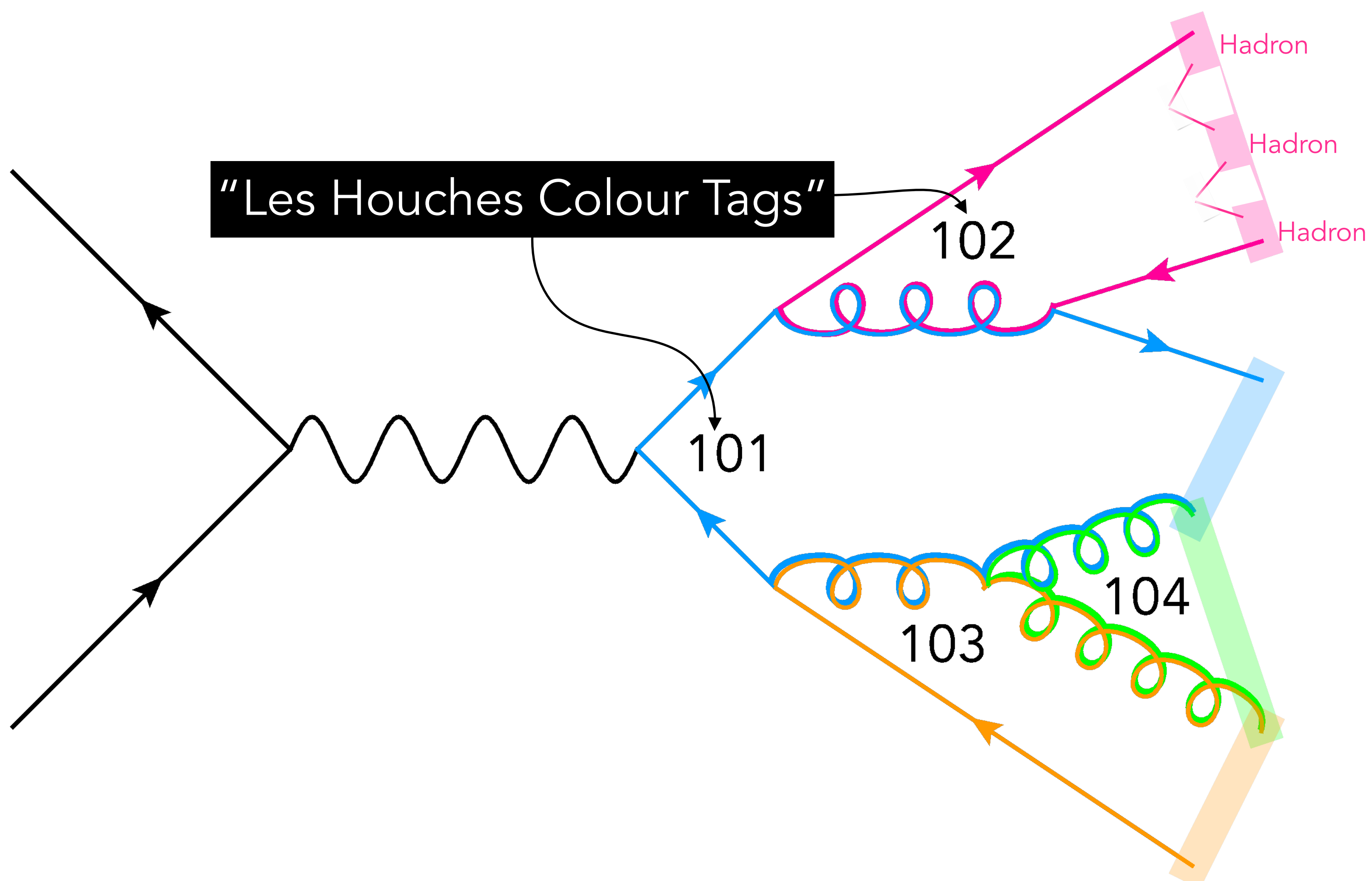
Coherence → Parton j really emitted by (i,k) "dipole" or **"antenna" (eikonal factors)**

