NNLO Matrix-Element Corrections

1. Brief overview of **current** (N)NLO matching approaches
(using off-the-shelf showers, **with LO Shower Kernels**)

2. **New:** fully-differential NNLO matching scheme
(based on "sectorised" **NLO Shower Kernels** ➜ `VinciaNNLO`)

3. Outlook

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Science Coffee • Lund • Sep 2022

Fixed Order Calculations & Parton Showers

Fixed Order pQCD

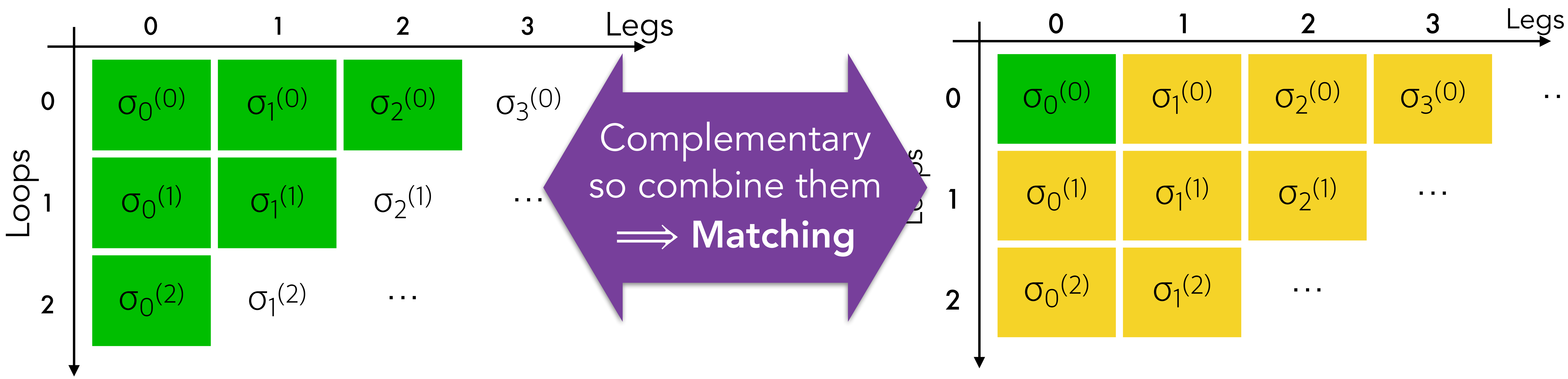
Hard QCD corrections

Well-resolved jets

Parton Showers

Jet substructure & soft radiation; recoil effects

Precursor for hadronisation, particle-level events



Definition: $\sigma_j^{(\ell)}$ = perturbative coefficient* for X + j jets, at order $(\alpha_s)^{j+\ell}\sigma_0^{(0)}$

[green] = The full perturbative coefficient

[yellow] = LO shower kernel (correct single-unresolved limits, leading poles)

*Glossing over 1/N$_C$ expansion

NLO + PS Matching

NLO singularity structure = single-unresolved limits

Matched by LO kernels in off-the-shelf showers*

*Still glossing over some colour subtleties, not the main point here.

Loops \ Legs	0	1	2	3	
0	$\sigma_0^{(0)}$	$\sigma_1^{(0)}$	$\sigma_2^{(0)}$	$\sigma_3^{(0)}$	…
1	$\sigma_0^{(1)}$	$\sigma_1^{(1)}$	$\sigma_2^{(1)}$	…	
2	$\sigma_0^{(2)}$	$\sigma_1^{(2)}$	…		

NLO+PS: two **general** approaches

- MC@NLO [Frixione, Webber hep-ph/0204244]
  modified subtraction with shower kernels
- POWHEG [Nason hep-ph/0409146] [Bengtsson, Sjöstrand, PLB185(1987)435]
  Born-local NLO weight + MEC in shower
- (refinements KRKNLO [Jadach et al. 1503.06849]
  and MACNLOPS [Nason, Salam 2111.03553])

Some "challenges" (largely well explored & understood by now, but relevant to remind before discussing NNLO)

[Frederix et al., 2002.12716]

MC@NLO: subtraction terms for each PS; negative weights (→ MC@NLO-Δ)

POWHEG: mismatches between POWHEG and PS evolution variables can be numerically important even when formally subleading (→ truncated showers)

Recent example: Powheg + Pythia for VBF [Höche et al., 2106.10987]

Slide adapted from C. Preuss

Status of NNLO + PS Matching

NNLO singularity structure = single- and double-unresolved limits

- Double-unresolved / 2nd-order singularities **not** matched by (iterated) LO kernels.
- These must be dealt with (regulated/unitarised) entirely on the non-shower side.

Loops \ Legs	0	1	2	3	
0	$\sigma_0^{(0)}$	$\sigma_1^{(0)}$	$\sigma_2^{(0)}$	$\sigma_3^{(0)}$	…
1	$\sigma_0^{(1)}$	$\sigma_1^{(1)}$	$\sigma_2^{(1)}$	…	
2	$\sigma_0^{(2)}$	$\sigma_1^{(2)}$	…		

NNLO+PS: first approaches, for some processes

- UN2LOPS [Höche et al. 1405.3607]
  inclusive NNLO + unitary merging
- NNLOPS/MiNNLOPS
  [Hamilton et al. 1212.4504]/[Monni et al. 1908.06987]
  regulated NLO POWHEG $1j$ + NNLO
- GENEVA [Alioli et al. 1211.7049]
  NNLO matched resummation + truncated shower

Some challenges (depending on your point of view):

UN2LOPS: Sudakov from explicit unitarisation → event-weight flips → low efficiencies?

MiNNLOPS/GENEVA: need analytic NNLL-NNLO Sudakov; done for several processes.

Resummation and shower p_T variables must be the same to LL. Effects of mismatches beyond controlled orders? Complex processes / “semi-unresolved” kinematics?

Much Recent Progress (since ~ 2021)

MiNNLOPS

Photon Pair Production [Gavardi et al., 2204.12602]

Top Pair Production [Mazitelli et al., 2112.12135]

VH production (with $H \to b\bar{b}$) [Zanoli et al., 2112.04168], [Haisch et al., 2204.00663]

VV & Vγ production [Buonocore et al., 2108.05337], [Lombardi et al., 2103.12077], [Lombardi et al., 2010.10478]

Full summary in Snowmass contribution [Buonocore et al., 2203.07240]

Geneva

Vγ production [Cridge et al., 2105.13214]

ZZ production [Alioli et al., 2103.01214]

Colour-singlet + N3LL [Alioli et al., 2102.08390]

Photon pair production [Alioli et al., 2010.10498]

UN2LOPS

Recently, mainly conceptual work on N3LO matching (TOMTE) [Prestel, 2106.03206] [Bertone, Prestel, 2202.01082]

New Approach: NNLO **Matrix-Element Corrections**

A Brief History of **Matrix-Element Corrections**

Historically, the oldest matching strategy

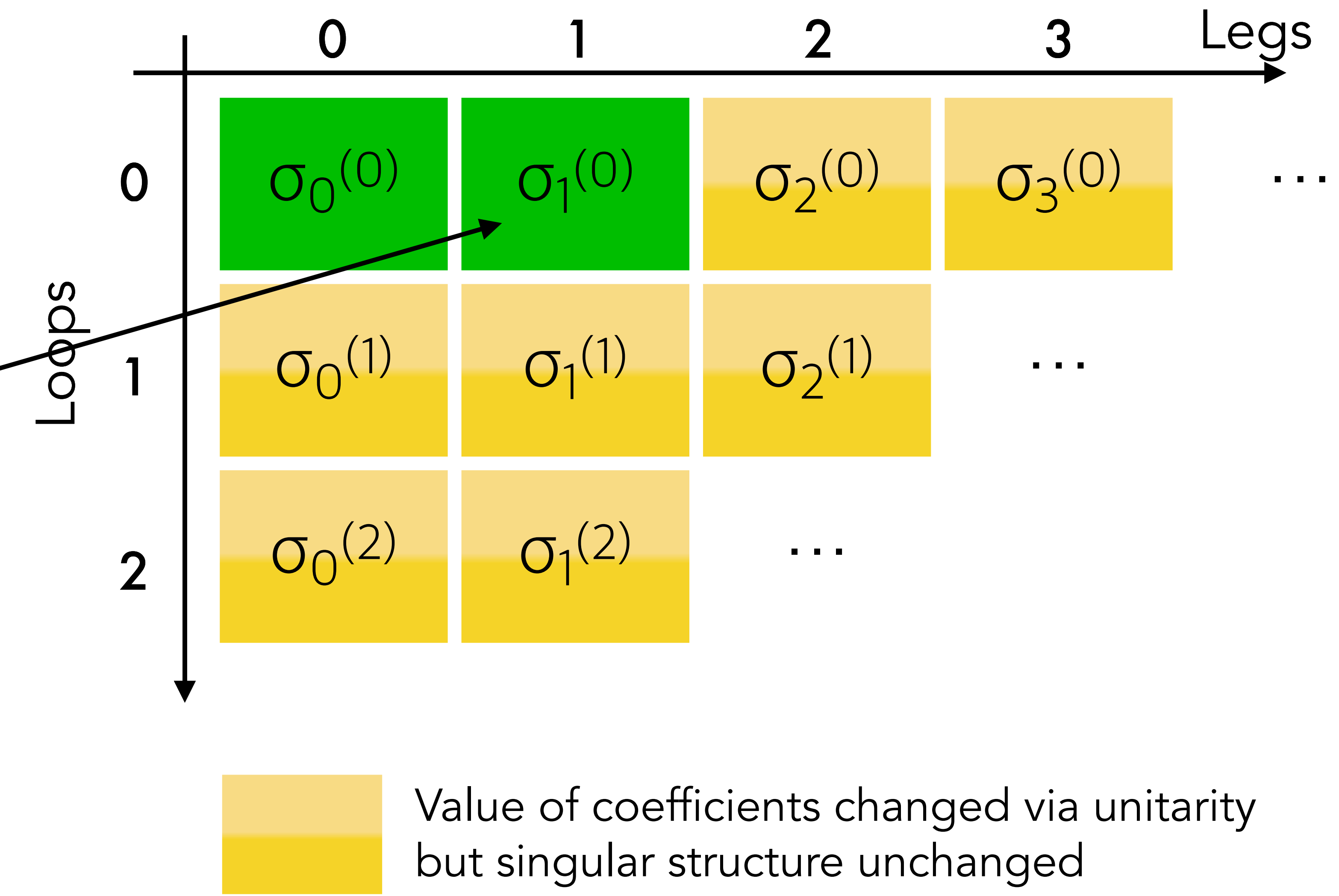
FSR: [Bengtsson, Sjöstrand, PLB185(1987)435];
ISR: [Miu, Sjöstrand, hep-ph/9812455]

Start from Born configuration; generate 1st shower emission as usual

But include real-emission ME/PS factor in accept probability

➜ PYTHIA default for hardest emission in single-H/V production processes & in most 2-body decays (incl BSM)

HERWIG also introduced MECs (+ ME events to populate a.o. dead zone) [Seymour, hep-ph/9410414]

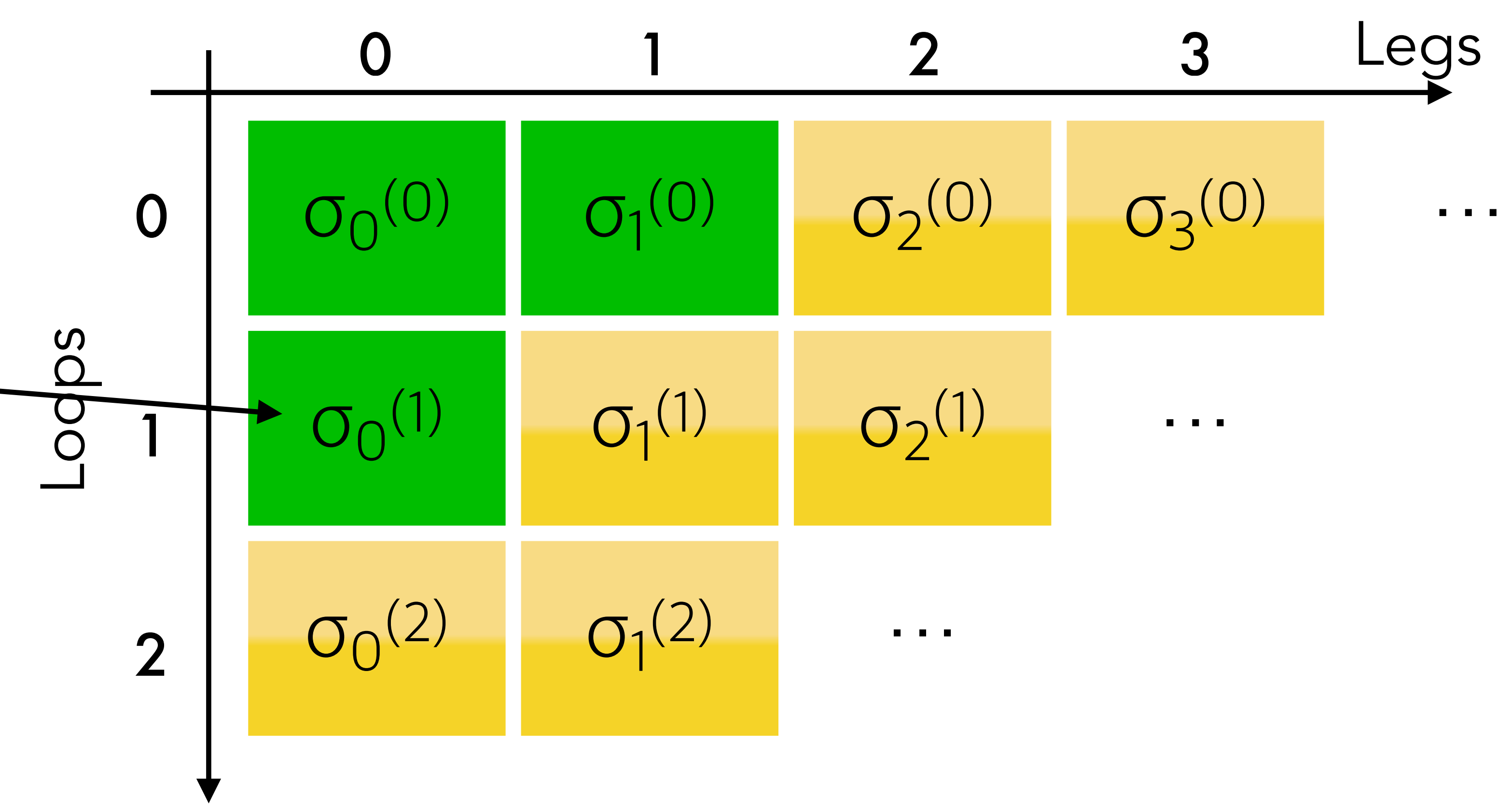


POWHEG: [Nason hep-ph/0409146]

Also include Born-local **NLO K-factor**

+ **Shower-agnostic** formulation applicable to **general processes**

➜ POWHEG BOX [Alioli et al, 1002.2581]



POWHEG as MECs

POWHEG master formula (for 2 Born jets):

LO Born |M|² Born One-loop MEC Shower off Born

$$\langle O \rangle^{\text{POWHEG}}_{\text{NLO+PS}} = \int \mathrm{d}\Phi_2 \, \boxed{\mathrm{B}(\Phi_2)} \, \boxed{k_{\text{NLO}}(\Phi_2)} \, \boxed{\mathcal{S}_2(t_0, O)}$$

local K-factor shower operator

Main trick: matrix-element correction (MEC) in first shower emission

$$\boxed{\mathcal{S}_2(t_0, O)} = \Delta_2(t_0, t_c) O(\Phi_2) + \int_{t_c}^{t_0} \boxed{\mathrm{d}\Phi_{+1} \, A_{2\mapsto 3}(\Phi_{+1})} \, \boxed{w^{\text{MEC}}_{2\mapsto 3}} \, \Delta_2(t, t_c) O(\Phi_2)$$

Shower PS and kernel Born + 1 Tree-level MEC

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Shower PS and kernel

Born + 1 Tree-level MEC

where $\boxed{w^{\text{MEC}}_{2\mapsto 3}} = \frac{\mathrm{R}(\Phi_2, \Phi_{+1})}{A_{2\mapsto 3}(\Phi_{+1})\mathrm{B}(\Phi_2)}$ and

Global showers: denominator is generally a sum of terms

Sector showers: denominator is normally a single term (discussed more later)

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Shower PS and kernel — Born + 1 Tree-level MEC — Unitarity

where $\boxed{w^{\text{MEC}}_{2\mapsto 3}} = \frac{\mathrm{R}(\Phi_2, \Phi_{+1})}{A_{2\mapsto 3}(\Phi_{+1}) \mathrm{B}(\Phi_2)}$ and

$$\Delta_2(t, t') = \exp\left(- \int_{t'}^{t} \boxed{\mathrm{d}\Phi_{+1} \, A_{2\mapsto 3}(\Phi_{+1})} \, \boxed{w^{\text{MEC}}_{2\mapsto 3}} (\Phi_2, \Phi_{+1}) \right)$$

