QED and EW showers in Vincia

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Ronald Kleiss, Peter Skands, Helen Brooks

Kleiss, RV 1709.04485
Skands, RV 2002.04939
Kleiss, RV 2002.09248
Brooks, Skands, RV 2108.10786

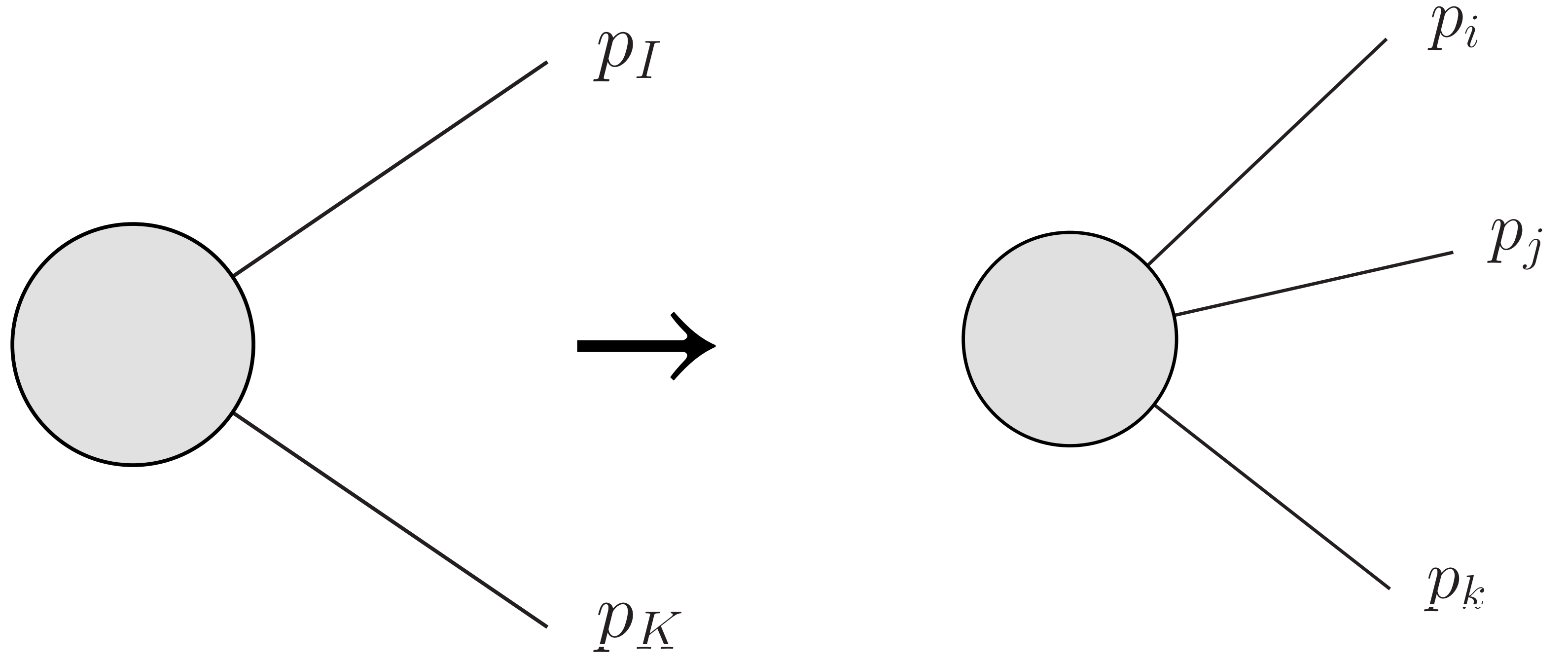
European Research Council
Established by the European Commission