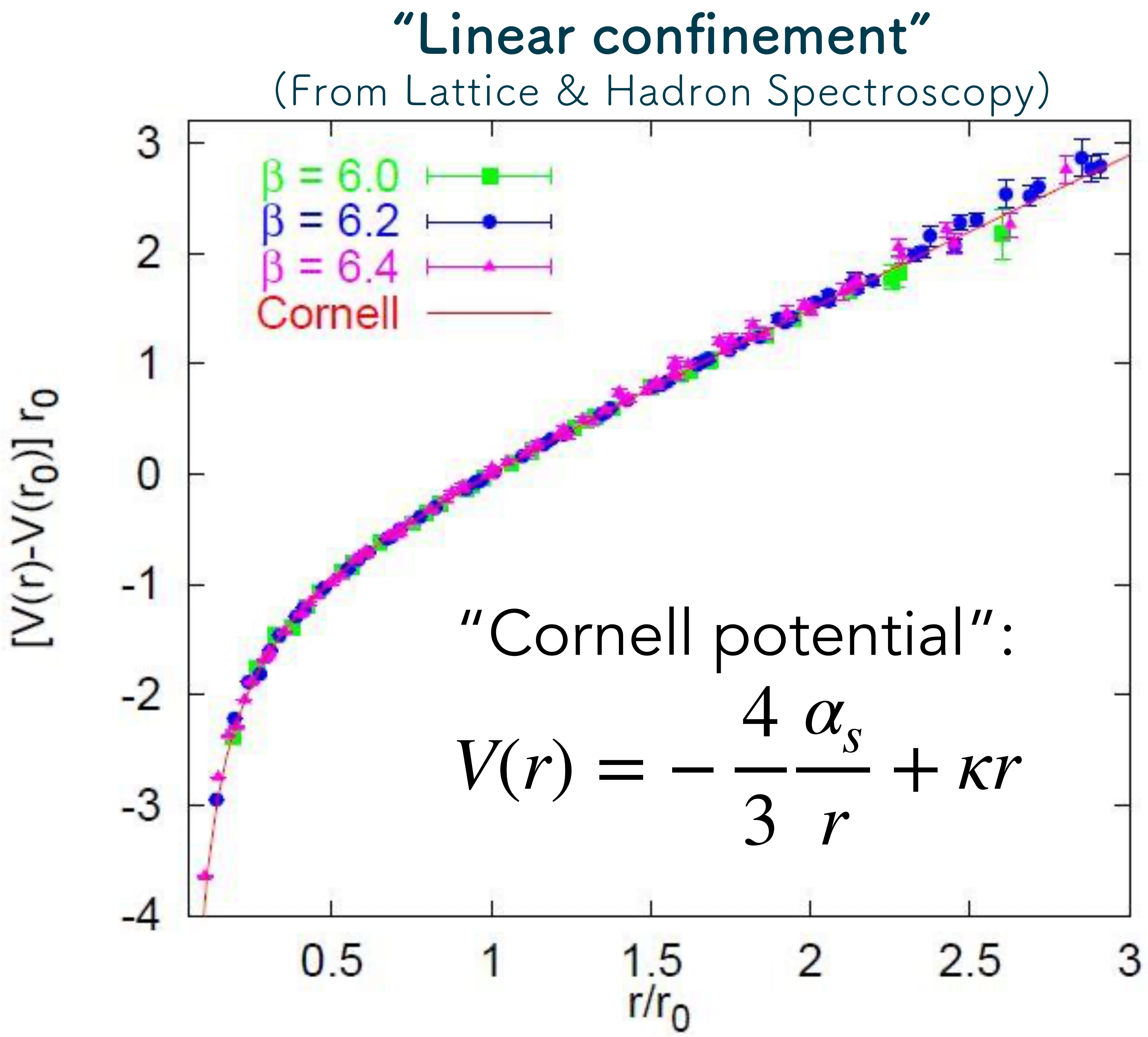
These are the **building blocks of parton showers** (DGLAP, dipole, antenna, …)

(+ running coupling, unitarity, and explicit energy-momentum conservation.)