Global showers: denominator is generally a sum of terms

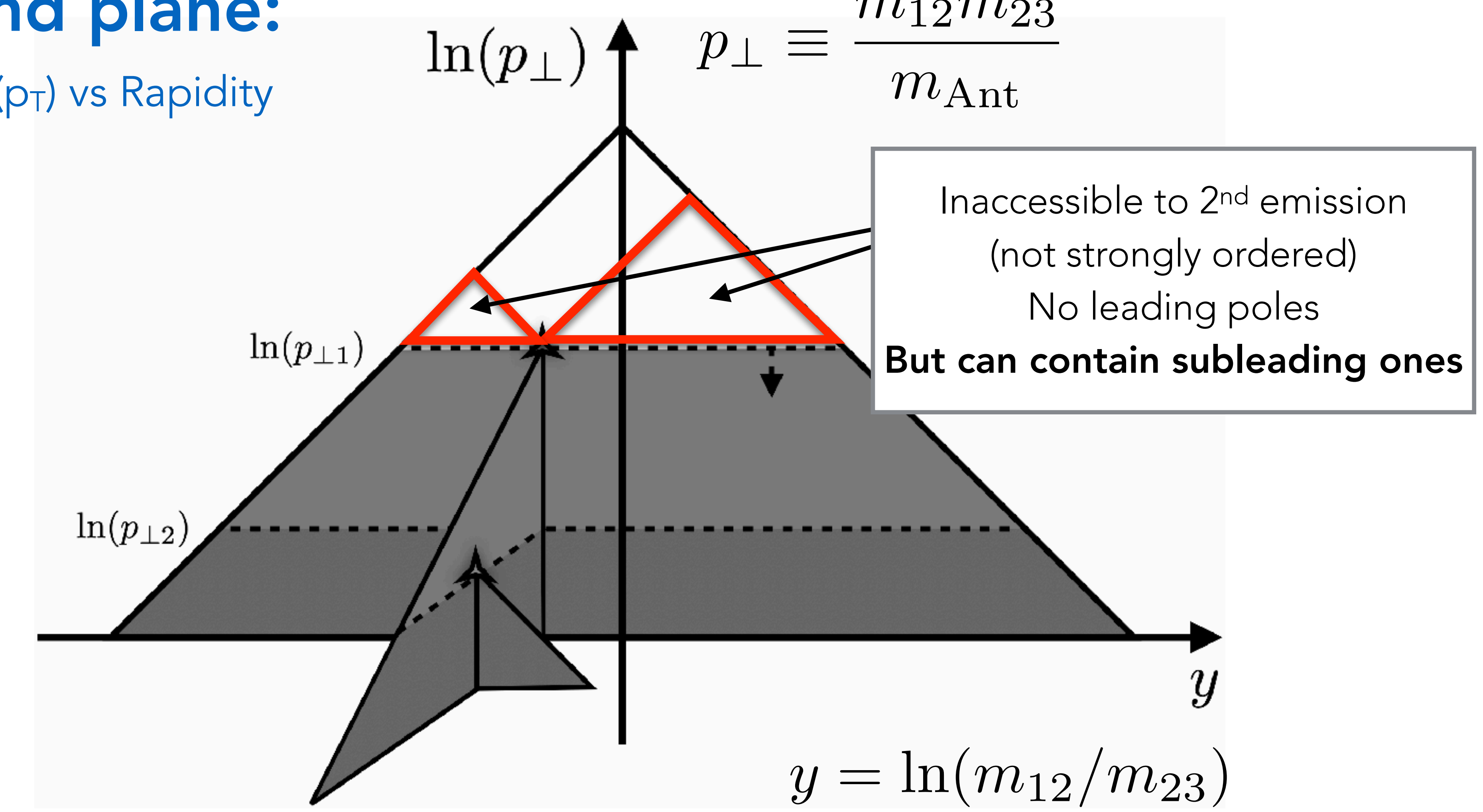
Sector showers: denominator is normally a single term (discussed more later)

Iterated MECs not possible with off-the-shelf showers

E.g., strong $p_\perp$-ordering **cuts out** part of the second-order phase space

Lund plane:

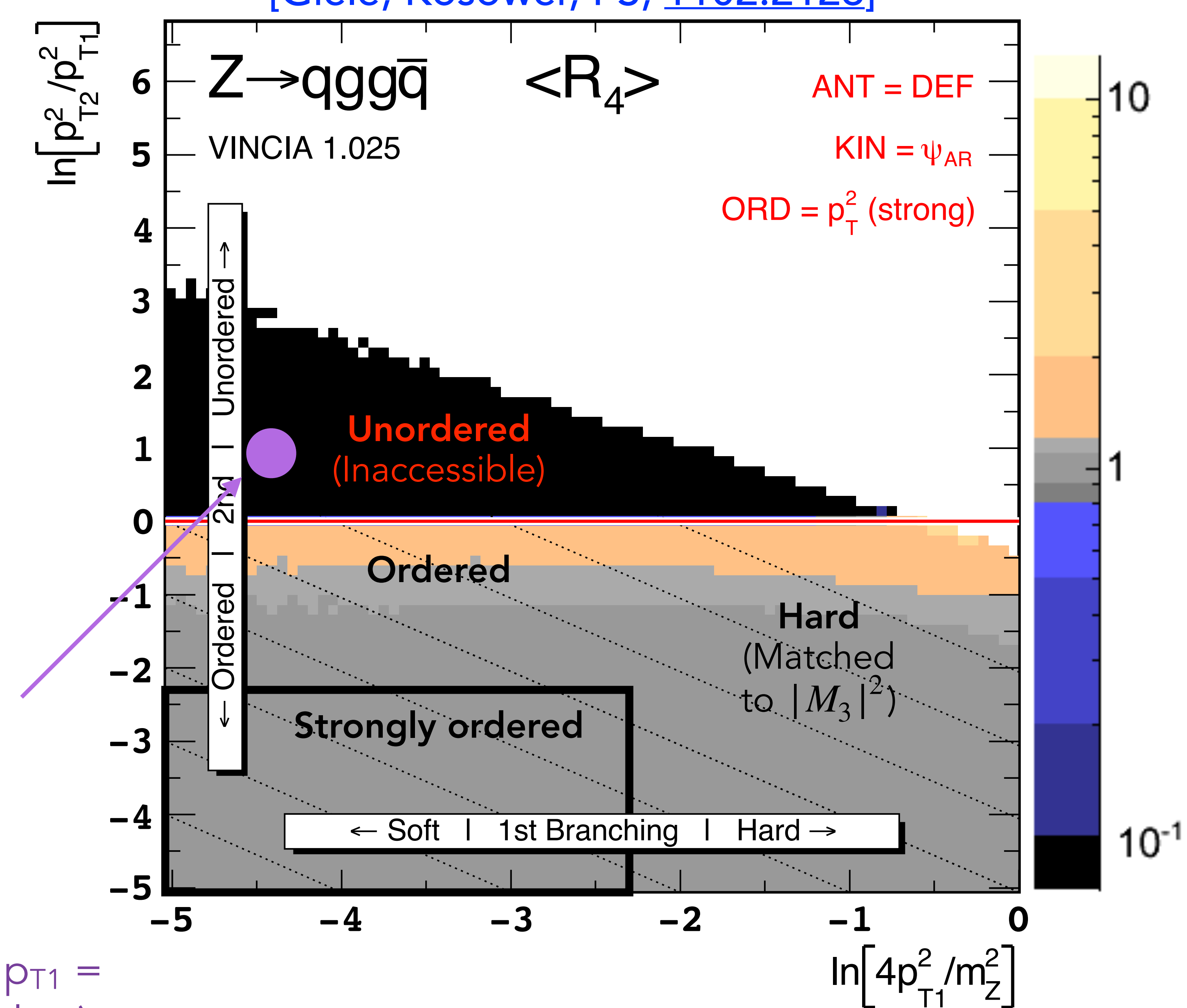
Log(p_T) vs Rapidity



Example: $Z \to qg\bar{g}\bar{q}$

$$R_4 = \frac{\text{Sum(shower histories)}}{|M^{(\rm LO,LC)}_{Z\to 4}|^2}$$

[Giele, Kosower, PS, 1102.2126]



Double-differential distribution in $\frac{p_{\perp 1}}{m_Z}$ & $\frac{p_{\perp 2}}{p_{\perp 1}}$ ➤

Example point: Q_0 = 91 GeV, p_{T1} = 5 GeV, p_{T2} = 8 GeV

Unordered but has $p_{\perp 2} \ll Q_0$: "Double Unresolved"

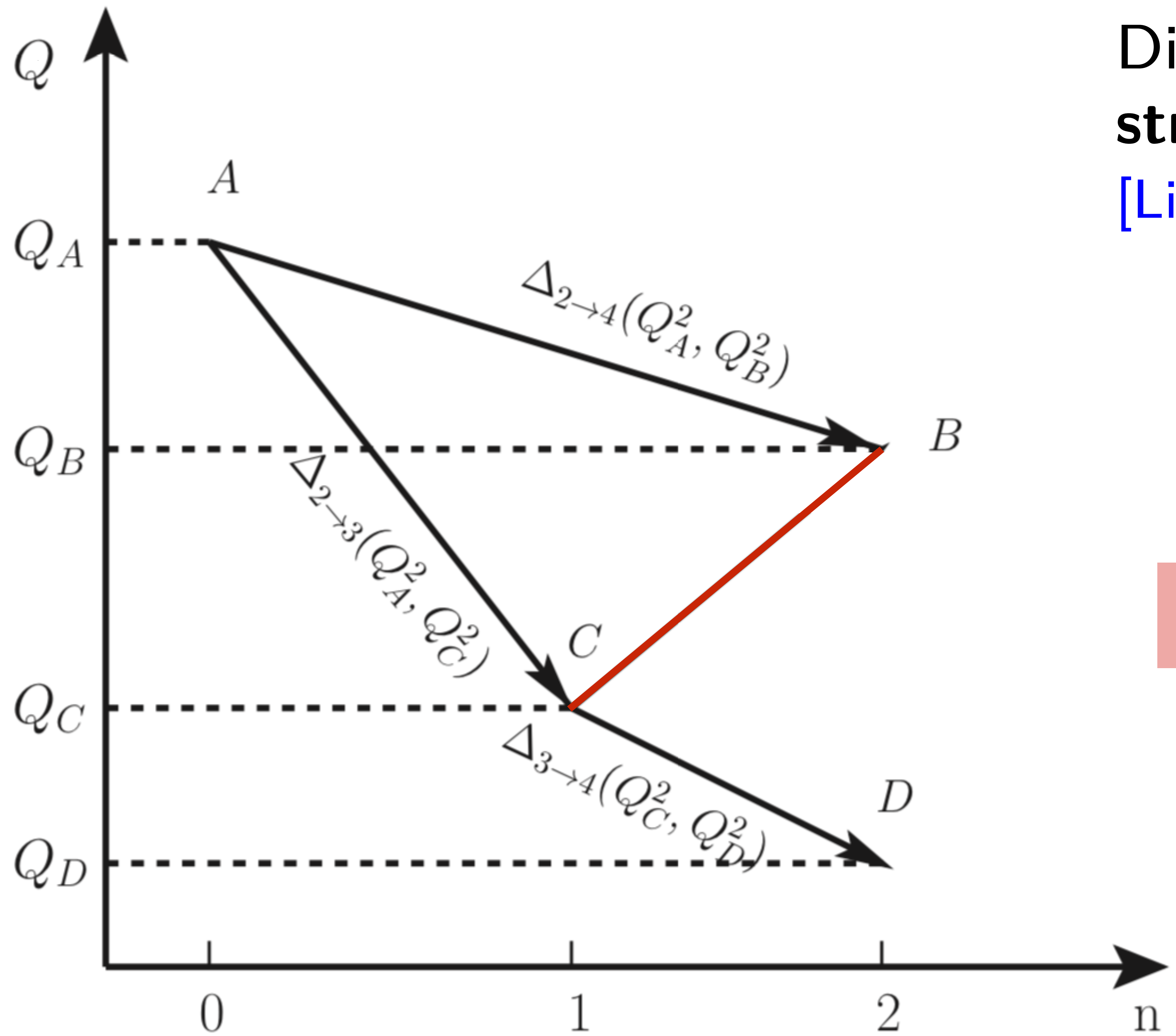
(Note: due to recoil effects, swapping the order of the two branchings does not simply give p_{T1} = 8 GeV, p_{T2} = 5 GeV but for this example point just produces a different unordered set of scales.)

(Averaged over other phase-space variables, uniform RAMBO scan)

Vice to Virtue: Define Ordered and Unordered Phase-Space **Sectors**

Ordered clusterings ⇔ **iterated single** branchings

Unordered clusterings ⇔ new **direct double** branchings



Divide double-emission phase space into **strongly-ordered** and **unordered** region: [Li, Skands 1611.00013]

$$\mathrm{d}\Phi_{+2} = \underbrace{\mathrm{d}\Phi^{>}_{+2}}_{\text{u.o.}} + \underbrace{\mathrm{d}\Phi^{<}_{+2}}_{\text{s.o.}}$$

Sector Definitions

"Ordered" $$\mathrm{d}\Phi^{<}_{+2} = \Theta\left(\hat{Q}^2_{+1} - Q^2_{+2}\right)\mathrm{d}\Phi_{+2}$$

"Unordered" $$\mathrm{d}\Phi^{>}_{+2} = \left(1 - \Theta\left(\hat{Q}^2_{+1} - Q^2_{+2}\right)\right)\mathrm{d}\Phi_{+2}$$

↗ ↗

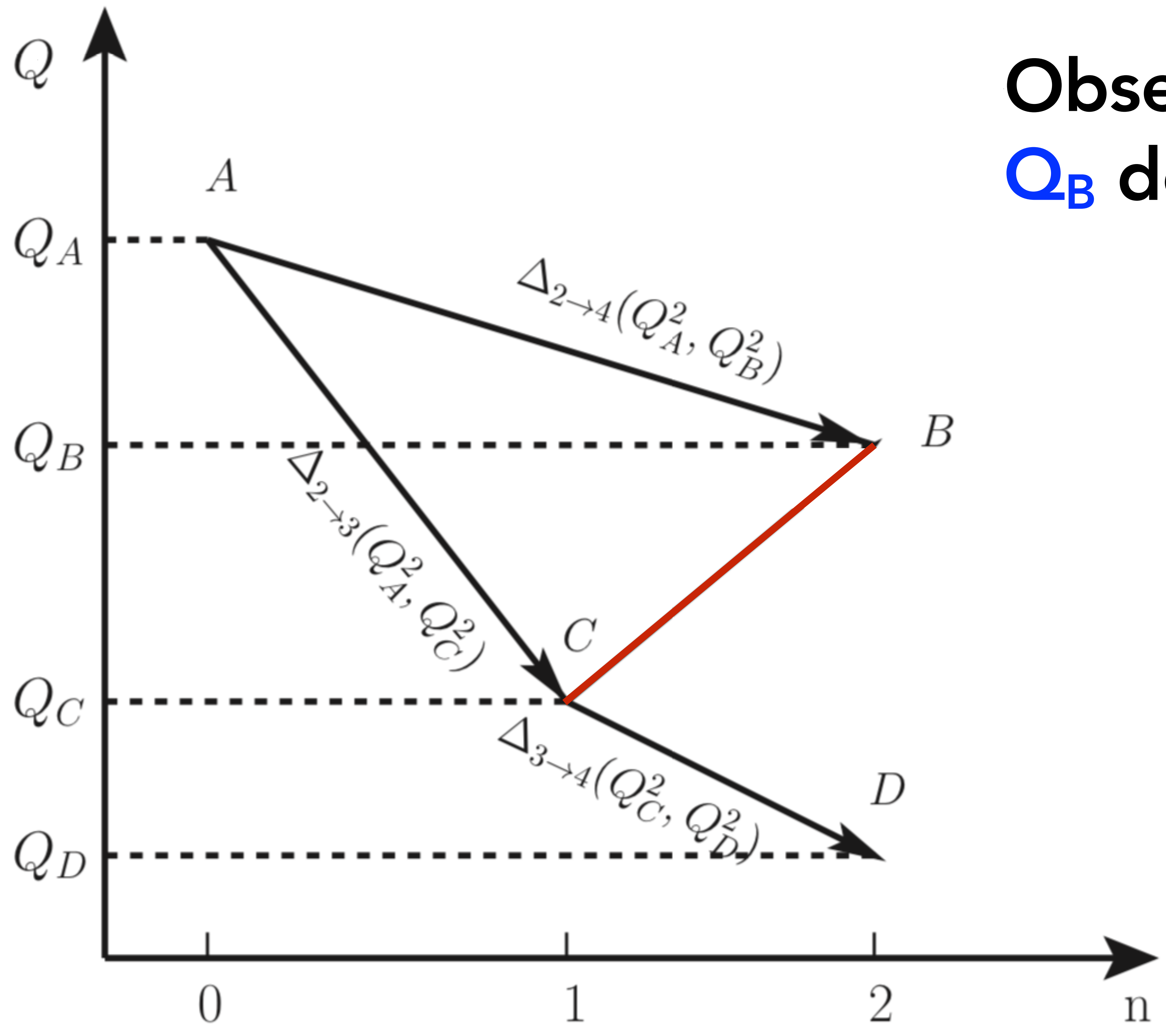
Unique scales provided by deterministic clustering algorithm

(In our case, the same as our sector-shower ordering variable)

Vice to Virtue: Define Ordered and Unordered Phase-Space **Sectors**

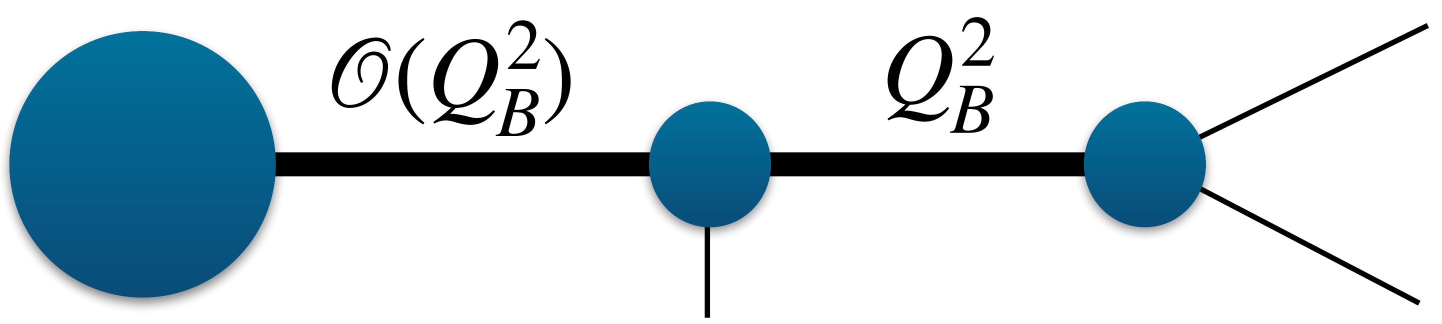
Ordered clusterings ⇔ **iterated single** branchings

Unordered clusterings ⇔ new **direct double** branchings



Observation: for direct double-branchings, Q_B defines the physical resolution scale

Corresponding Feynman diagram(s) have highly **off-shell** intermediate propagator



Intermediate "clustered" **on-shell** 3-parton state at (C) is merely a convenient stepping stone in phase space ⇨ integrate out

Direct (unordered) Double-Branching Generator

[Li & PS: PLB771 (2017) 59]

Sudakov integral for direct double branchings above scale $Q_B < Q_A$:

Unordered Sector

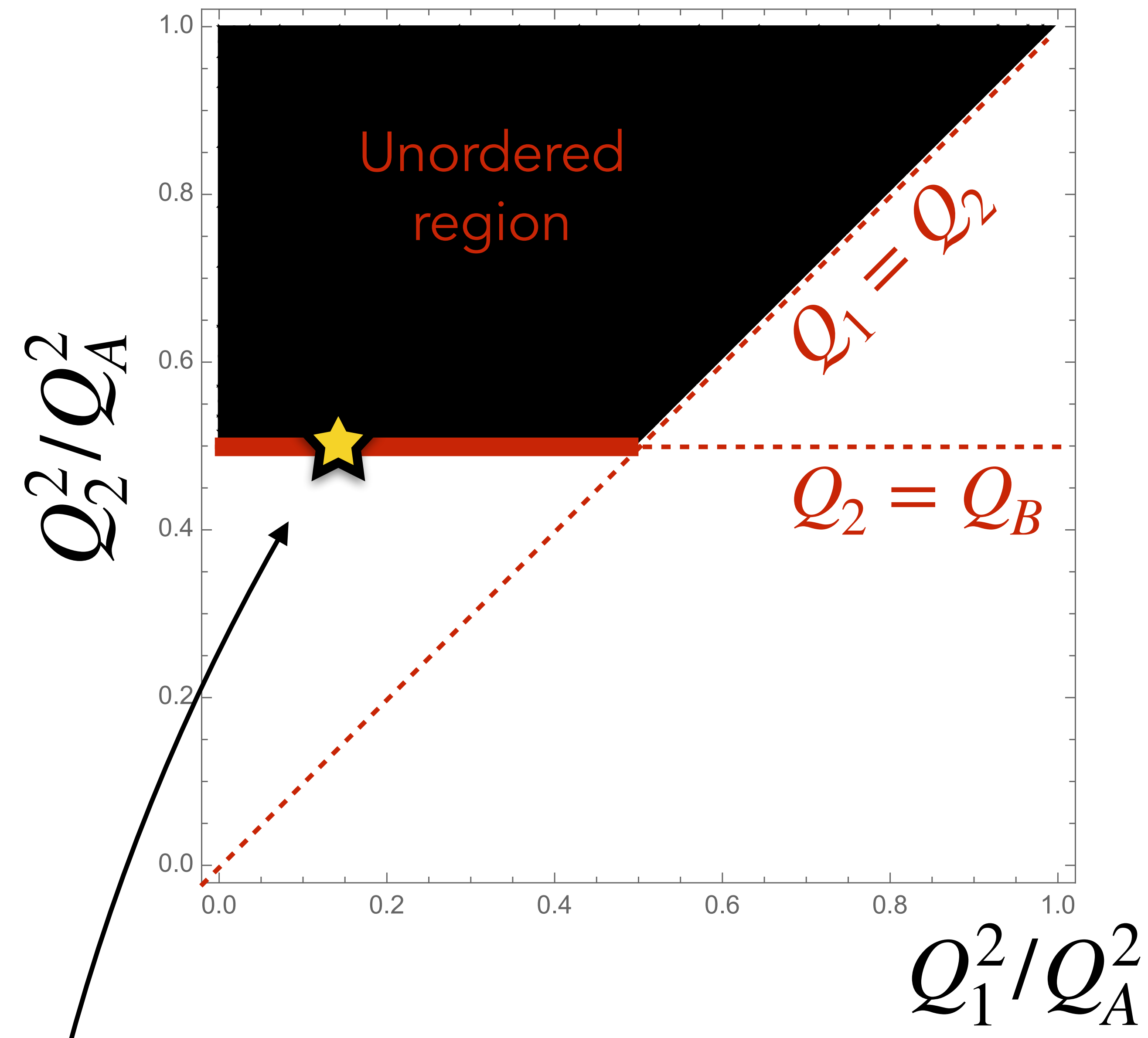
Generic double-branching kernel (overestimate)

$$-\ln \Delta(Q_A^2, Q_B^2) = \int_0^{Q_A^2} \mathrm{d}Q_1^2 \int_{Q_B^2}^{Q_A^2} \mathrm{d}Q_2^2 \; \Theta(Q_2^2 - Q_1^2) \; f(Q_1^2, Q_2^2)$$

We use: [Li & PS (2017); Giele, Kosower, PS (2011)]

$$f(Q_1^2, Q_2^2) \propto \frac{\alpha_s^2(Q_2^2)}{Q_2^2 \; (Q_1^2 + Q_2^2)}$$

see also backup slides



Swap integration order: outer integral along Q₂

$$= \int_{Q_B^2}^{Q_A^2} \mathrm{d}Q_2^2 \int_0^{Q_2^2} \mathrm{d}Q_1^2 \; f(Q_1^2, Q_2^2) = \int_{Q_B^2}^{Q_A^2} \mathrm{d}Q_2^2 \, F(Q_2^2)$$

➜ First generate physical scale Q_B, then generate $0 < Q_1 < Q_B$ + two z and φ choices

NNLO MECs

Iterated + Direct double branchings allows to fill all of phase space

$\Longrightarrow$ Can now consider NNLO MECs

Proof of concept for hadronic Z decays in VINCIA: [Campbell, Höche, Li, Preuss, PS, 2108.07133]

Idea: "POWHEG at NNLO" (focus here on $e^+e^- \to 2j$)

"Two-loop MEC"

$$\langle O \rangle^{\text{VINCIA}}_{\text{NNLO+PS}} = \int \mathrm{d}\Phi_2 \, \mathrm{B}(\Phi_2) \, \boxed{k_{\text{NNLO}}(\Phi_2)} \, \boxed{\mathcal{S}_2(t_0, O)}$$

local K-factor — shower operator

Need:

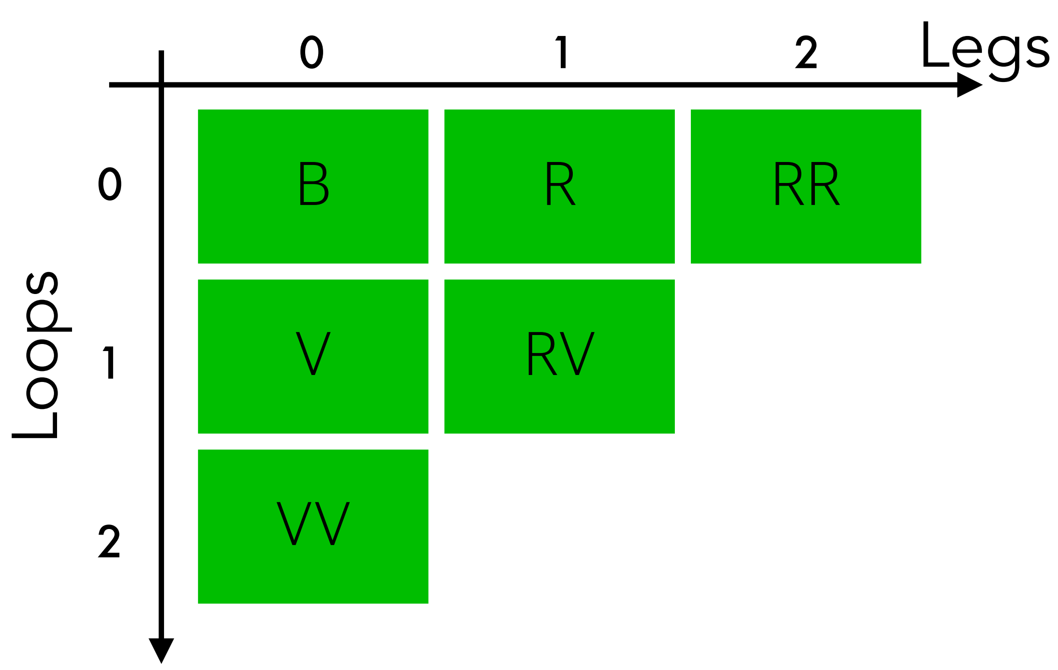
1. (Born-local) NNLO K-factors
2. shower filling strongly-ordered and unordered regions of 1- and 2-emission phase space
3. tree-level MECs in strongly-ordered and unordered shower paths
4. NLO MECs in the first emission

1. Born-Local NNLO K-factor

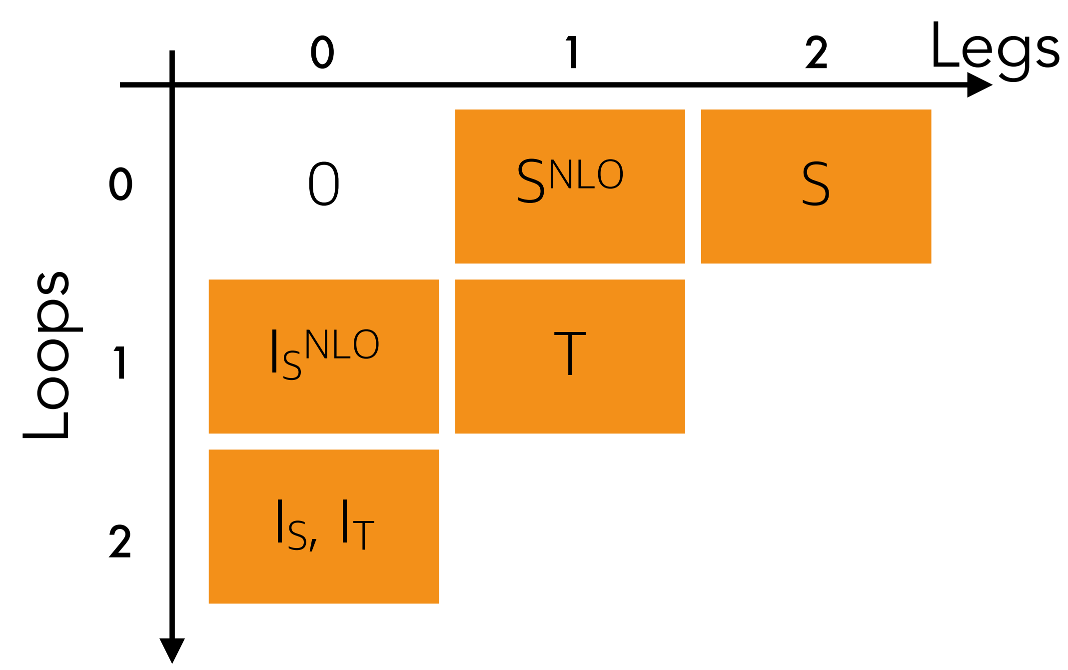
Reweight each Born-level event by **local** *K*-factor

$$k_{\rm NNLO}(\Phi_2) = 1 + \frac{\rm V(\Phi_2)}{\rm B(\Phi_2)} + \frac{\rm I_S^{NLO}(\Phi_2)}{\rm B(\Phi_2)} + \frac{\rm VV(\Phi_2)}{\rm B(\Phi_2)} + \frac{\rm I_T(\Phi_2)}{\rm B(\Phi_2)} + \frac{\rm I_S(\Phi_2)}{\rm B(\Phi_2)}$$
$$+ \int {\rm d}\Phi_{+1} \left[\frac{\rm R(\Phi_2,\Phi_{+1})}{\rm B(\Phi_2)} - \frac{\rm S^{NLO}(\Phi_2,\Phi_{+1})}{\rm B(\Phi_2)} + \frac{\rm RV(\Phi_2,\Phi_{+1})}{\rm B(\Phi_2)} - \frac{\rm T(\Phi_2,\Phi_{+1})}{\rm B(\Phi_2)} \right]$$
$$+ \int {\rm d}\Phi_{+2} \left[\frac{\rm RR(\Phi_2,\Phi_{+2})}{\rm B(\Phi_2)} - \frac{\rm S(\Phi_2,\Phi_{+2})}{\rm B(\Phi_2)} \right]$$

Fixed-Order Coefficients:



Subtraction Terms (not tied to shower formalism):



Note: requires "Born-local" NNLO subtraction terms. Currently only for simplest cases.
Some ideas what to do in meantime — but anticipate such subtractions in near future

⊗ Shower Operator with Second-Order MECs

Key aspect

up to matched order, include **process-specific** **NLO corrections** into shower evolution:

1. correct first branching to exclusive ($< t'$) NLO rate:

$$\Delta^{\mathrm{NLO}}_{2\mapsto3}(t_0, t') = \exp\left\{ -\int_{t'}^{t_0} \mathrm{d}\Phi_{+1}\, \underline{\mathrm{A}_{2\mapsto3}(\Phi_{+1}) w^{\mathrm{NLO}}_{2\mapsto3}}(\Phi_2, \Phi_{+1}) \right\}$$

2. correct second branching to LO ME:

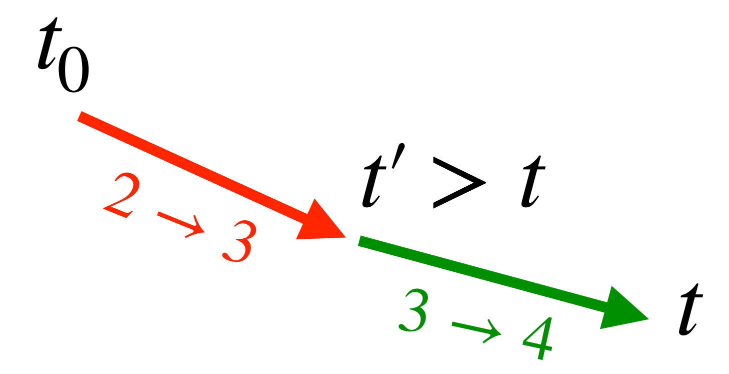
$$\Delta^{\mathrm{LO}}_{3\mapsto4}(t', t) = \exp\left\{ -\int_{t}^{t'} \mathrm{d}\Phi'_{+1}\, \underline{\mathrm{A}_{3\mapsto4}(\Phi'_{+1}) w^{\mathrm{LO}}_{3\mapsto4}}(\Phi_3, \Phi'_{+1}) \right\}$$

3. add direct $2 \mapsto 4$ branching and correct it to LO ME:

$$\Delta^{\mathrm{LO}}_{2\mapsto4}(t_0, t) = \exp\left\{ -\int_{t}^{t_0} \mathrm{d}\Phi^{>}_{+2}\, \underline{\mathrm{A}_{2\mapsto4}(\Phi_{+2}) w^{\mathrm{LO}}_{2\mapsto4}}(\Phi_2, \Phi_{+2}) \right\}$$

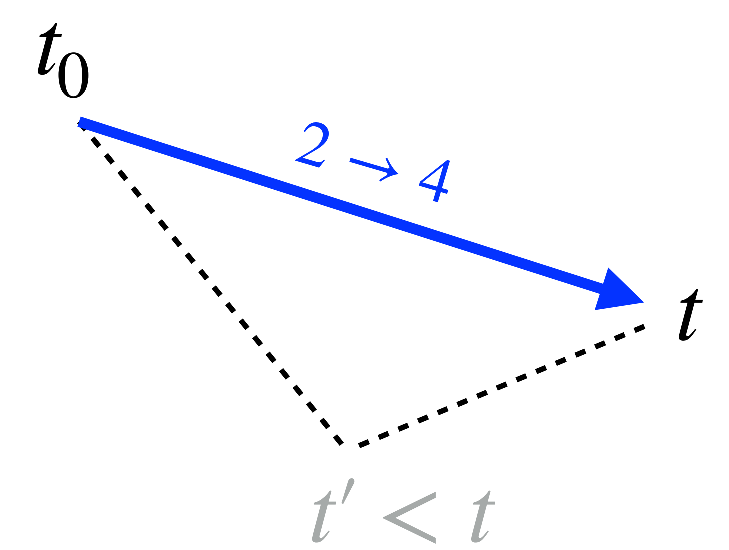
Iterated:

(Ordered)



Direct:

(Unordered)



⇒ entirely based on **MECs** and **sectorisation**

⇒ **by construction**, expansion of extended shower **matches** NNLO singularity structure

But shower kernels **do not** define **NNLO subtraction terms*** (!)

*This would be required in an "Mc@Nnlo" scheme, but difficult to realise in antenna showers.

Slide adapted from C. Preuss

2. Shower Filling both Single- and Double-Branching Phase Space

✓ Based on **Sector Antennae**

Suggested by Kosower [Kosower, PRD57(1998)5410; PRD71(2005)045016]; also used in [Larkoski & Peskin, PRD81(2010)054010; PRD84(2011)034034]

Divide n-gluon phase space into n non-overlapping sectors, inside each of which **only the most singular** kernel is allowed to contribute.

$\Longrightarrow$ Each sector branching kernel must contain the **full** soft-collinear singular structure of its sector ✅

Lorentz-invariant def of "most singular" gluon:

Based on ARIADNE $p^2_{\perp j} = \frac{s_{ij}s_{jk}}{s_{ijk}}$ with $s_{ij} \equiv 2(p_i \cdot p_j)$

(+ generalisations for heavy-quark emitters)

Suitable for **antenna approach**. Vanishes linearly when either $s_{ij} \to 0$ or $s_{jk} \to 0$, quadratically when both $\to 0$.