One-slide Vincia summary

$$s_{ab} = 2p_a \cdot p_b$$
$$m_{ab}^2 = (p_a + p_b)^2$$

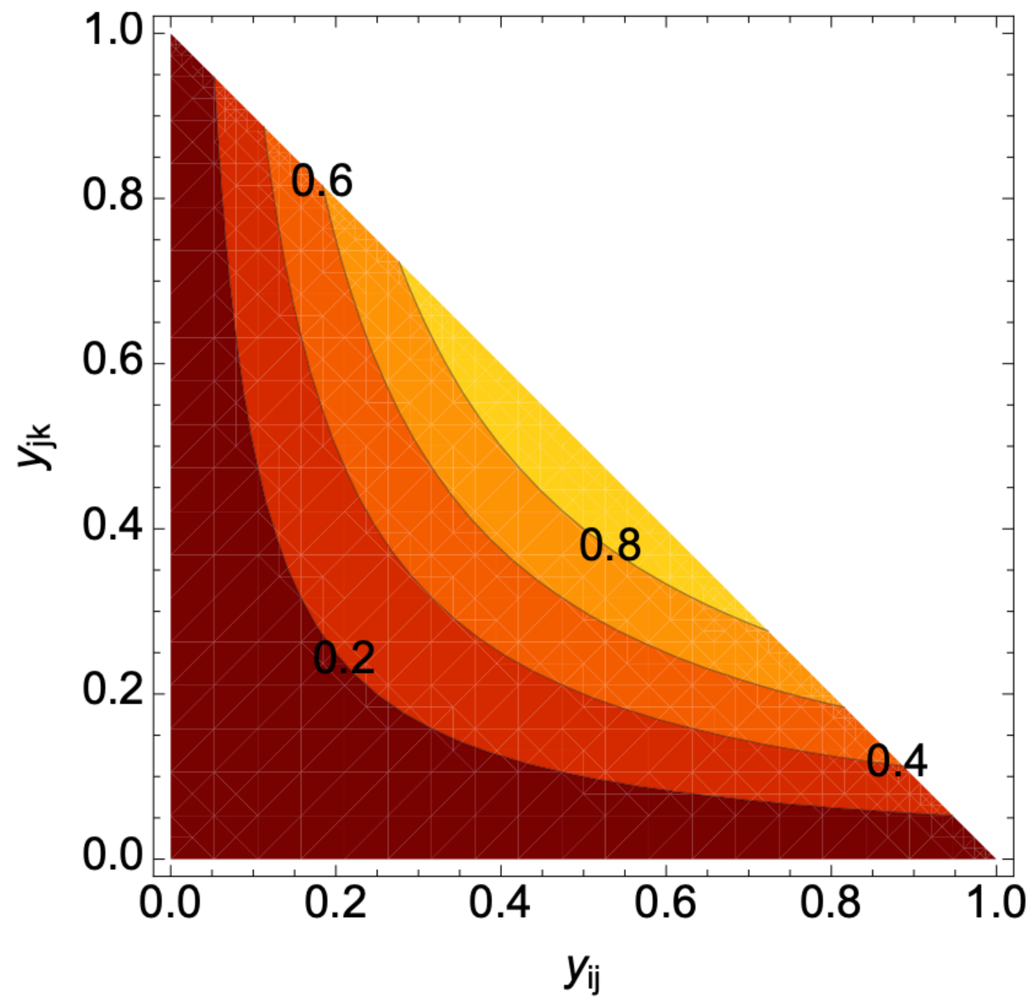
1. Phase space factorisation

$$d\Phi_{\rm ps} = \frac{1}{16\pi^2}\lambda^{\frac{1}{2}}(m_{IK}^2, m_I^2, m_K^2)\, ds_{ij}\, ds_{jk}\, \frac{d\varphi}{2\pi}$$