Confinement in PYTHIA: *The Lund String Model*

Simplified (leading-N_C) **"colour flow"** ➜ determine between which partons to set up confining potentials





Map from Partons to Strings:

Quarks ➡ string endpoints; gluons ➡ transverse "kinks"

System then evolves as a string world sheet

+ String breaks via spontaneous $q\bar{q}$ pair creation ("Schwinger mechanism") → **hadrons**

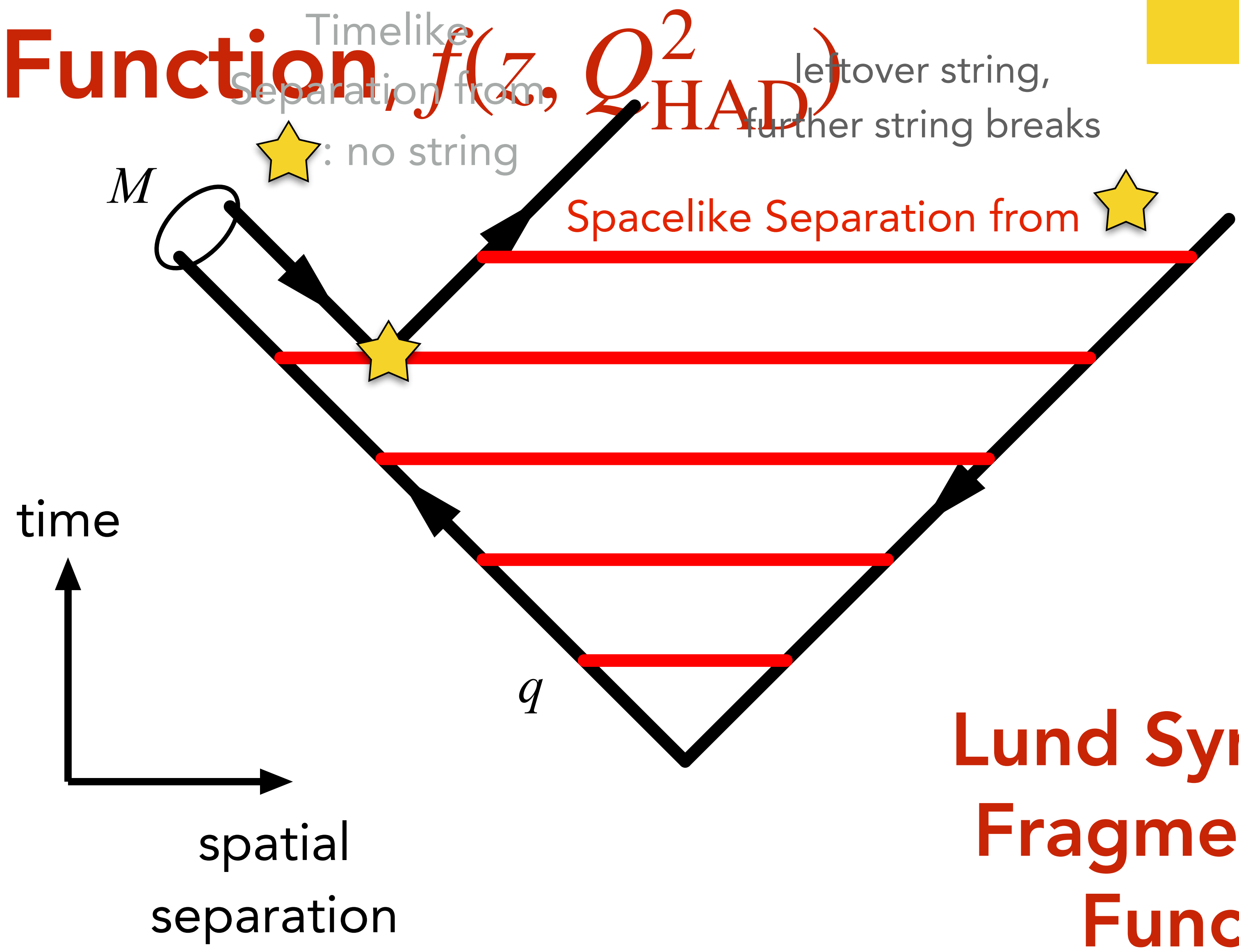
The String Fragmentation Function

Consider a string break ☆, producing a meson M, and a leftover string piece

The meson M takes a fraction z of the quark momentum,

Probability distribution in $z \in [0,1]$ parametrised by **Fragmentation Function**, $f(z, Q^2_{\mathrm{HAD}})$