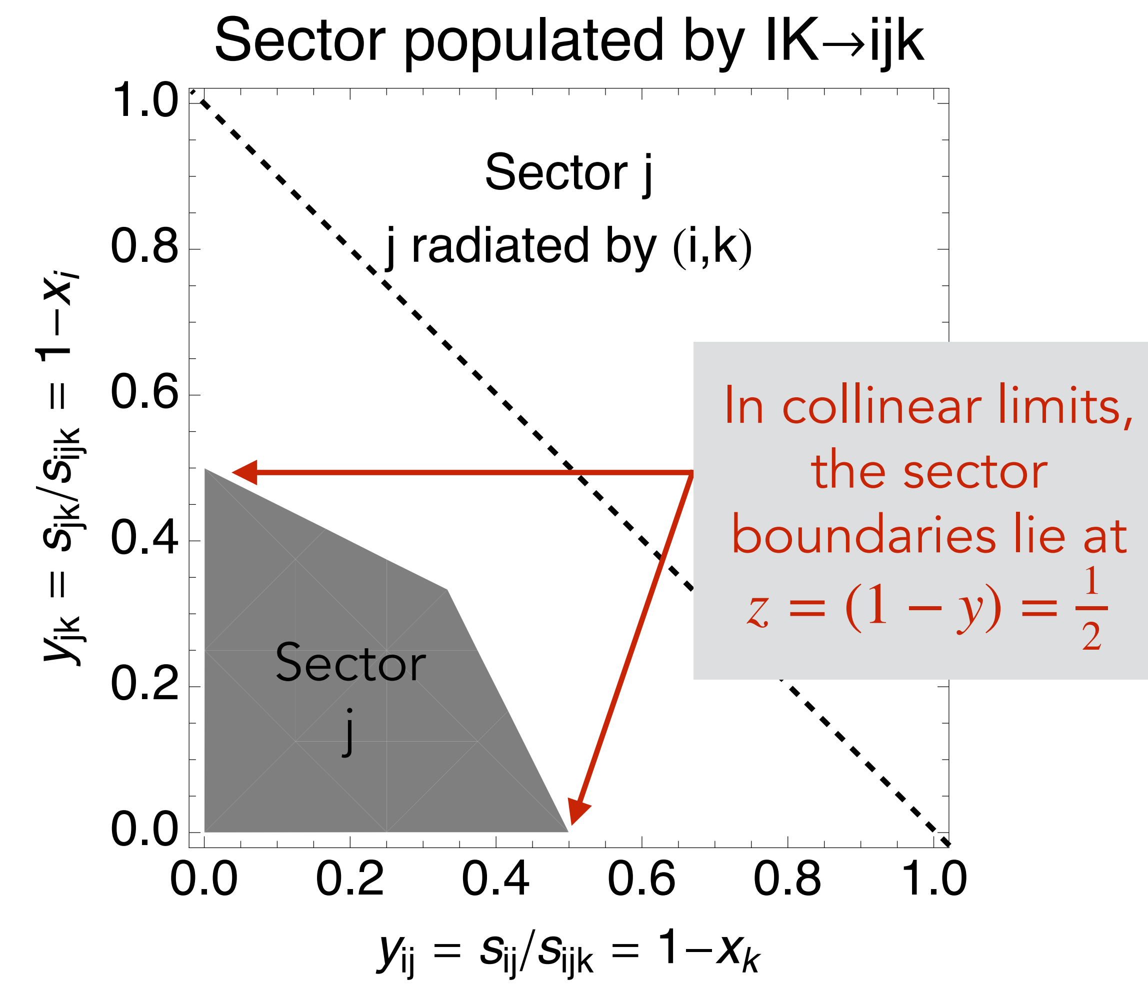
(Avoids splitting collinear and soft into separate sectors).

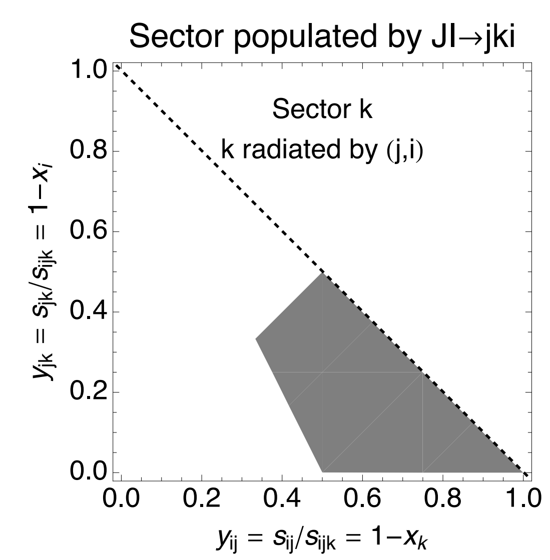
Produces same singularity structure as global approach, with a single history.

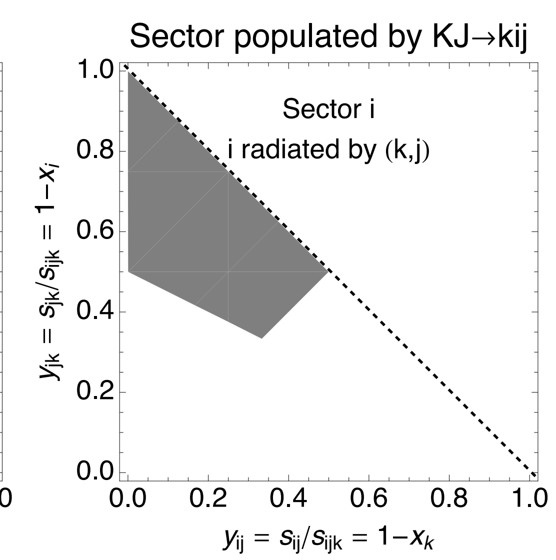
$\Longrightarrow$ with **a single unique scale**

(Generalisation to $g \to q\bar{q} \Longrightarrow$ factorial growth in same-flavour quark pairs.)

Example: single-branching sectors in $H \to g_i g_j g_k$







Single-Branching Sector Kernels

Sector antenna functions have to incorporate full single-unresolved limits for given PS point

- e.g. (FF) $qg \mapsto qgg$ ($s_{ij} = 2p_i \cdot p_j$):

$$A^{\mathrm{sct}}_{qg\mapsto qgg}(i_q, j_g, k_g) \to \begin{cases} \frac{2s_{ik}}{s_{ij}s_{jk}} & \text{if } j_g \text{ soft} \\ \frac{1}{s_{ij}}\frac{1+z^2}{1-z} & \text{if } i_q \parallel j_g \\ \frac{1}{s_{jk}}\frac{2(1-z(1-z))^2}{z(1-z)} & \text{if } j_g \parallel k_g \end{cases}$$

Compare to global antenna functions:

- only "half" of the $j_g \parallel k_g$ limit contained in the splitting kernel:

$$A^{\mathrm{gl}}_{qg\mapsto qgg}(i_q, j_g, k_g) \to \begin{cases} \frac{2s_{ik}}{s_{ij}s_{jk}} & \text{if } j_g \text{ soft} \\ \frac{1}{s_{ij}}\frac{1+z^2}{1-z} & \text{if } i_q \parallel j_g \\ \frac{1}{s_{jk}}\frac{1+z^3}{1-z} & \text{if } j_g \parallel k_g \end{cases}$$

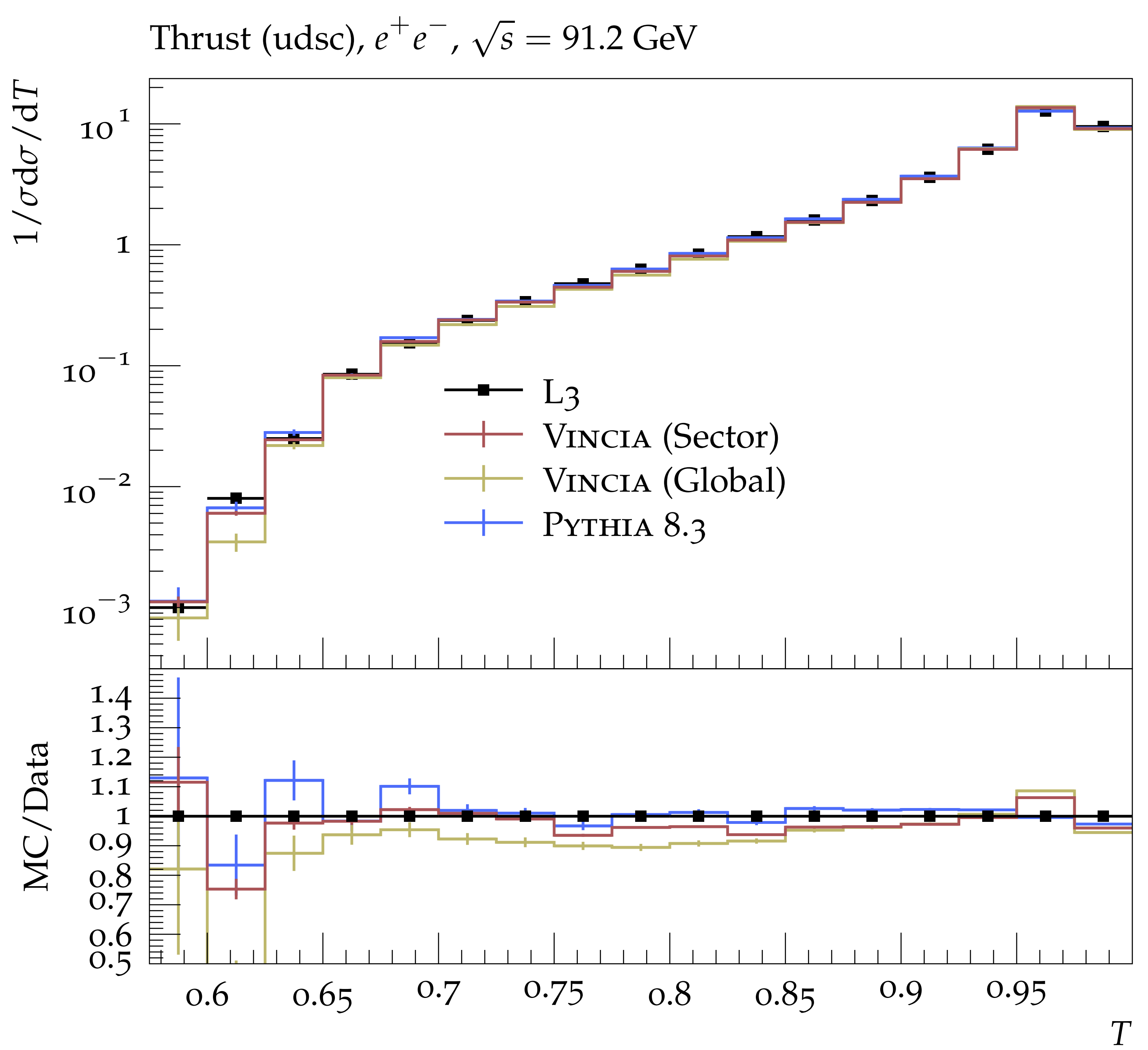
- "rest" of the jk-collinear limit reproduced by neighbouring antenna ($z \leftrightarrow 1 - z$)

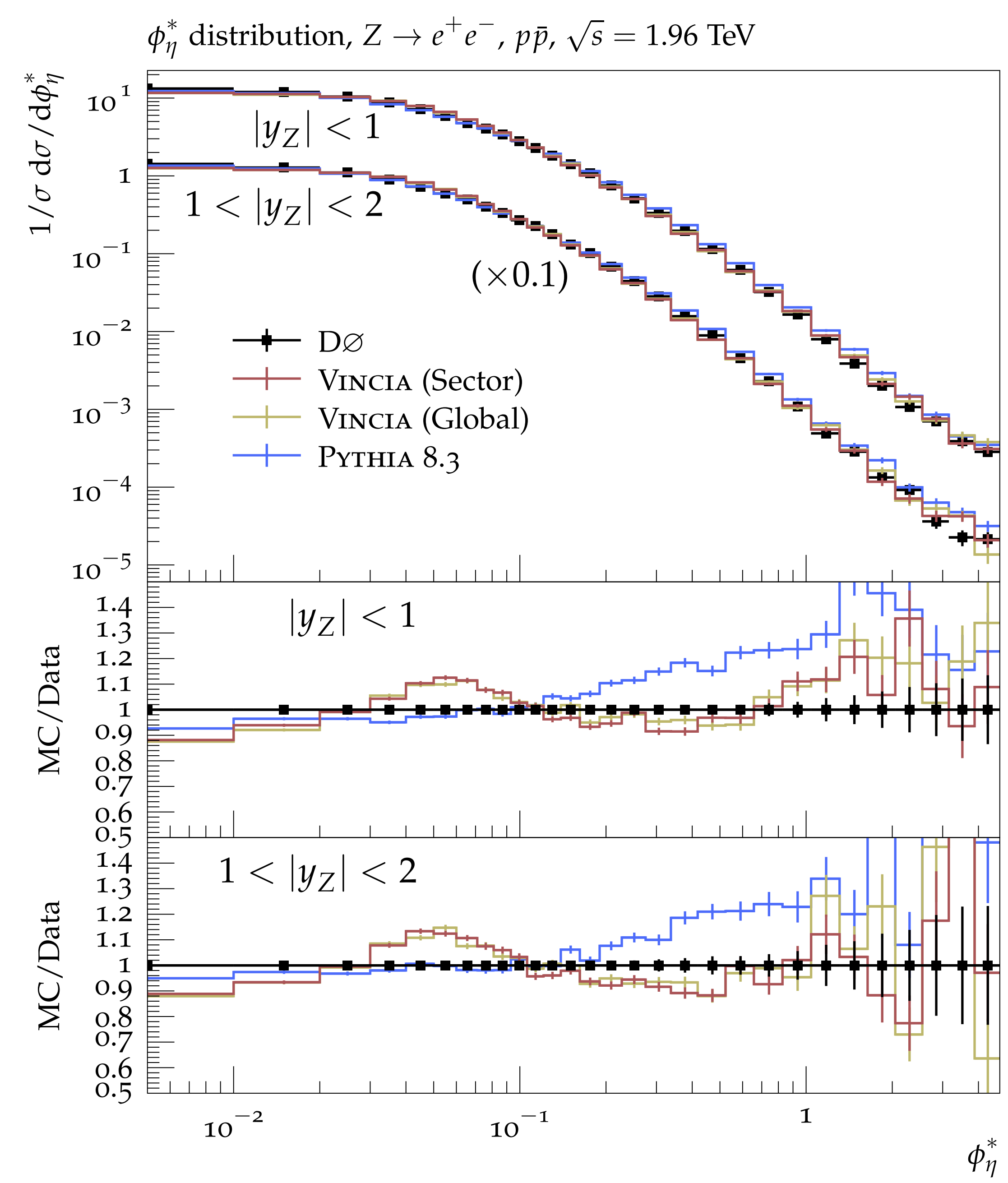
Slide adapted from C. Preuss

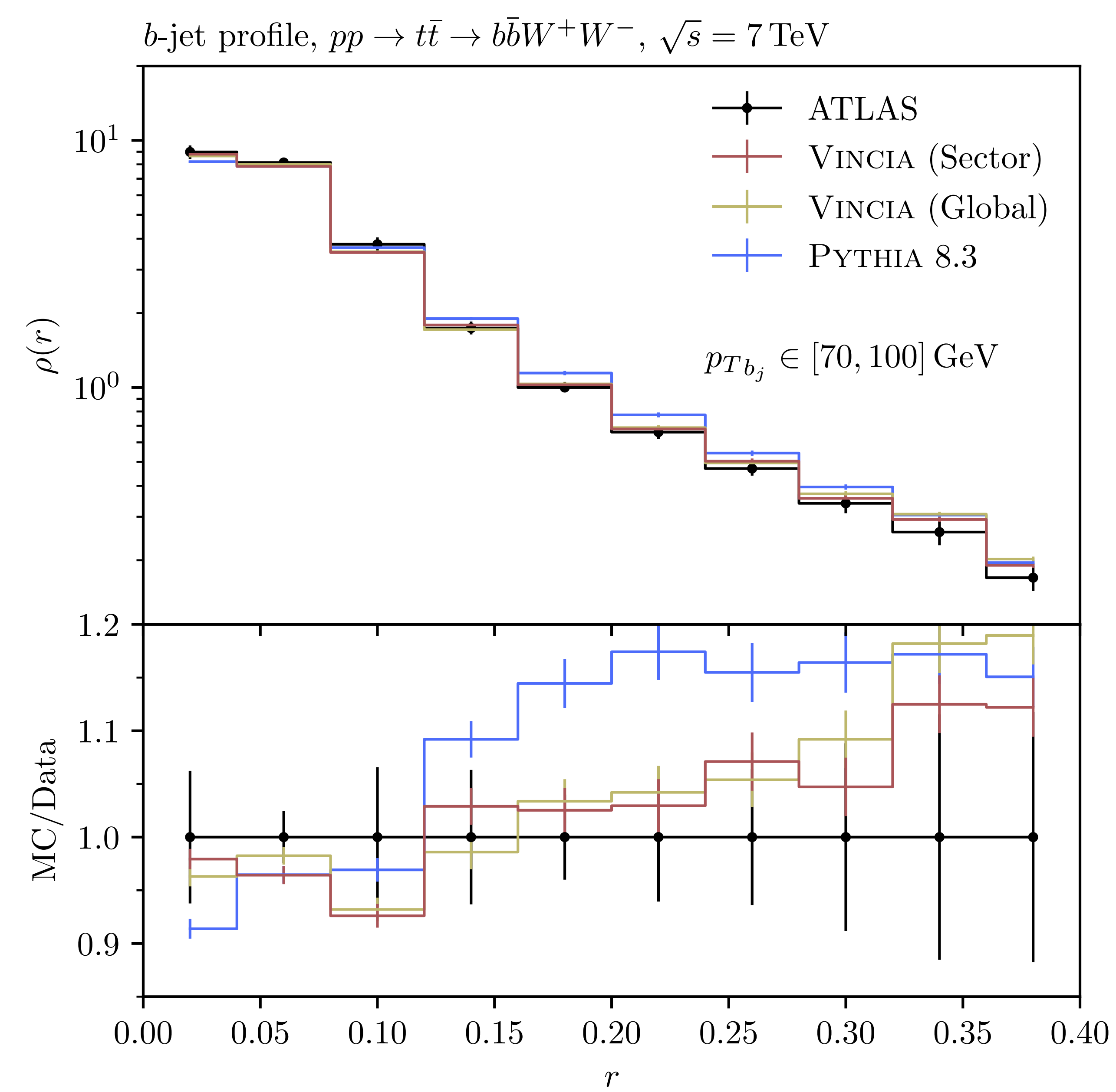
Full-fledged sector-antenna shower implemented in Pythia 8.304

`PartonShowers:Model = 2` [Brooks, Preuss & PS 2003.00702]

Sector approach is merely an **alternative way** to fraction singularities, so **formal accuracy*** of the shower should be **retained**.







Note: same (global) tune parameters used for sector runs with Vincia

[Hoche et al., 2106.10987]

NB: also fully compatible with POWHEG Box for NLO Matching (dedicated Vincia POWHEG UserHooks).

*We have not yet quantified the formal logarithmic accuracy of VINCIA.