2. Ordering scale: Ariadne $p_\perp^2$

$$p_\perp^2 = \frac{s_{ij} s_{jk}}{s_{IK}}$$

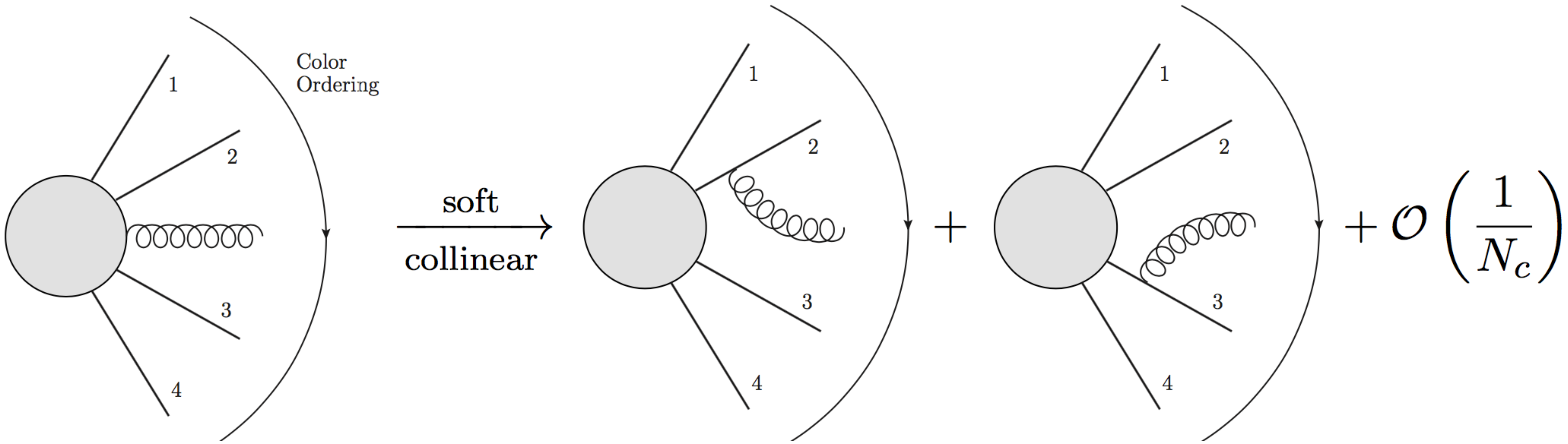


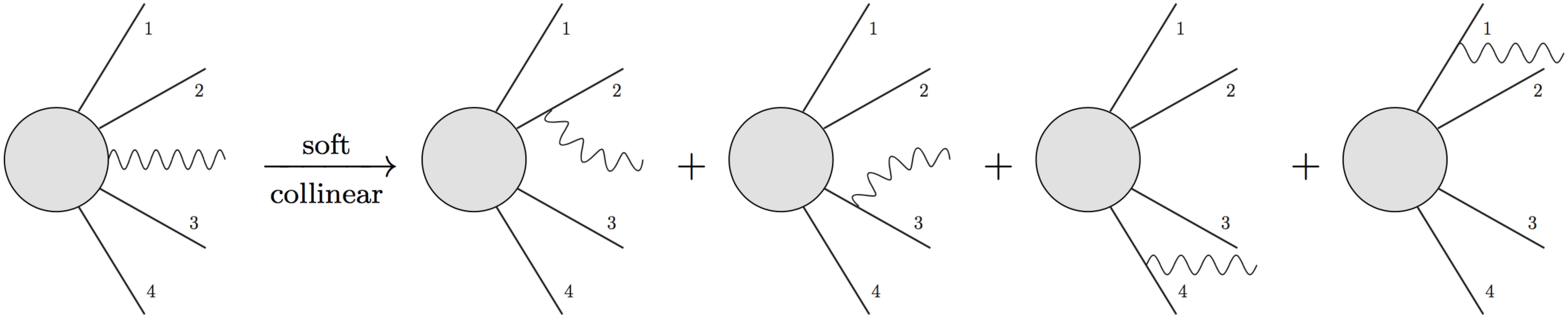
3. Branching kernel: Antenna functions

$$a_{q\bar{q}}(s_{ij}, s_{jk}) = 4\pi\alpha_s C_F \left(2\frac{s_{ik}}{s_{ij}s_{jk}} - 2\frac{m_i^2}{s_{ij}^2} - 2\frac{m_k^2}{s_{jk}^2} + \frac{1}{s_{IK}}\left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}}\right)\right)$$

QED Showers

QCD vs. QED


Color
Ordering
1
2
3
4
soft
collinear
+
$+\mathcal{O}\left(\frac{1}{N_c}\right)$


1
2
3
4
soft
collinear
+
+
+

Coherent Photon Radiation

Soft limit

$$|M_{n+1}(\{p\},p_j)|^2 = -8\pi\alpha \sum_{x,y} \sigma_x Q_x \sigma_y Q_y \frac{s_{xy}}{s_{xj}s_{yj}} |M_n(\{p\}|^2$$

Collinear limit

$$|M_{n+1}(p_1,..,p_i,..,p_n,p_j)|^2 = 4\pi\alpha Q_i^2 \frac{2}{s_{ij}} P_{I\to ij}(z) |M_{n+1}(p_1,..,p_i+p_j,..,p_n)|^2$$

Single branching kernel $\bar{a}^{\mathrm{QED}}(\{p\},p_j) = -\sum_{\{x,y\}} \sigma_x Q_x \sigma_y Q_y a^{\mathrm{QED}}_{f\bar{f}}(s_{xj},s_{yj})$