Observation: All string breaks are **causally disconnected**



Lorentz invariance $\Longrightarrow$ string breaks can be considered in *any order*. Imposes "left-right symmetry" on the **FF**

$\Longrightarrow$ **FF** constrained to a form with **two free parameters**, a & b: constrained by fits to measured hadron spectra

Lund Symmetric Fragmentation Function

$$f(z) \propto \frac{1}{z}(1-z)^a \exp\left(-\frac{b(m_h^2 + p_{\perp h}^2)}{z}\right)$$

Supresses high-z hadrons (↑ a)

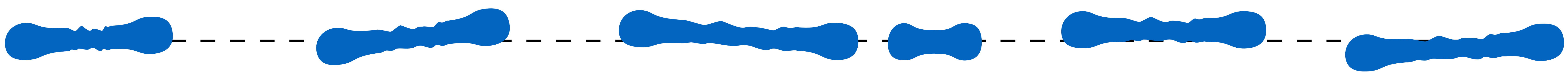
Supresses low-z hadrons (↑ b)

Automated Hadronization Uncertainties

NEW

Problem:

Given a colour-singlet system that (randomly) broke up into a specific set of hadrons:



What is the **relative probability** that same system would have resulted, if the fragmentation parameters had been **different?**

Would this particular final state become **more likely** ($w' > 1$)? Or **less likely** ($w' < 1$)

Crucially: **maintaining unitarity** $\Longrightarrow$ inclusive cross section remains unchanged!

August 2023: Bierlich, Ilten, Menzo, Mrenna, Szewc, Wilkinson, Youssef, Zupan

[Reweighting MC Predictions & Automated Fragmentation Variations in Pythia 8, 2308.13459]

Method is general; demonstrated on variations of the 7 main parameters governing longitudinal and transverse fragmentation functions in PYTHIA 8

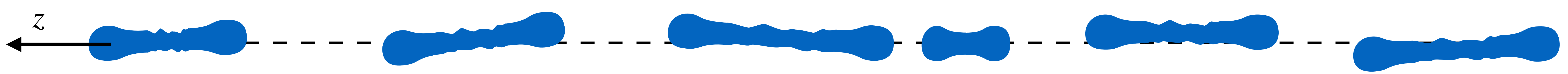
https://gitlab.com/uchep/mlhad-weights-validation

Pythia 8.311

Demonstration

[*Reweighting MC Predictions & Automated Fragmentation Variations in Pythia 8*, 2308.13459]

Example: Longitudinal Fragmentation Function (Lund Symmetric FF)



$f(z)$ ~ ***scaled* light-cone hadron momentum fraction**

$$\propto \frac{1}{z^{1+r_Q b m_Q^2}} (1-z)^a \exp\left(-\frac{b m_\perp^2}{z}\right)$$

variations

Reweighting Methodology:

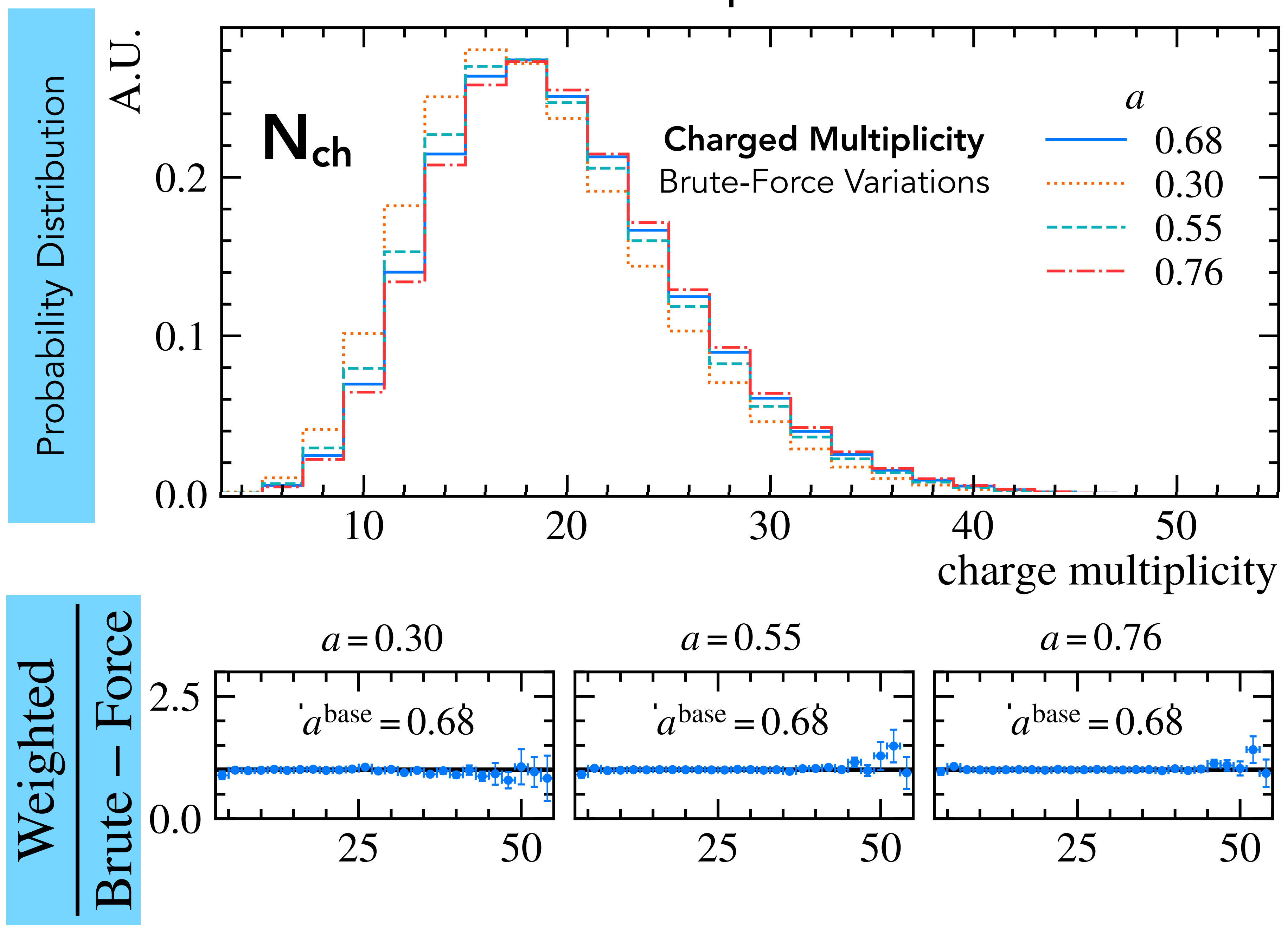
Accept-Reject Algorithm (analogous to shower variations):

$$w' = w \prod_{i \in \text{accepted}} R'_{i,\text{accept}}(z) \prod_{j \in \text{rejected}} R'_{j,\text{reject}}(z),$$

with

$$R'_{\text{accept}}(z) = \frac{P'_{\text{accept}}(z)}{P_{\text{accept}}(z)} \qquad R'_{\text{reject}}(z) = \frac{P'_{\text{reject}}(z)}{P_{\text{reject}}(z)} = \frac{1-P'_{\text{accept}}(z)}{1-P_{\text{accept}}(z)}$$