Slide adapted from C. Preuss

3. Tree-Level MECs
(for both iterated-single and direct-double branchings)

MECs are **extremely** simple in sector showers

Sector kernels can be replaced by ratios of (colour-ordered) **tree-level MEs:**

- **Global** shower: $A^{\mathrm{glb}}_{IK\to ijk}(i,j,k) \to A^{\mathrm{glb}}_{IK\to ijk}\dfrac{|M_{n+1}(\ldots,i,j,k,\ldots)|^2}{\sum_{h\in\mathrm{histories}} A_h |M_n(\ldots I_h, K_h,\ldots)|^2}$ = **complicated** [Fischer & Prestel 1706.06218]

- **Sector** shower: $A^{\mathrm{sct}}_{IK\to ijk}(i,j,k) \to \dfrac{|M_{n+1}(\ldots,i,j,k,\ldots)|^2}{|M_n(\ldots I, K,\ldots)|^2}$ = **simple** [Lopez-Villarejo & PS 1109.3608]

Can also incorporate (fixed-order) sub-leading colour effects by "colour MECs":

[Giele, Kosower, PS, 1102.2126]

$$w_{\mathrm{col}} = \frac{\sum_{\alpha,\beta} \mathcal{M}_\alpha \mathcal{M}^*_\beta}{\sum_\alpha |\mathcal{M}_\alpha|^2}$$

Example: $Z \to q\bar{q} + 2g$

$$P_{\mathrm{MEC}} = w_{\mathrm{col}} \frac{A_4^0(1_q, 3_g, 4_g, 2_{\bar{q}})}{A_3^0(\widetilde{13}_q, \widetilde{34}_g, 2_{\bar{q}})}\theta(p^2_{\perp,134} < p^2_{\perp,243}) + w_{\mathrm{col}} \frac{A_4^0(1_q, 3_g, 4_g, 2_{\bar{q}})}{A_3^0(1_q, \widetilde{34}_g, \widetilde{23}_{\bar{q}})}\theta(p^2_{\perp,243} < p^2_{\perp,134})$$

$$w_{\mathrm{col}} = \frac{A_4^0(1,3,4,2) + A_4^0(1,4,3,2) - \frac{1}{N_C^2}\tilde{A}_4^0(1,3,4,2)}{A_4^0(1,3,4,2) + A_4^0(1,4,3,2)}$$

Real and Double-Real MEC factors

Separation of double-real integral defines tree-level MECs:

$$\int_t^{t_0} \mathrm{d}\Phi_{+2} \frac{\mathrm{RR}(\Phi_2, \Phi_{+2})}{\mathrm{B}(\Phi_2)} = \int_t^{t_0} \mathrm{d}\Phi_{+2}^{>} \frac{\mathrm{RR}(\Phi_2, \Phi_{+2})}{\mathrm{B}(\Phi_2)} + \int_t^{t_0} \mathrm{d}\Phi_{+2}^{<} \frac{\mathrm{RR}(\Phi_2, \Phi_{+2})}{\mathrm{B}(\Phi_2)}$$

$$= \int_t^{t_0} \mathrm{d}\Phi_{+2}^{>} \underbrace{\mathrm{A}_{2\mapsto 4}(\Phi_{+2}) w_{2\mapsto 4}^{\mathrm{LO}}(\Phi_2, \Phi_{+2})}_{\text{direct/unordered } n \to n+2}$$

$$+ \int_{t'}^{t_0} \mathrm{d}\Phi_{+1} \underbrace{\mathrm{A}_{2\mapsto 3}(\Phi_{+1}) w_{2\mapsto 3}^{\mathrm{LO}}(\Phi_2, \Phi_{+1})}_{\text{Iterated/ordered branching \#1}} \int_t^{t'} \mathrm{d}\Phi'_{+1} \underbrace{\mathrm{A}_{3\mapsto 4}(\Phi'_{+1}) w_{3\mapsto 4}^{\mathrm{LO}}(\Phi_3, \Phi'_{+1})}_{\text{Iterated/ordered branching \#2}}$$

Iterated tree-level MECs in **ordered** region:

$$w_{2\mapsto 3}^{\mathrm{LO}}(\Phi_2, \Phi_{+1}) = \frac{\mathrm{R}(\Phi_2, \Phi_{+1})}{\mathrm{A}_{2\mapsto 3}(\Phi_{+1})\mathrm{B}(\Phi_2)}$$

$$w_{3\mapsto 4}^{\mathrm{LO}}(\Phi_3, \Phi'_{+1}) = \frac{\mathrm{RR}(\Phi_3, \Phi'_{+1})}{\mathrm{A}_{3\mapsto 4}(\Phi'_{+1})\mathrm{R}(\Phi_3)}$$

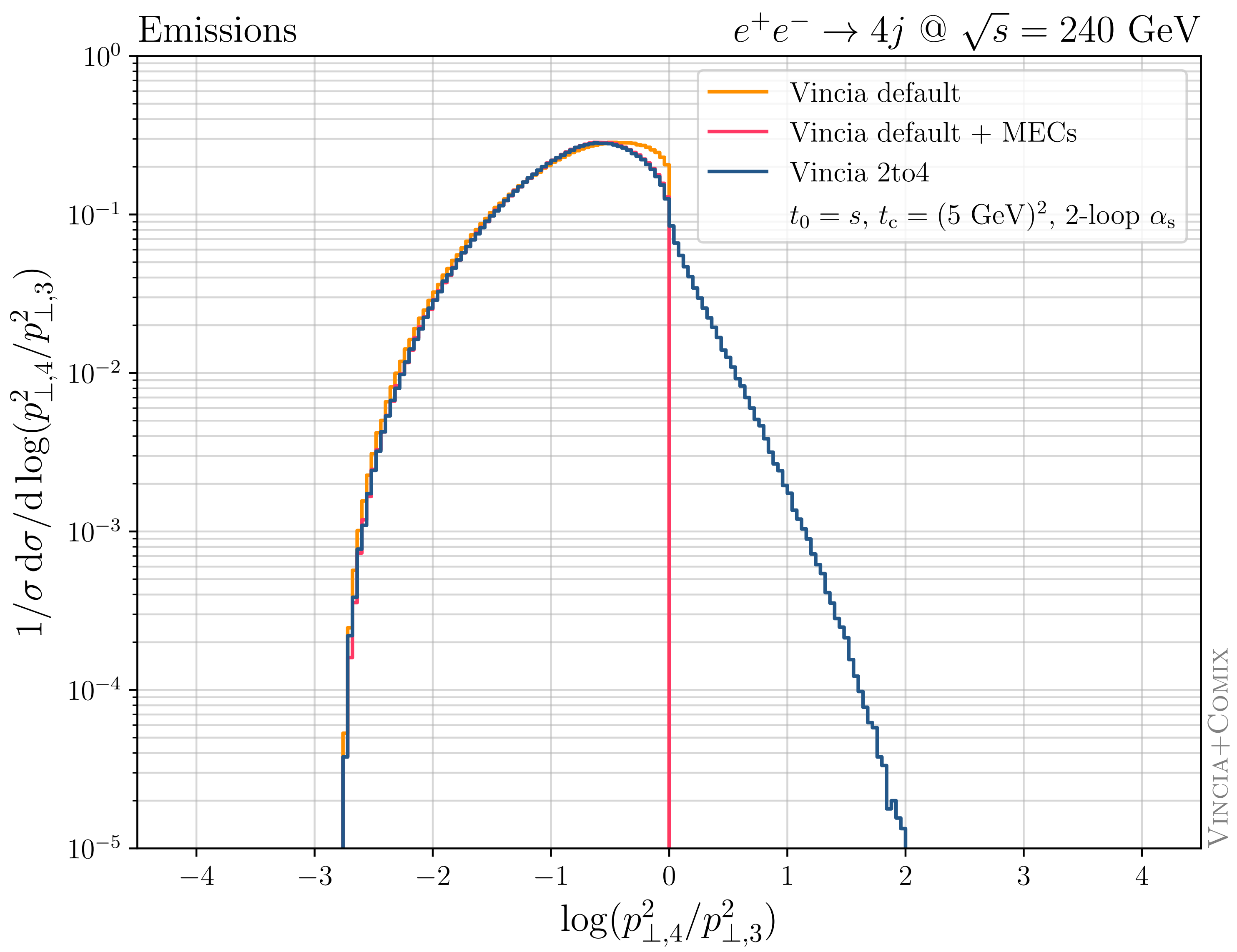
Tree-level MECs in **unordered** region:

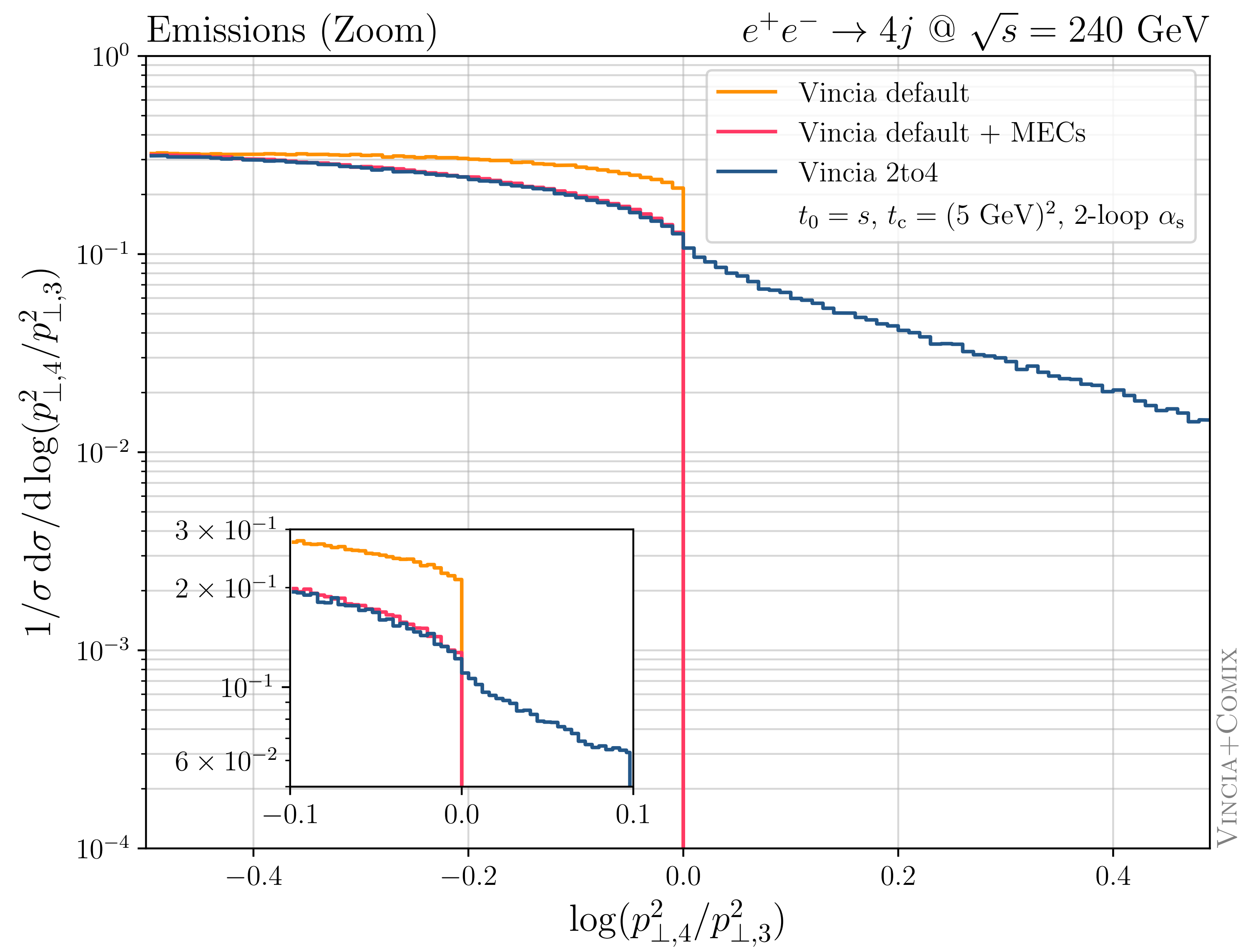
$$w_{2\mapsto 4}^{\mathrm{LO}}(\Phi_2, \Phi_{+2}) = \frac{\mathrm{RR}(\Phi_2, \Phi_{+2})}{\mathrm{A}_{2\mapsto 4}(\Phi_{+2})\mathrm{B}(\Phi_2)}$$

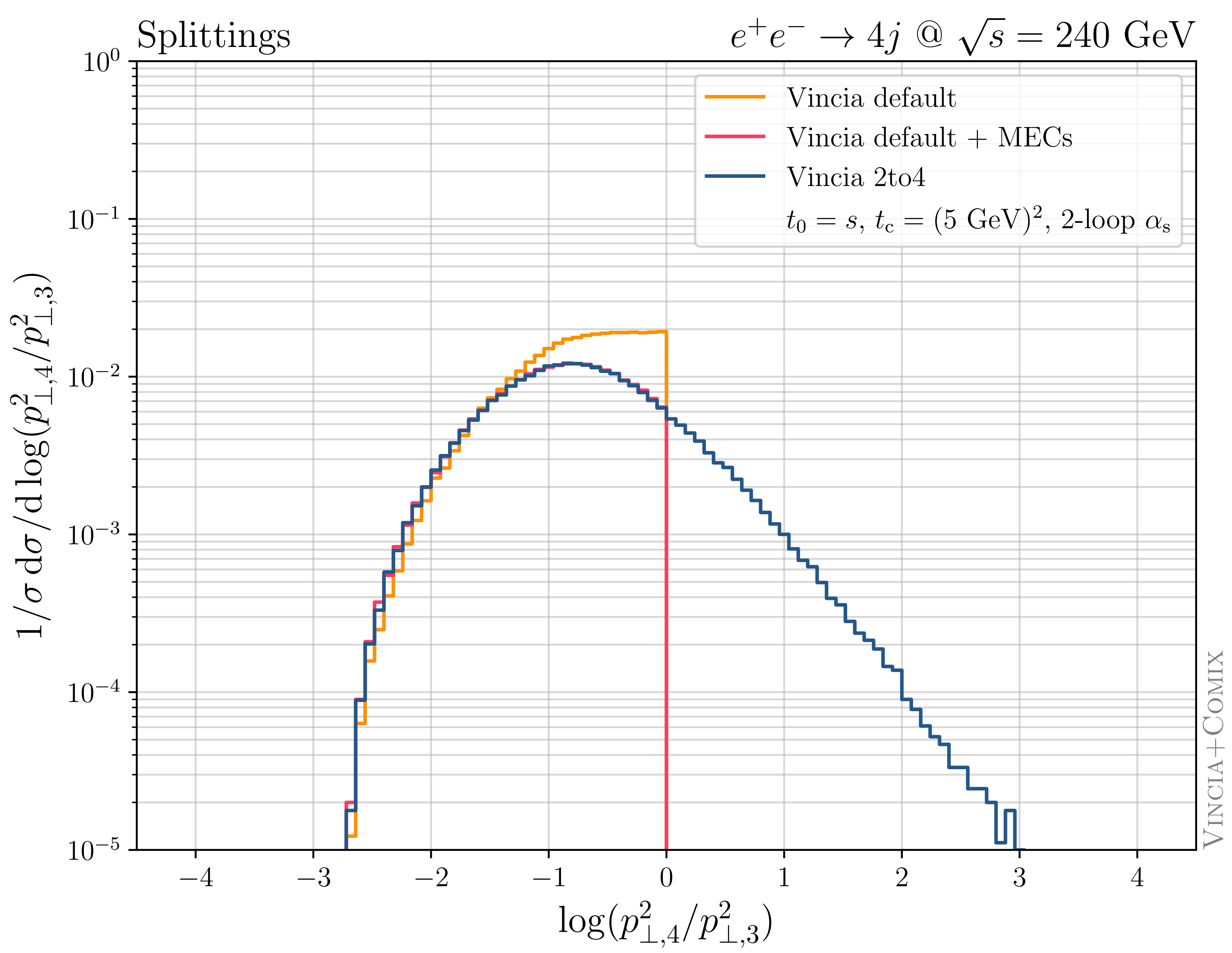
Thus, the full tree-level 4-parton matrix element is imposed

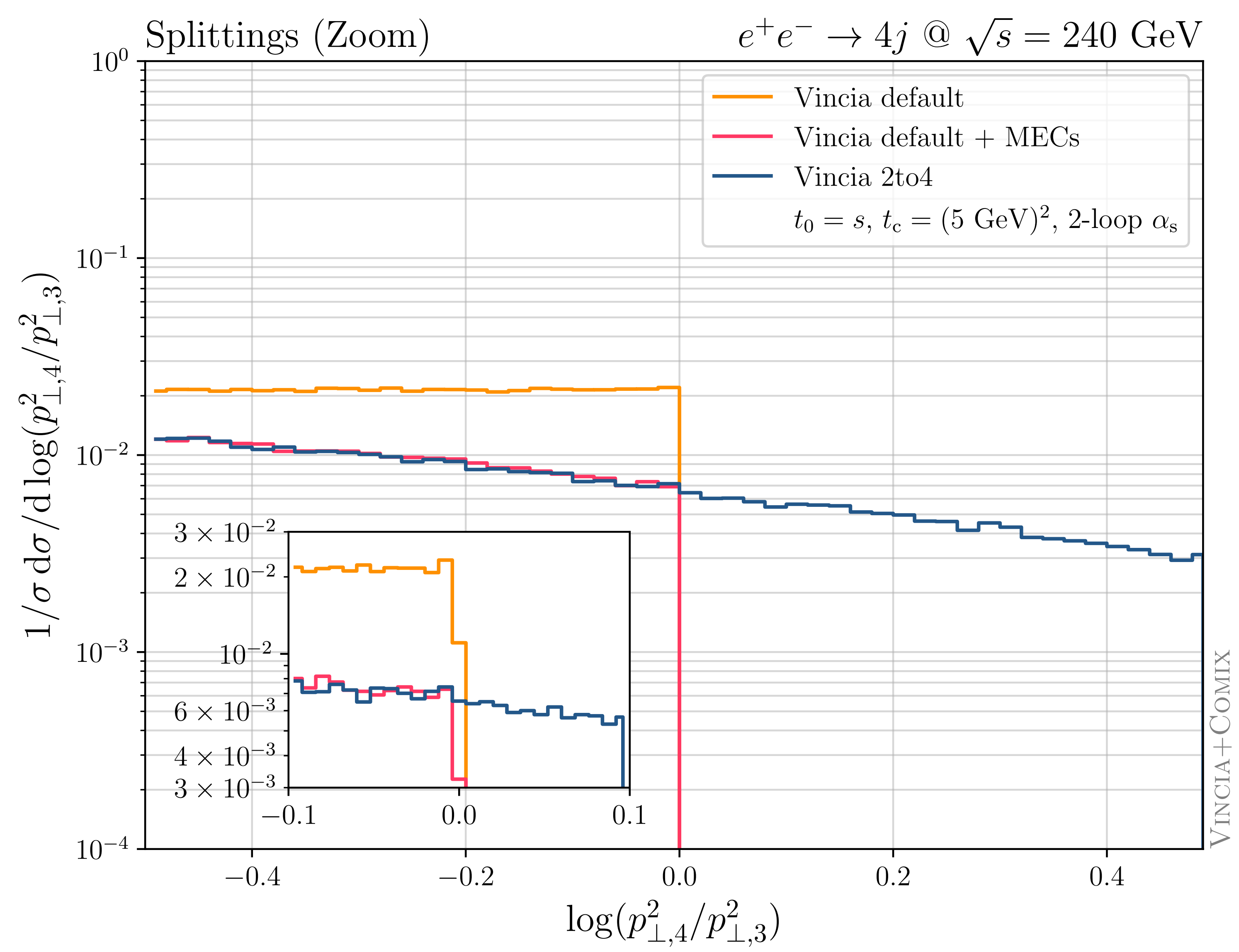
Not only in the direct/unordered phase-space sector, but **also** in the iterated/ordered sector