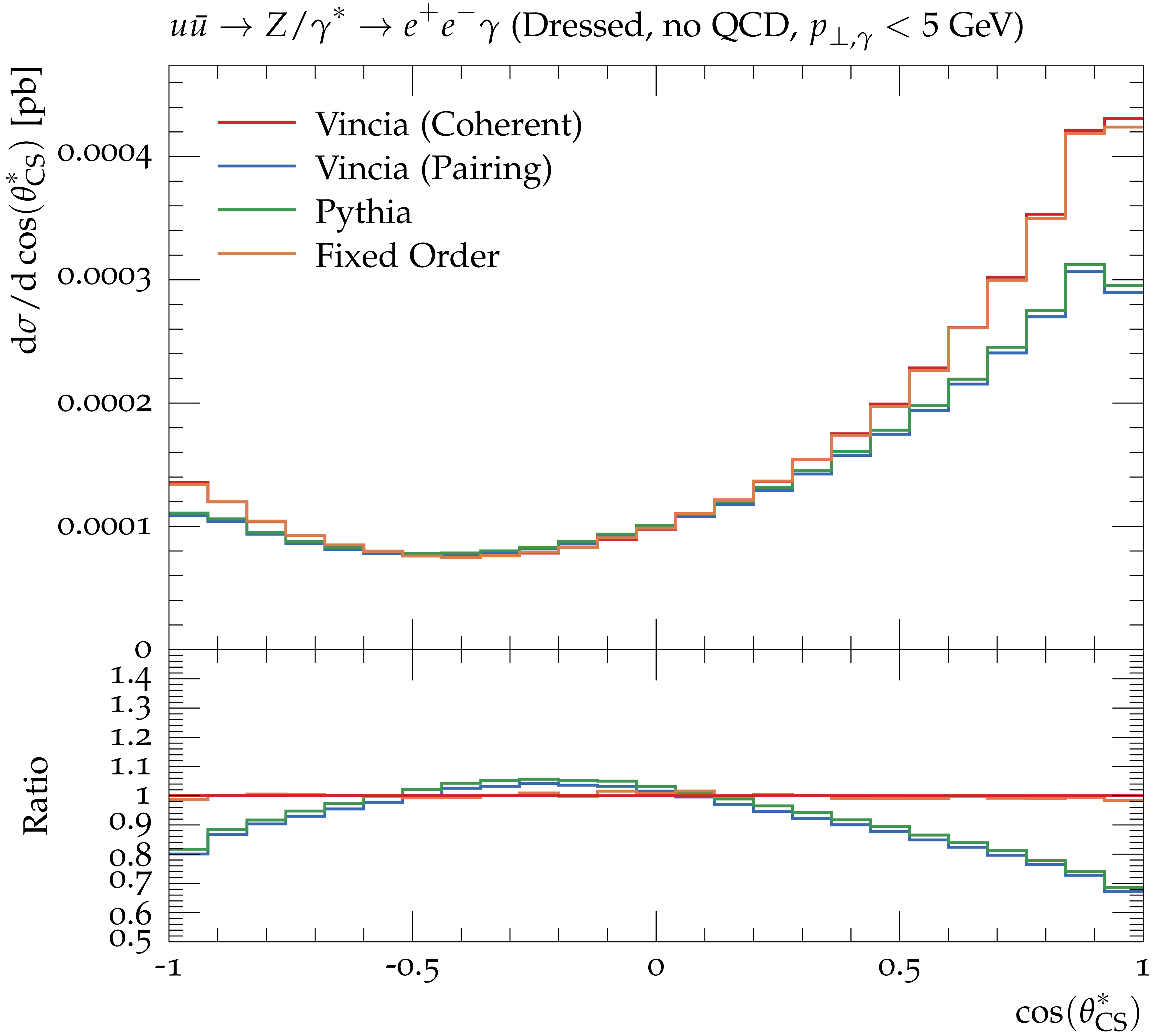
Sectorize the phase space

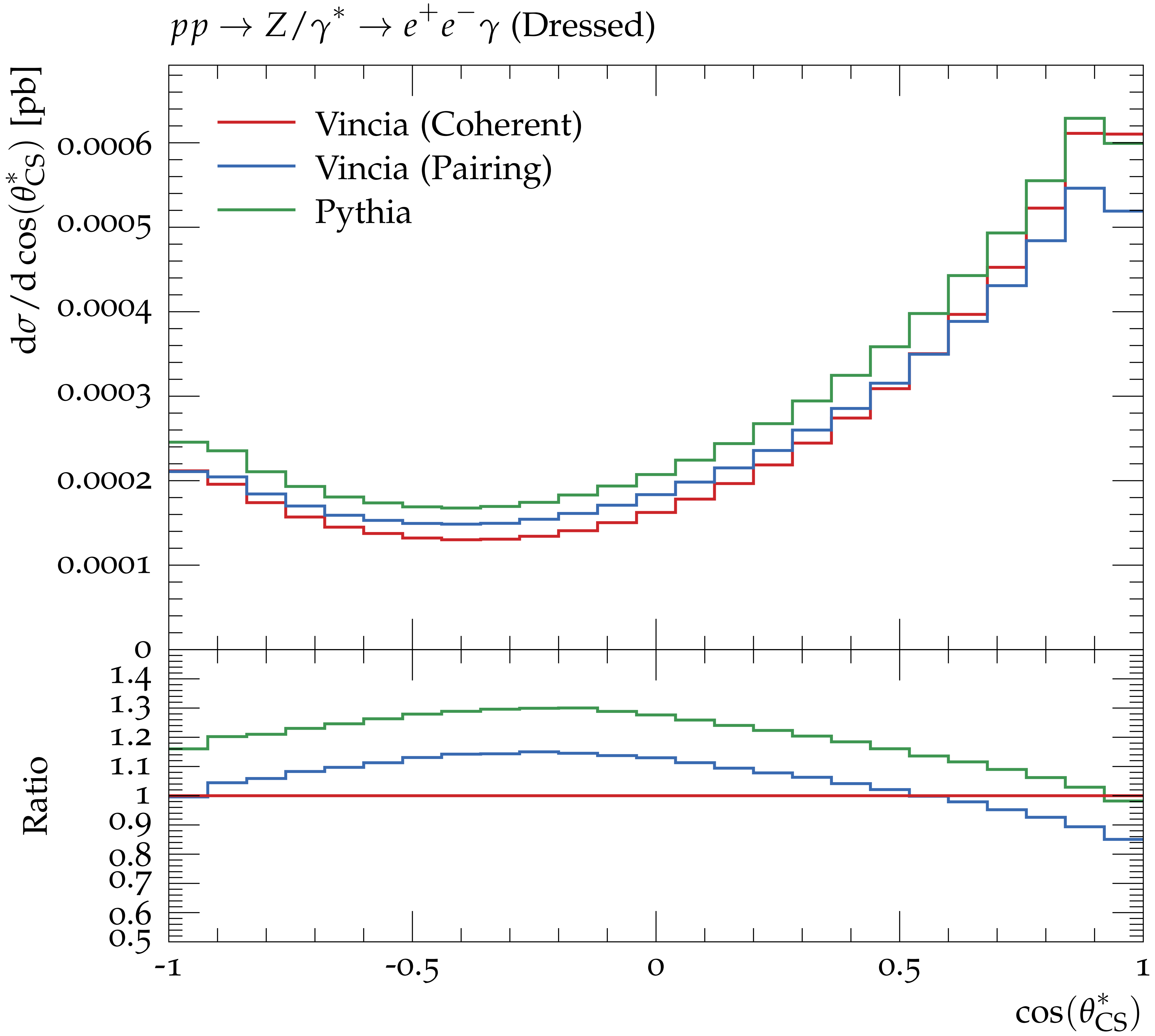
$$|M_{n+1}(\{p\},p_j)|^2 = \bar{a}^{\mathrm{QED}}(\{p\},p_j) \sum_{\{x,y\}} \Theta(p^2_{\perp,xy}) |M_n(\{\bar{p}\}_{xy})|^2$$

← x,y does the emission

↑ $p^2_{\perp,xy}$ is the smallest of all $p^2_{\perp}$

High-mass Drell-Yan





$$\cos\theta^*_{CS} = 2\frac{p^z_{ee}}{|p^z_{ee}|}\frac{p^+_{e^+}p^-_{e^-} - p^-_{e^+}p^+_{e^-}}{m_{ee}\sqrt{m^2_{ee} + p^2_{\perp,ee}}},$$