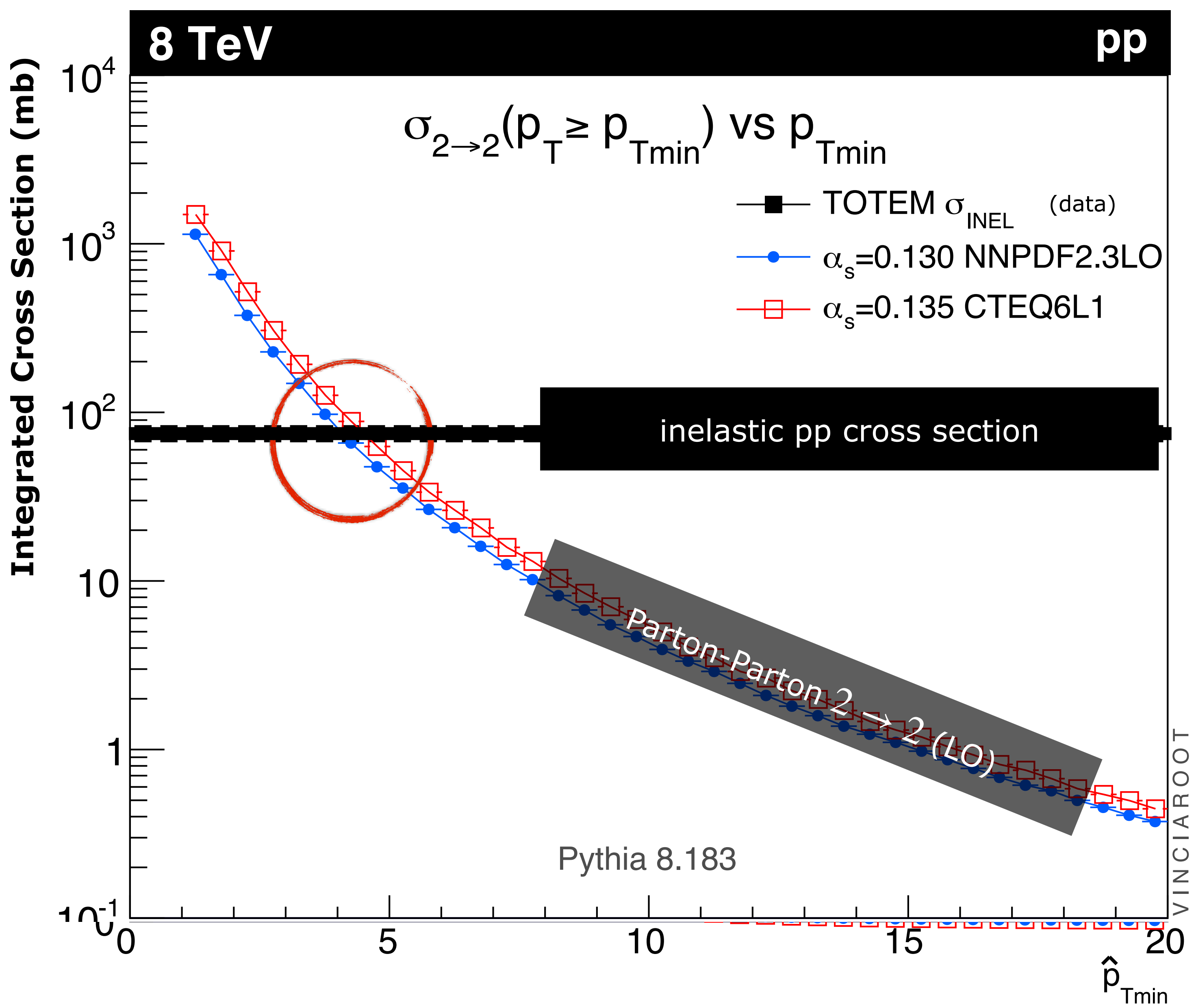
Example



A Brief History of MPI in PYTHIA

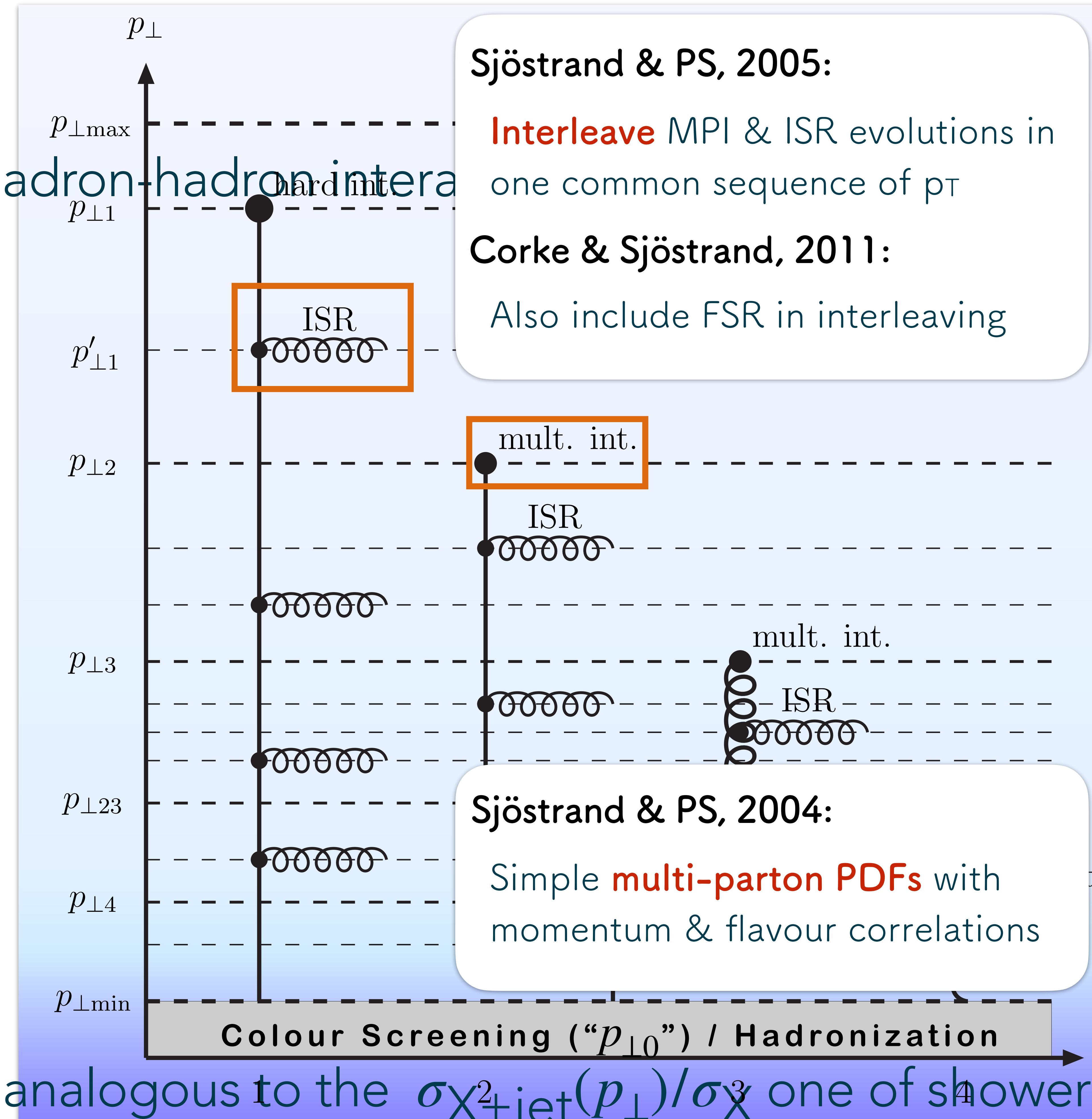
$$\frac{\sigma_{\text{parton-parton}}(\hat{p}_\perp)}{\sigma_{\text{hadron-hadron}}} > 1$$

$\Longrightarrow$ several parton-parton interactions *per* hadron-hadron intera



Sjöstrand & van Zijl, 1985:

Cast as **Sudakov-style evolution equation**, analogous to the one of showers



Sjöstrand & PS, 2005:

Interleave MPI & ISR evolutions in one common sequence of pT

Corke & Sjöstrand, 2011:

Also include FSR in interleaving

Sjöstrand & PS, 2004:

Simple **multi-parton PDFs** with momentum & flavour correlations

Figure from Sjöstrand & PS, 2005