Validation: Real and Double-Real Corrections







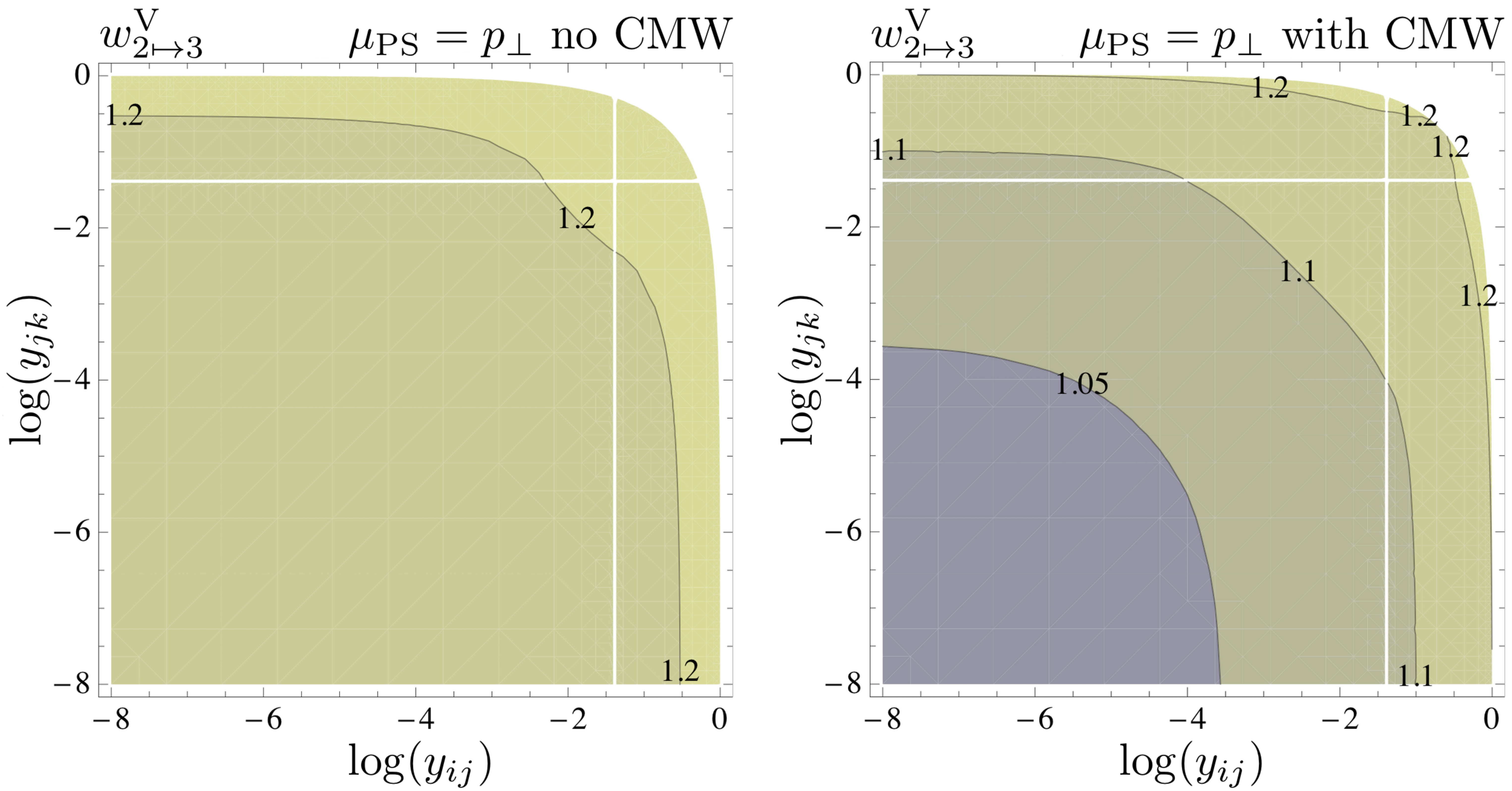


4. NLO MECs for the First Emission

The Real-Virtual Correction Factor

$$w^{\mathrm{NLO}}_{2\mapsto3} = w^{\mathrm{LO}}_{2\mapsto3}\left(1 + w^{\mathrm{V}}_{2\mapsto3}\right)$$

studied **analytically** in detail for $Z \to q\bar{q}$ in [Hartgring, Laenen, Skands 1303.4974]:



⇒ now: **generalisation & (semi-)automation** in VINCIA in form of NLO MECs

Slide adapted from C. Preuss

Real-Virtual Corrections: NLO MECs

Rewrite **NLO MEC** as product of **LO MEC** and **"Born"-local K-factor** $1 + w^{\mathrm{V}}$ ("Powheg in the exponent"):

$$w^{\mathrm{NLO}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}) = w^{\mathrm{LO}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}) \times \left(1 + w^{\mathrm{V}}_{2\mapsto 3}(\Phi_2, \Phi_{+1})\right)$$

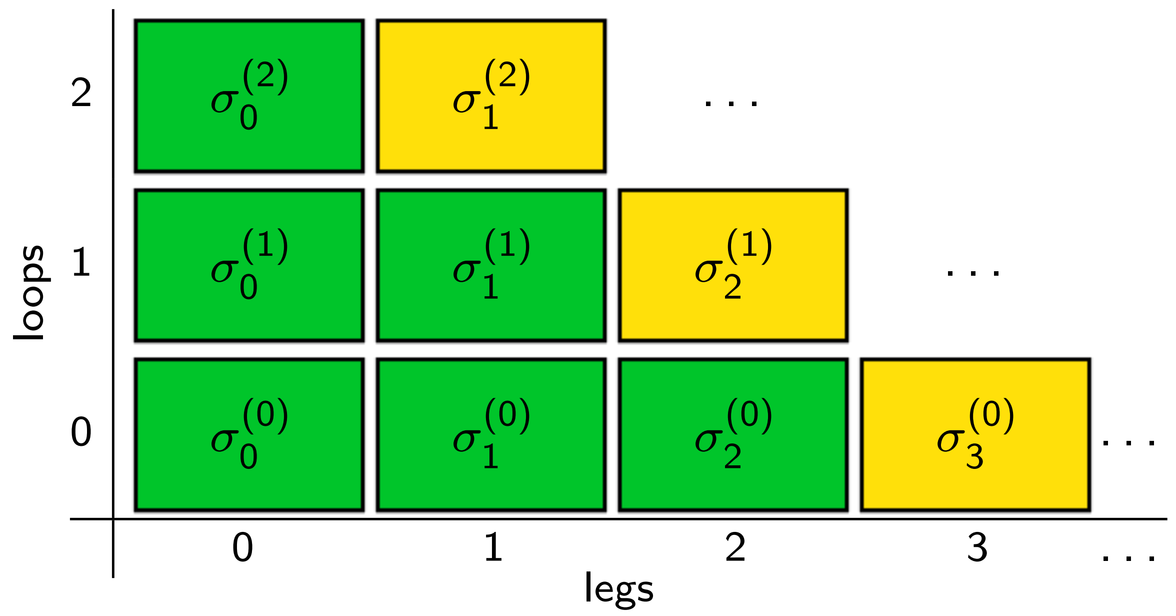
Local correction given by **three terms**:

$$\begin{aligned}
w^{\mathrm{V}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}) = & \left(\frac{\mathrm{RV}(\Phi_2, \Phi_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} + \frac{\mathrm{I}^{\mathrm{NLO}}(\Phi_2, \Phi_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} \right. \\
\text{NLO Born+1}j \quad & \left. + \int_0^t \mathrm{d}\Phi'_{+1} \left[\frac{\mathrm{RR}(\Phi_2, \Phi_{+1}, \Phi'_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} - \frac{\mathrm{S}^{\mathrm{NLO}}(\Phi_2, \Phi_{+1}, \Phi'_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} \right] \right) \\
\text{NLO Born} \quad & - \left(\frac{\mathrm{V}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}^{\mathrm{NLO}}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \int_0^{t_0} \mathrm{d}\Phi'_{+1} \left[\frac{\mathrm{R}(\Phi_2, \Phi'_{+1})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{S}^{\mathrm{NLO}}(\Phi_2, \Phi'_{+1})}{\mathrm{B}(\Phi_2)} \right] \right) \\
\text{shower} \quad & + \left(\frac{\alpha_{\mathrm{S}}}{2\pi} \log\left(\frac{\kappa^2 \mu^2_{\mathrm{PS}}}{\mu^2_{\mathrm{R}}} \right) + \int_t^{t_0} \mathrm{d}\Phi'_{+1}\, \mathrm{A}_{2\mapsto 3}(\Phi'_{+1}) w^{\mathrm{LO}}_{2\mapsto 3}(\Phi_2, \Phi'_{+1}) \right)
\end{aligned}$$

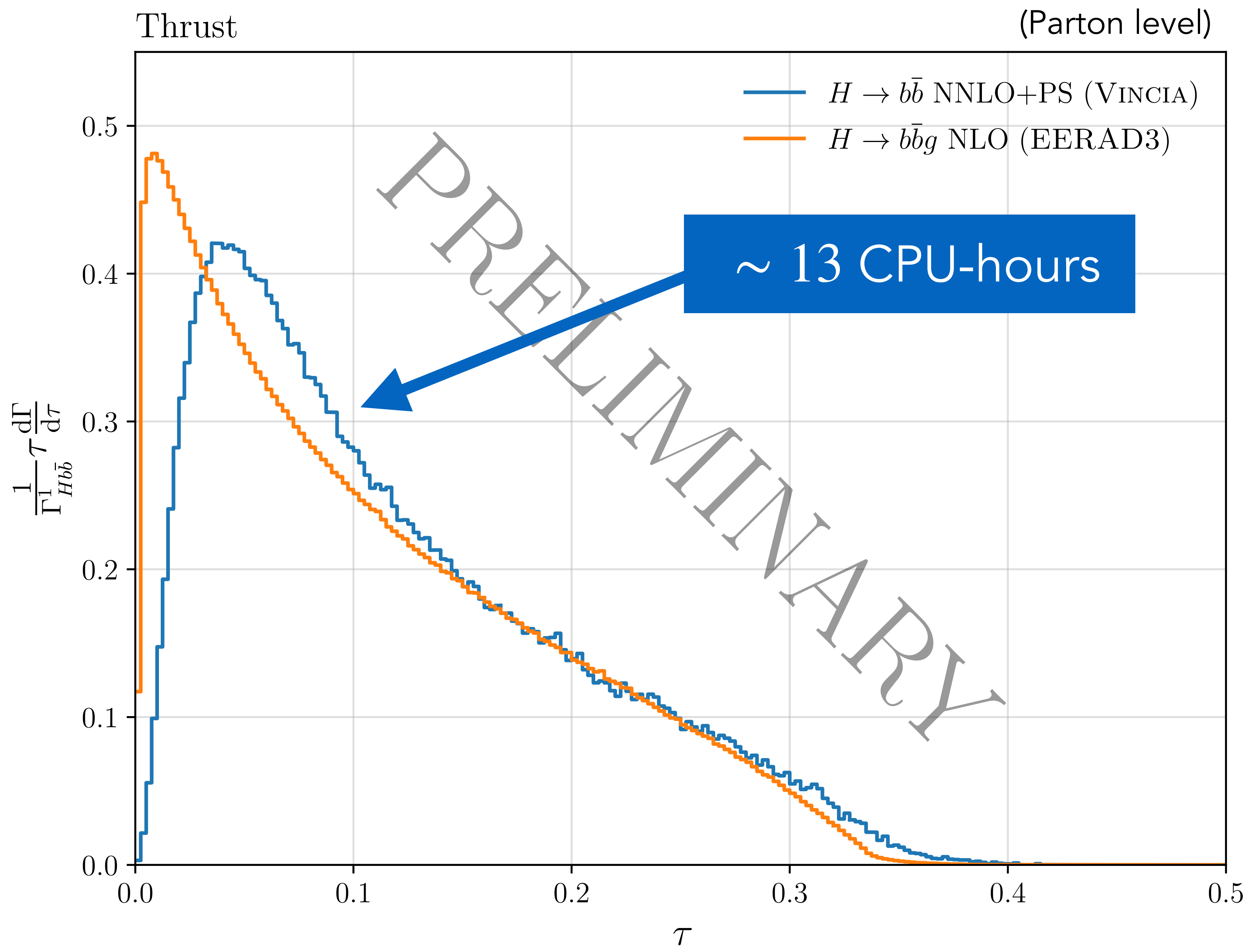
- First and third term from **NLO shower evolution**, second from **NNLO matching**
- Calculation can be **(semi-)automated**, given a suitable NLO subtraction scheme

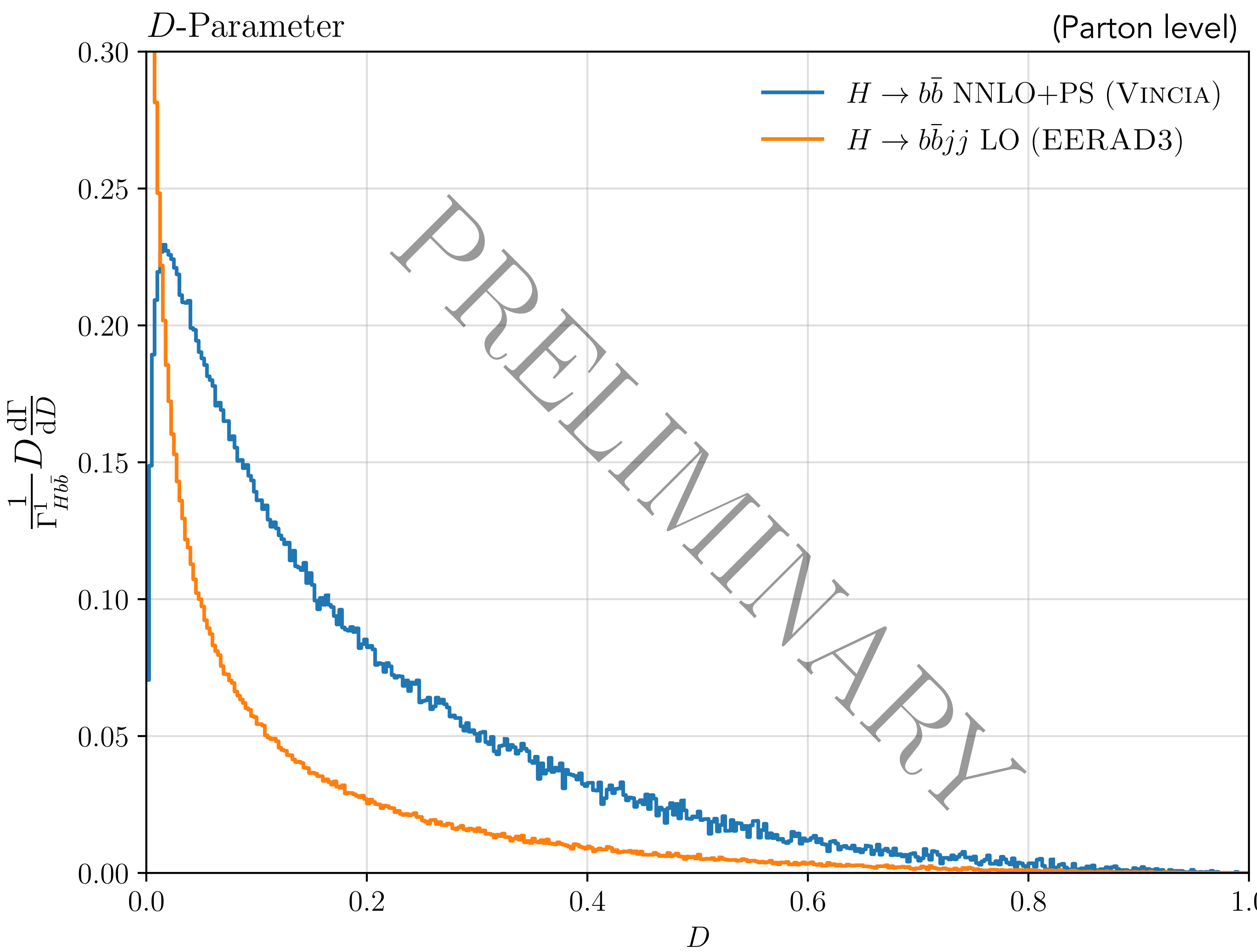
Slide adapted from C. Preuss

Slide adapted from C. Preuss (HP2, Newcastle, Sept 2022)



NNLO accuracy in $H \to 2j$ implies **NLO correction in first emission** and **LO correction in second emission.**





Outlook

NNLO MECs: Generalisations and Limitations

The VINCIANNLO method (aka NNLO MECs) is in principle general

First fully-differential NNLO matching; built on shower with NNLO-accurate pole structure

No dependence on any auxiliary scales (and/or external analytic input other than matrix elements)

Addition of colour singlets trivial; automation on the level of "process classes".

E.g., if $e^+e^- \to 2j$ implemented, also $e^+e^- \to 2j + X$ with any set of colour singlets X.

Additional final-state partons straightforward. In practice, some pitfalls:

Born-local NNLO weight not available in general.

Quark-gluon double-branching antenna functions develop spurious singularities, but:

No exact knowledge of double-branching kernels required.

Sector-antenna functions can effectively be replaced by matrix-element ratios.

Subtractions via "colour-ordered projectors" under development.

For hadronic initial states, the technique remains structurally the same.