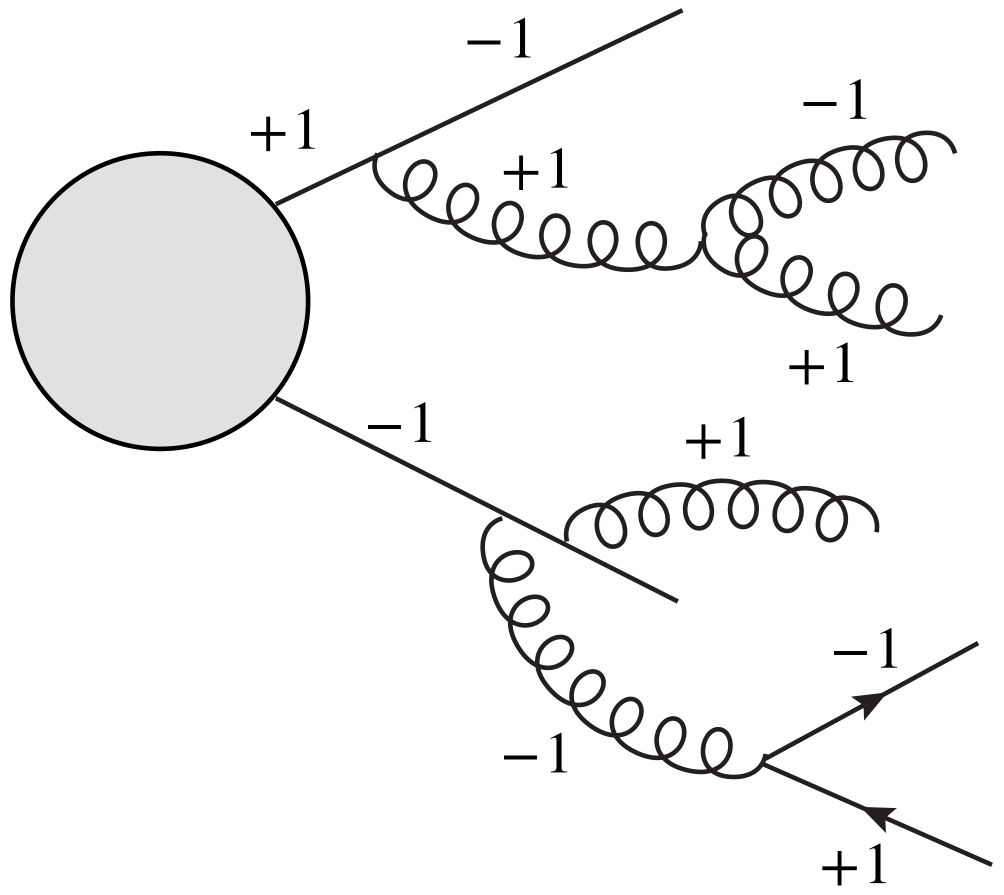
$m^2_{ee} > 1$ TeV, $p_{\perp,e} > 25$ GeV and $|\eta_e| < 3.5$

$p_{\perp,\gamma} > 0.5$ GeV and $|\eta_\gamma| < 3.5$

Electroweak Showers

EW Showers

- **Real corrections: EW gauge bosons, tops, Higgs part of jets**
- **Virtual corrections: Universal incorporation of Sudakov logs** $\frac{\alpha}{\pi} \ln^2 \left(s/Q^2_{\mathrm{EW}}\right)$

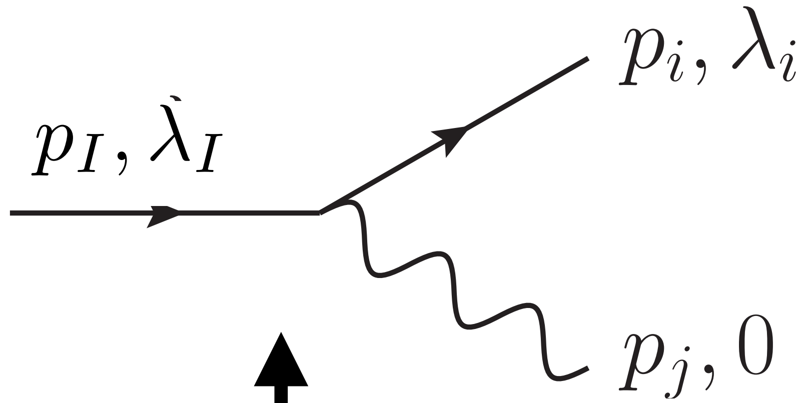


Features of the EW sector

- Chiral ⟶ Helicity showers

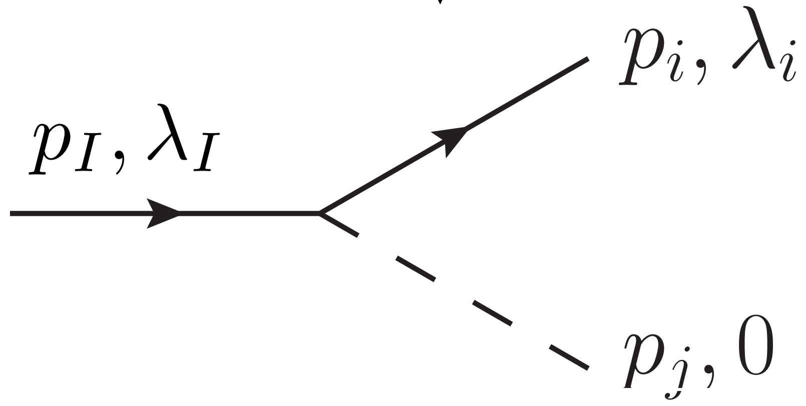
Larkoski, Lopez-Villarejo, Skands 1301.0933