Interplay of NLO parton evolution and NLO shower evolution needs clarification.

Further questions on phase-space coverage ("power showers" needed to fill full PS?)

Further Work

Current status

[Brooks, Preuss, PS, 2003.00702] [PS, Verheyen, 2002.04939]

Full-fledged sector shower for ISR and FSR, including multipole-coherent QED shower

Efficient sector-based CKKW-L style LO merging & POWHEG Hooks

[Brooks, Preuss, 2008.09468] [Hoche, Mrenna, Payne, Preuss, PS, 2106.10987]

Soon …

VINCIANNLO Implementation of SM colour-singlet decays ($V/H \to q\bar{q}$, $H \to gg$)

Automation of iterated tree-level MECs. Using interfaces to MadGraph & Comix.

Final-Final double-branchers ($2 \to 4$ antenna branchers; QG parents still need work).

Next few years (post doc opening soon at Monash)

Iterated NLO MECs for final-state radiators. Using MCFM interface [Campbell, Hoche, Preuss 2107.04472]

Incoming Partons (double-branchings, interplay with PDFs, initial-state phase space, …)

Required from fixed-order community (anticipated on ~ short time scale)

Born-local NNLO k-factors for "arbitrary" processes; in reasonable CPU time?

Final Remarks: Perspectives for Matching at N3LO

TOMTE *(similar in spirit to UN2LOPS)* [Prestel, 2106.03206] & [Bertone, Prestel, 2202.01082]

Starts from NNLO+PS matched cross section for X + jet ~ UN2LOPS

Allow jet to become unresolved, regulated by shower Sudakov

Remove unwanted NNLO terms and subtract projected 1-jet bin from 0-jet bin

Include N3LO jet-vetoed zero-jet cross section

Some challenges:

- Large amount of book-keeping ➜ complex code & computational bottlenecks?
- Many counter-events, counter-counter-events, etc ➜ many weight sign flips.
- $\Longrightarrow$ Huge computing resources for relatively slow convergence?

N3LO MECs? (hypothetical extension of VINCIANNLO MECs)

Method in principle generalises.

Add direct-triple ($2 \to 5$) branchings to cover all of phase space: in principle **simple**.

Challenging: need local NNLO subtractions for Born + 1.

…

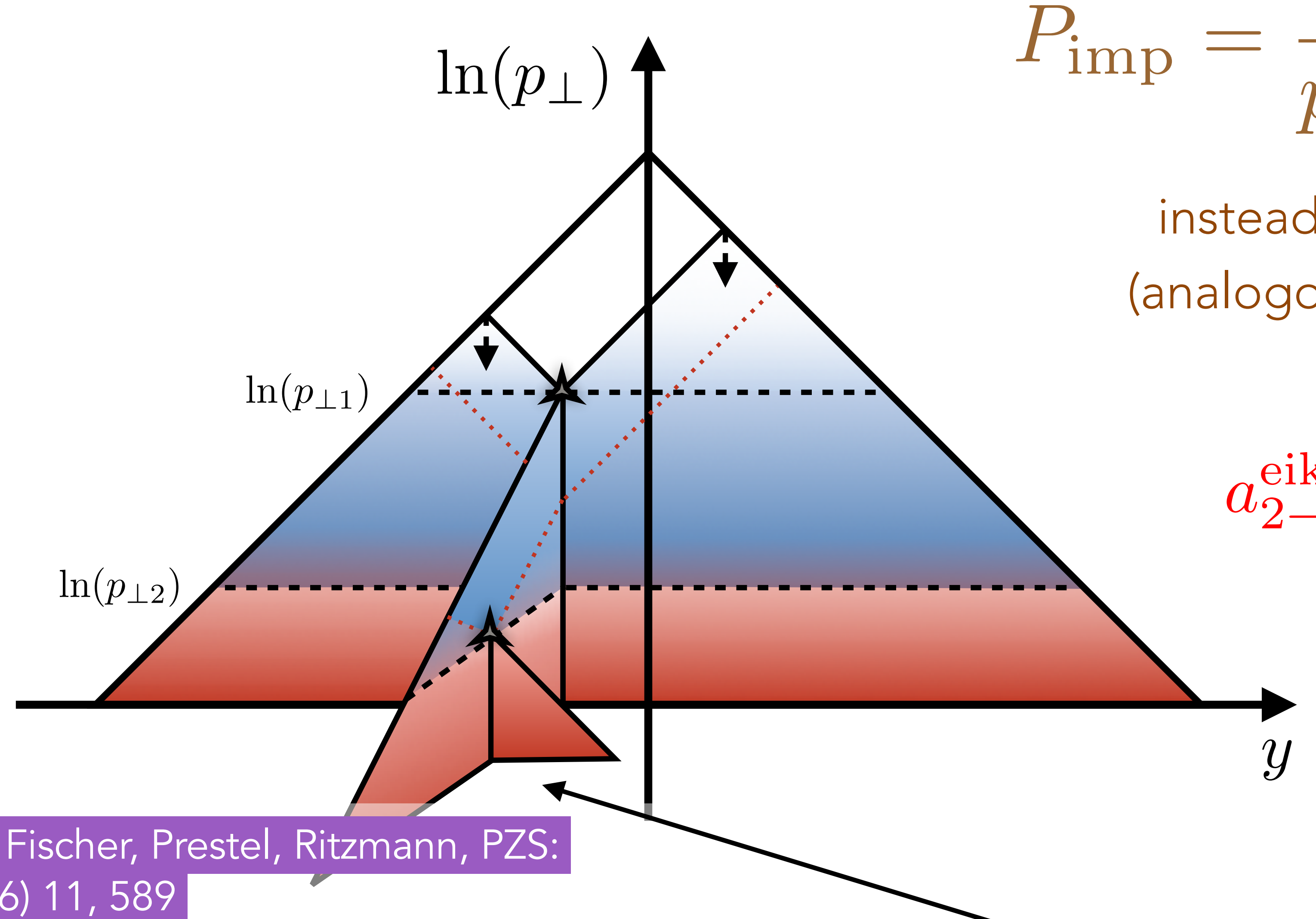
Extra Slides

The Solution that worked at LO: Smooth Ordering

Wanted starting point for (LO) matrix-element corrections over all of phase space (good approx → small corrections)

Allow newly created antennae to evolve over their full phase spaces, with suppressed (beyond-LL) probability: **smooth ordering**

Giele, Kosower, PZS: PRD84 (2011) 054003



Figures from Fischer, Prestel, Ritzmann, PZS: EPJC76 (2016) 11, 589

(b) Smooth Ordering

$$P_{\mathrm{imp}} = \frac{p^2_{\perp n-1}}{p^2_{\perp n-1} + p^2_{\perp n}}$$

→ 1 for $p_{\perp n} \ll p_{\perp, n-1}$

→ 1/2 for $p_{\perp n} \sim p_{\perp, n-1}$

→ 0 for $p_{\perp n} \gg p_{\perp, n-1}$

instead of strong ordering
(analogous to POWHEG hfact)

$$a^{\mathrm{eik}}_{2\to 4} \sim \frac{1}{p^2_{\perp n-1}} P_{\mathrm{imp}} \frac{1}{p^2_{\perp n}} \propto \begin{cases} 1/p^2_{\perp n} & \text{ordered} \\ 1/p^4_{\perp n} & \text{unordered} \end{cases}$$

Leading Logs unchanged

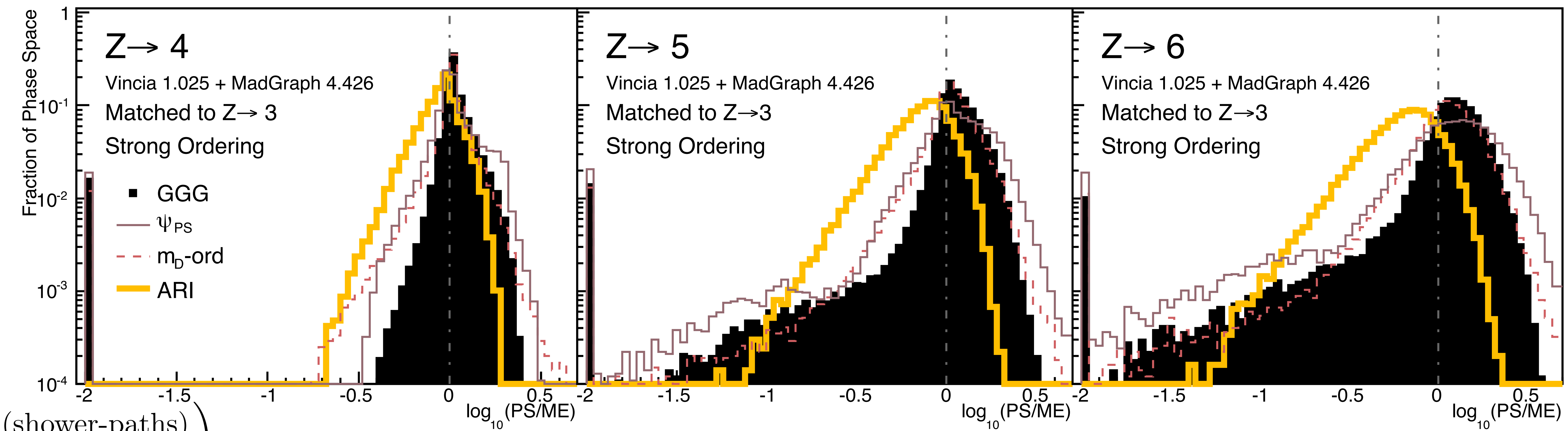
Fischer, Prestel, Ritzmann, PZS: EPJC76 (2016) 11, 589

$$-\ln\Delta \propto \int_{p_\perp^2}^{m^2} \frac{1}{1+\frac{q_\perp^2}{Q_\perp^2}} \frac{\mathrm{d}q_\perp^2}{q_\perp^2} \ln\left[\frac{m^2}{q_\perp^2}\right] \sim \left(\frac{1}{2}\ln^2\left[\frac{Q_\perp^2}{p_\perp^2}\right] + \ln\left[\frac{Q_\perp^2}{p_\perp^2}\right]\ln\left[\frac{m^2}{Q_\perp^2}\right]\right)$$

Note: this conclusion appears to differ from that of Bellm et al., Eur.Phys.J. C76 (2016) no.1
My interpretation is that, in the context of a partonic angular ordering, they neglect the additional rapidity range from the extra origami folds

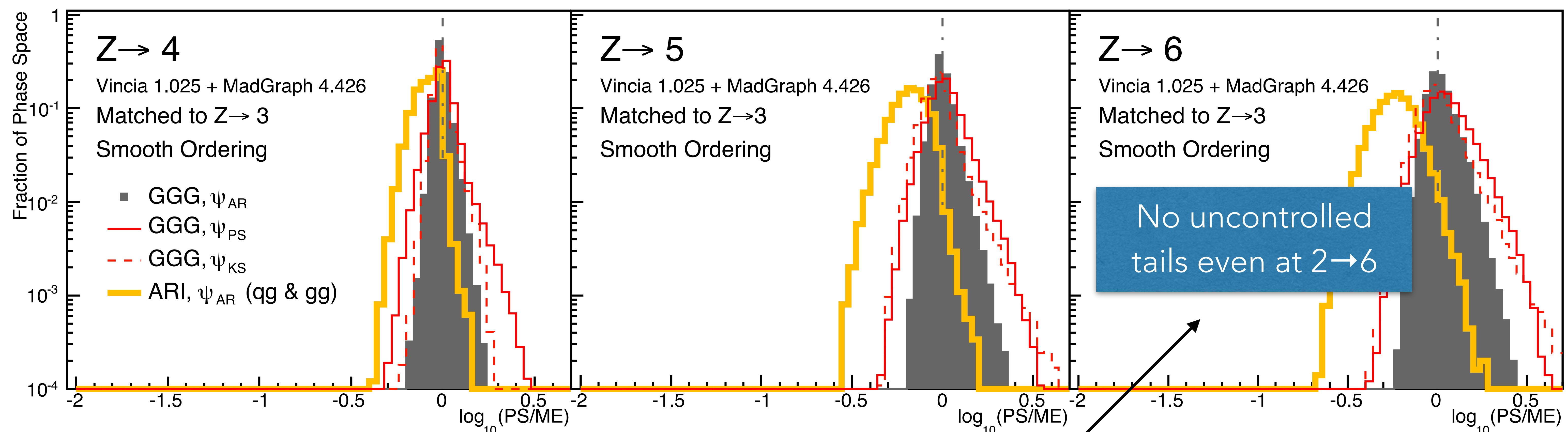
Smooth ordering: An excellent approximation
(at tree level)

Strong



$$R_N = \log_{10}\left(\frac{\mathrm{Sum(shower\text{-}paths)}}{|M_N^{(\mathrm{LO,LC})}|^2}\right)$$

Smooth



No uncontrolled tails even at 2→6

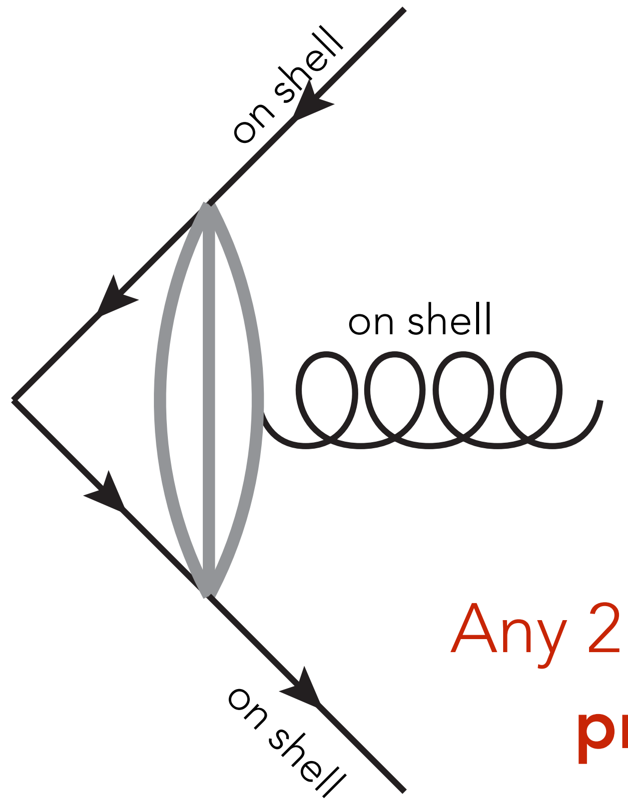
Even after three sequential shower emissions, the smooth shower approximation is still a very close approximation to the matrix element **over all of phase space**

(Why it works?)

The antenna factorisations are on shell

n on-shell partons ➞ **n+1** on-shell partons

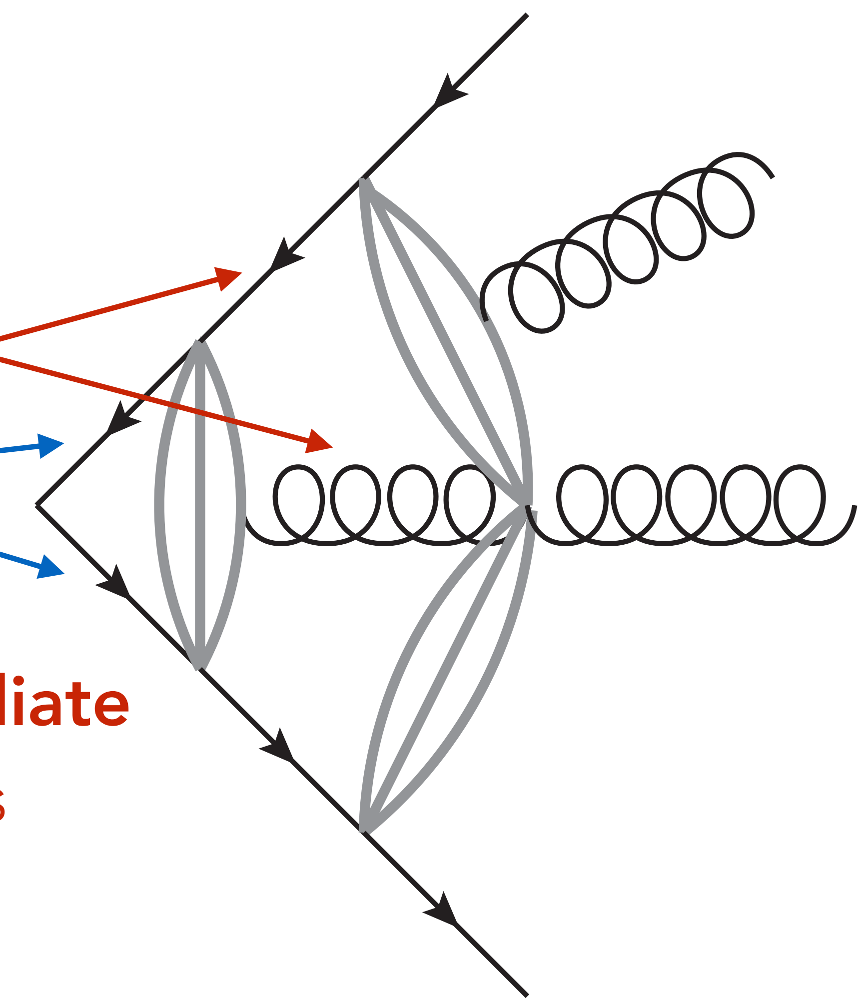
In the first 2➞3 branching, final-leg virtualities assumed ~ 0



Strong Ordering:

these virtualities

small compared to these virtualities



Any 2➞4 Feynman diagrams we draw will involve **intermediate propagators** with virtualities of order the last p_T^2 scales

Cannot be neglected in unordered part of phase space

Interpretation: off-shell effect

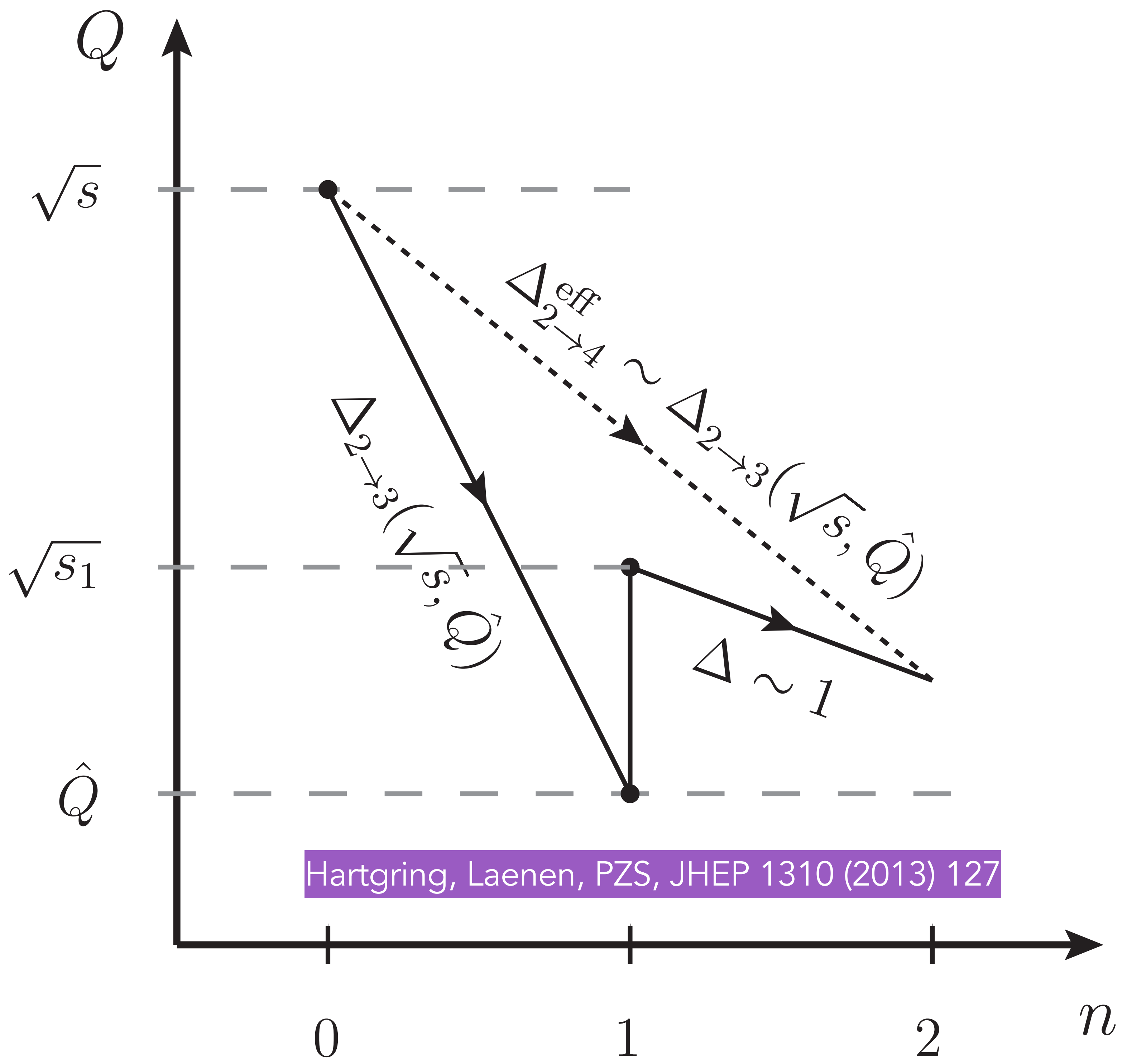
$$\frac{1}{2p_i \cdot p_j} \rightarrow \frac{P_{\mathrm{imp}}(n \rightarrow n+1)}{2p_i \cdot p_j} = \frac{1}{2p_i \cdot p_j + \mathcal{O}(p^2_{\perp n+1})}$$

Good agreement with ME ➞ good starting point for 2➞4

The problem with Smooth Ordering

Smooth ordering: nice tree-level expansions (small ME corrections) ⇨ good 2→4 starting point

But we worried the Sudakov factors were "wrong" ⇨ not good starting point for 2→3 virtual corrections? Not good exponentiation?



For unordered branchings (e.g., double-unresolved)

effective 2→4 Sudakov factor effectively → LL Sudakov for intermediate (unphysical) 3-parton point

2→4 Trial Generation

$$\frac{1}{(16\pi^2)^2} a_{\rm trial}^{2\to4} = \frac{2}{(16\pi^2)^2} a_{\rm trial}^{2\to3}(Q_3^2) P_{\rm imp} a_{\rm trial}^{2\to3}(Q_4^2)$$
$$= C \left(\frac{\alpha_s}{4\pi}\right)^2 \frac{128}{(Q_3^2 + Q_4^2)Q_4^2} \,. \qquad (15)$$

In particular, the trial function for sector A (B) is independent of momentum p_6 (p_3) which makes it easy to translate the 2 → 4 phase spaces defined in eq. (6) to shower variables. Technically, we generate these phase spaces by oversampling, vetoing configurations which do not fall in the appropriate sector.

Accept ratio: $P_{\rm trial}^{2\to4} = \frac{\alpha_s^2}{\hat{\alpha}_s^2} \frac{a_4}{a_{\rm trial}^{2\to4}}$

Solution for constant trial α_s

$$\mathcal{A}_{2\to4}^{\rm trial}(Q_0^2, Q^2) = C\, I_\zeta \frac{\ln(2)\hat{\alpha}_s^2}{8\pi^2} \ln\frac{Q_0^2}{Q^2} \ln\frac{m^4}{Q_0^2 Q^2}$$

$$\Rightarrow \quad Q^2 = m^2 \exp\left(-\sqrt{\ln^2(Q_0^2/m^2) + 2 f_R/\hat{\alpha}_s^2}\right)$$

where $f_R = -4\pi^2 \ln R/(\ln(2) C I_\zeta)$. (Same $I_{\rm zeta}$ as in GKS)

Solution for first-order running α_s (also used as overestimate for 2-loop running):

$$Q^2 = \frac{4\Lambda^2}{k_\mu^2} \left(\frac{k_\mu^2 m^2}{4\Lambda^2}\right)^{-1/W_{-1}(-y)} . \qquad (20)$$

Lambert W

where

$$y = \frac{\ln k_\mu^2 m^2/4\Lambda^2}{\ln k_\mu^2 Q_0^2/4\Lambda^2} \exp\left[-f_R b_0^2 - \frac{\ln k_\mu^2 m^2/4\Lambda^2}{\ln k_\mu^2 Q_0^2/4\Lambda^2}\right] ,$$

Scale Definitions

Conventional ("global") **shower-branching (and subtraction) formalisms:**

Each phase-space point receives contributions from several branching "histories" = clusterings

~ sum over (singular) kernels $\Longrightarrow$ full singularity structure ✅

Number of Histories for n Branchings (Colour-ordered; starting from a single $q\bar{q}$ pair)

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
CS Dipole	2	8	48	384	3840	46080	645120
Global Antenna	1	2	6	24	120	720	5040
	NLO	NNLO	N^{3}LO	… (relevant for iterated MECs & multi-leg merging)			

Fewer partial-fractionings, but still factorial growth

When these are generated by a shower-style formalism (a la POWHEG):

Each term has its own value of the shower scale = scale of last branching

Complicates the definition of an unambiguous matching condition between the (multi-scale) shower and the (single-scale) fixed-order calculation.

- 1st attempt: define matching condition via fully exclusive jet cross sections [Hartgring, Laenen, PS, 1303.4974]
- 2nd attempt: define double-branching "sectors" with unique scales [Li, PS, 1611.00013]
- 3rd attempt: **sectorise everything** [Campbell, Höche, Li, Preuss, PS, 2108.07133]

Sector-Antenna Subtraction

Borrow some concepts from FKS to calculate "Born"-local real integral in NLO MECs:

- Decompose (colour-ordered) real correction into **shower sectors**:

$$\int_0^{t'} \mathrm{d}\Phi'_{+1} \left[\frac{\mathrm{RR}(\Phi_2, \Phi_{+1}, \Phi'_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} - \frac{\mathrm{S}^{\mathrm{NLO}}(\Phi_2, \Phi_{+1}, \Phi'_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} \right]$$

$$= \sum_j \int_0^{t'} \mathrm{d}\Phi_{ijk}^{\mathrm{ant}}\, \Theta_{ijk}^{\mathrm{sct}} \left[\frac{\mathrm{RR}(\Phi_3, \Phi_{ijk}^{\mathrm{ant}})}{\mathrm{R}(\Phi_3)} - A_{IK \mapsto ijk}^{\mathrm{sct}}(i,j,k) \right]$$

- Integral over shower sector $\Theta_{ijk}^{\mathrm{sct}}$ in general **not analytically calculable**
- Need to add/subtract integral over "simple" sector with **known integral**:

$$\int_0^{t'} \mathrm{d}\Phi_{ijk}^{\mathrm{ant}} \left[\Theta_{ijk}^{\mathrm{sct}} - \Theta_{ijk}^{\mathrm{simple}} \right] A_{IK \mapsto ijk}^{\mathrm{sct}}(i,j,k) + \int_0^{t'} \mathrm{d}\Phi_{ijk}^{\mathrm{ant}}\, \Theta_{ijk}^{\mathrm{simple}} A_{IK \mapsto ijk}^{\mathrm{sct}}(i,j,k)$$

$\Rightarrow$ Adds **bottleneck**, as difference of step functions not ideal for MC integration

Slide adapted from C. Preuss

Colour-Ordered Projectors

Better: use smooth projectors [Frixione et al. 0709.2092]

$$\mathrm{RR}(\Phi_3, \Phi'_{+1}) = \sum_j \frac{C_{ijk}}{\sum_m C_{\ell mn}} \mathrm{RR}(\Phi_3, \Phi^{\mathrm{ant}}_{ijk}) \,, \quad C_{ijk} = A_{IK \mapsto ijk} \mathrm{R}(\Phi_3)$$

- **But**: antenna-subtraction term **not positive-definite**!
- To render this well-defined, need to work on **colour-ordered** level

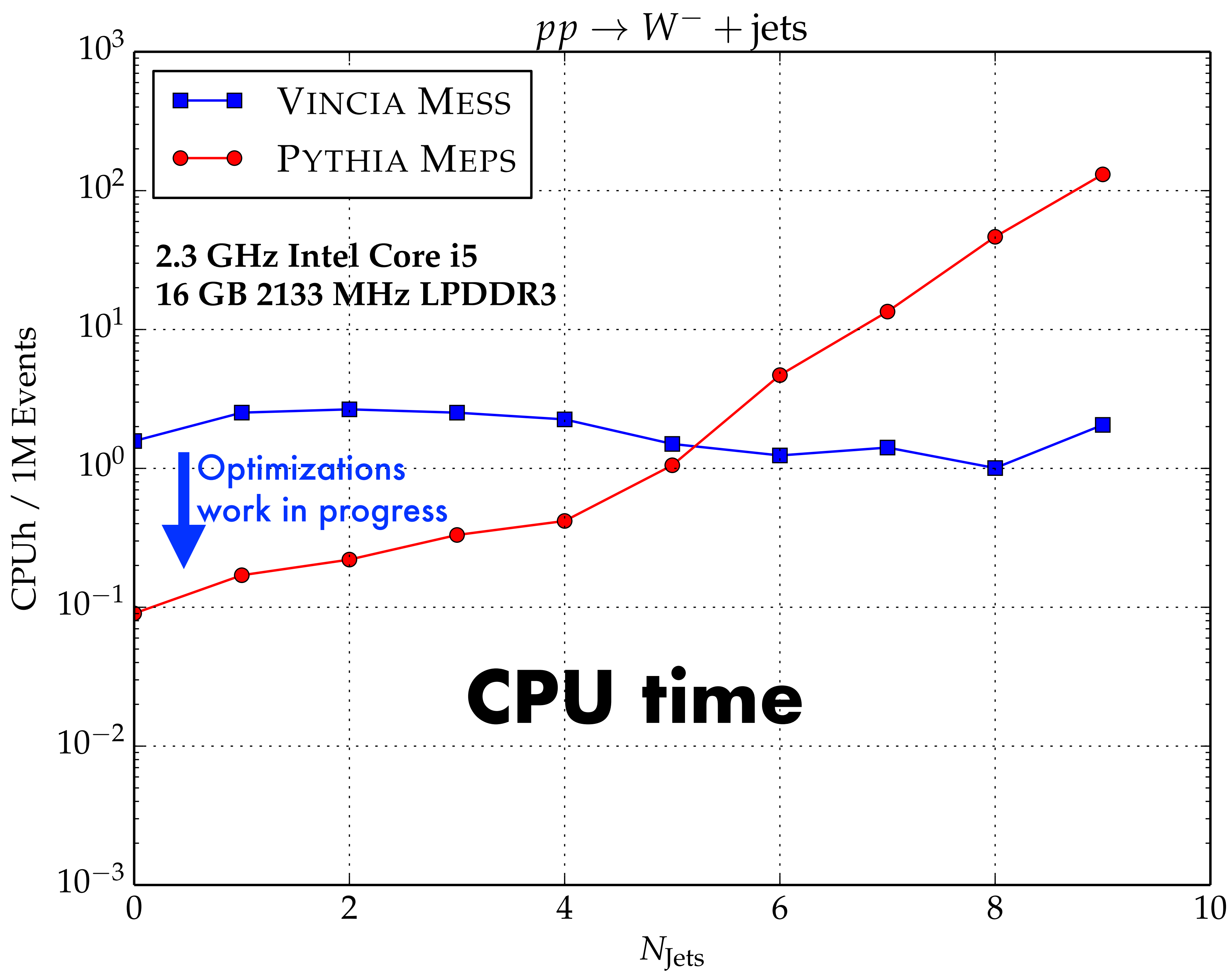
$$\mathrm{RR} = \mathcal{C} \sum_\alpha \mathrm{RR}^{(\alpha)} - \frac{\mathcal{C}}{N_\mathrm{C}^2} \sum_\beta \mathrm{RR}^{(\beta)} \pm \ldots$$

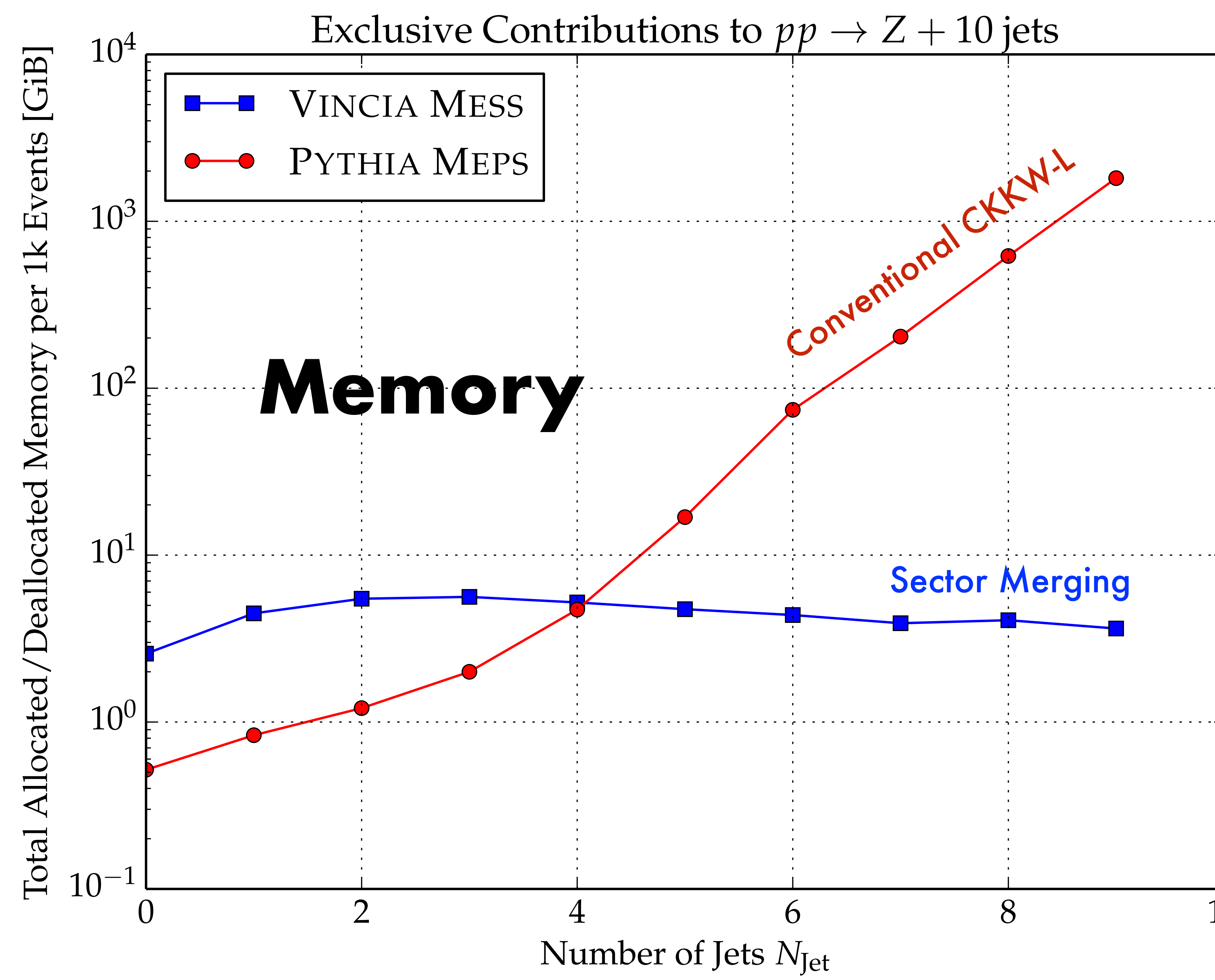
- Different colour factors enter with different sign, but **no sign changes** within one term

$$\mathcal{C} \left[\frac{C_{ijk}}{\sum_m C_{\ell mn}} \frac{\mathrm{RR}^{(\alpha)}(\Phi_3, \Phi^{\mathrm{ant}}_{ijk})}{\mathrm{R}(\Phi_3)} - A_{IK \mapsto ijk} \right]$$

$\Rightarrow$ Numerically **better behaved**, uses **standard antenna-subtraction** terms

New: Sectorized CKKW-L Merging in Pythia 8.306





Brooks & Preuss, "Efficient multi-jet merging with the VINCIA sector shower", 2008.09468

Ready for serious applications (Note: Vincia also has dedicated POWHEG hooks)

Work ongoing to optimise baseline algorithm.

Work at Fermilab: **NNLO** matching, $2 \to 4$ sector antennae, **MCFM** interface, …

Vincia tutorial: http://skands.physics.monash.edu/slides/files/Pythia83-VinciaTute.pdf