Fischer, Lifson, Stands, 1708.01736



- EW-scale mass corrections
- Longitudinal polarisations / Goldstone bosons

$$\epsilon_0^\mu(p) = \frac{1}{m}\left(p^\mu - \frac{m^2}{p\cdot k}k^\mu\right)$$



- Neutral boson interference
- Double-counting between QCD and EW
- Resonance-like branchings

Lots of Antenna Functions

$$a^{FF}_{f_\lambda\mapsto f_\lambda V_\lambda} = 2(v-\lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}\frac{1}{x_j}$$

$$a^{FF}_{f_\lambda\mapsto f_\lambda V_{-\lambda}} = 2(v-\lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}\frac{x_i^2}{x_j}$$

$$a^{FF}_{f_\lambda\mapsto f_{-\lambda} V_\lambda} = 2\frac{1}{(m_{ij}^2-m_I^2)^2}\left((v-\lambda a)m_i\frac{1}{\sqrt{x_i}} - (v+\lambda a)m_I\sqrt{x_i}\right)^2$$

$$a^{FF}_{f_\lambda\mapsto f_\lambda V_0} = \frac{1}{(m_{ij}^2-m_I^2)^2}\left[(v-\lambda a)\left(\frac{m_I^2}{m_j}\sqrt{x_i} - \frac{m_i^2}{m_j}\frac{1}{\sqrt{x_i}} - 2m_j\frac{\sqrt{x_i}}{x_j}\right) + (v+\lambda a)\frac{m_I m_i}{m_j}\frac{x_j}{\sqrt{x_i}}\right]^2$$

$$a^{FF}_{f_\lambda\mapsto f_{-\lambda} V_0} = \frac{(m_I(v+\lambda a) - m_i(v-\lambda a))^2}{m_j^2}\frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}x_j.$$

$$a^{FF}_{V_\lambda\mapsto f_\lambda \bar{f}_{-\lambda}} = 2(v-\lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}x_j^2$$

$$a^{FF}_{V_\lambda\mapsto f_{-\lambda} \bar{f}_{\lambda}} = 2(v+\lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}x_i^2$$

$$a^{FF}_{V_\lambda\mapsto f_{-\lambda} \bar{f}_{-\lambda}} = 2\frac{1}{(m_{ij}^2-m_I^2)^2}\left((v+\lambda a)m_i\sqrt{\frac{x_j}{x_i}} + (v-\lambda a)m_j\sqrt{\frac{x_i}{x_j}}\right)^2$$

$$a^{FF}_{V_0\mapsto f_\lambda \bar{f}_{\lambda}} = \frac{((v+\lambda a)m_i - (v-\lambda a)m_j)^2}{m_I^2}\frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}$$

$$a^{FF}_{V_0\mapsto f_\lambda \bar{f}_{-\lambda}} = \frac{1}{(m_{ij}^2-m_I^2)^2} \times \left[(v-\lambda a)\left(2m_I\sqrt{x_ix_j} - \frac{m_i^2}{m_I}\sqrt{\frac{x_j}{x_i}} - \frac{m_j^2}{m_I}\sqrt{\frac{x_i}{x_j}}\right) + (v+\lambda a)\frac{m_i m_j}{m}\frac{1}{\sqrt{x_ix_j}}\right]^2.$$

$$a^{FF}_{f_\lambda f_\lambda H} = \frac{e^2}{4s_w^2}\frac{m_i^4}{s_w^2}\frac{1}{(m_{ij}^2-m_I^2)^2}\left(\sqrt{x_i} + \frac{1}{\sqrt{x_i}}\right)^2$$

$$a^{FF}_{f_\lambda f_{-\lambda} H} = \frac{e^2}{4s_w^2}\frac{m_i^2}{s_w^2}\frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}x_j.$$

$$a^{FF}_{V_\lambda\mapsto V_\lambda V_\lambda} = 2g_v^2\frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}\frac{1}{x_ix_j}$$

$$a^{FF}_{V_\lambda\mapsto V_\lambda V_{-\lambda}} = 2g_v^2\frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}\frac{x_i^3}{x_j}$$

$$a^{FF}_{V_\lambda\mapsto V_{-\lambda} V_\lambda} = 2g_v^2\frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}\frac{x_j^3}{x_i}$$

$$a^{FF}_{V_\lambda\mapsto V_\lambda V_0} = g_v^2\frac{1}{(m_{ij}^2-m_I^2)^2}\frac{(m_I^2 - m_i^2 - \frac{1+x_i}{x_j}m_j^2)^2}{m_j^2}$$

$$a^{FF}_{V_\lambda\mapsto V_0 V_\lambda} = g_v^2\frac{1}{(m_{ij}^2-m_I^2)^2}\frac{(m_I^2 - m_j^2 - \frac{1+x_j}{x_i}m_i^2)^2}{m_i^2}$$

$$a^{FF}_{V_\lambda\mapsto V_0 V_0} = \frac{g_v^2}{2}\frac{(m_I^2 - m_i^2 - m_j^2)^2}{m_i^2m_j^2}\frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}x_ix_j.$$

$$a^{FF}_{V_\lambda\mapsto V_\lambda H} = \frac{e^2}{s_w^2}\frac{m_v^4}{m_w^2}\frac{1}{(m_{ij}^2-m_I^2)^2}$$

$$a^{FF}_{V_\lambda\mapsto V_0 H} = \frac{e^2}{2s_w^2}\frac{m_v^2}{m_w^2}\frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}x_ix_j$$

$$a^{FF}_{V_0\mapsto V_\lambda H} = \frac{e^2}{2s_w^2}\frac{m_v^2}{m_w^2}\frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}\frac{x_j}{x_i}$$

$$a^{FF}_{V_0\mapsto V_0 H} = \frac{e^2}{4s_w^2}\frac{1}{m_w^2}\frac{1}{(m_{ij}^2-m_I^2)^2}\left(m_I^2 - 2m_i^2\left(x_i + \frac{1}{x_i}\right)\right)^2.$$

$$a^{FF}_{H\mapsto f_\lambda \bar{f}_\lambda} = \frac{e^2}{4s_w^2}\frac{m_i^2}{s_w^2}\frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}$$

$$a^{FF}_{H\mapsto f_\lambda \bar{f}_{-\lambda}} = \frac{e^2}{4s_w^2}\frac{m_i^4}{s_w^2}\frac{1}{(m_{ij}^2-m_I^2)^2}\left(\sqrt{\frac{x_i}{x_j}} - \sqrt{\frac{x_j}{x_i}}\right)^2.$$

Lots of Antenna Functions (pt. 2)

$$a^{FF}_{V_0 \mapsto V_\lambda V_{-\lambda}} = g_v^2 \frac{1}{(m_{ij}^2 - m_I^2)^2} \frac{(m_I^2(1-2x_i) + m_i^2 - m_j^2)^2}{m_I^2}$$

$$a^{FF}_{V_0 \mapsto V_\lambda V_0} = \frac{g_v^2}{2} \frac{(m_I^2 - m_i^2 + m_j^2)^2}{m_I^2 m_j^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{x_j}{x_i}$$

$$a^{FF}_{V_0 \mapsto V_0 V_\lambda} = \frac{g_v^2}{2} \frac{(m_I^2 + m_i^2 - m_j^2)^2}{m_I^2 m_i^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{x_i}{x_j}$$

$$a^{FF}_{V_0 \mapsto V_0 V_0} = \frac{g_v^2}{4} \frac{1}{m_I^2 m_i^2 m_j^2} \frac{1}{x_i^2 x_j^2} \times \Big[m_I^4 x_i x_j (x_i - x_j) + 2 m_I^2 (m_i^2 x_j^2 (1 + x_i) - m_j^2 x_i^2 (1 + x_j)) - (m_i^2 - m_j^2)(m_i^2 x_j (1 + x_j) + m_j^2 x_i (1 + x_i)) \Big]^2 .$$

$$a^{II}_{f_\lambda \mapsto f_\lambda V_\lambda} = 2(v - \lambda a)^2 \frac{\tilde{q}_{aj}^2}{(m_A^2 - q_{ai}^2)^2} \frac{1}{x_A} \frac{1}{x_j}$$

$$a^{II}_{f_\lambda \mapsto f_\lambda V_{-\lambda}} = 2(v - \lambda a)^2 \frac{\tilde{q}_{aj}^2}{(m_A^2 - q_{ai}^2)^2} \frac{x_A}{x_j}$$

$$a^{II}_{f_\lambda \mapsto f_{-\lambda} V_\lambda} = 2 \frac{1}{(m_A^2 - q_{ai}^2)^2} \left((v - \lambda a) \frac{m_A}{\sqrt{x_A}} - (v + \lambda a) \sqrt{x_A} m_a \right)^2$$

$$a^{II}_{f_\lambda \mapsto f_\lambda V_0} = \frac{1}{(m_A^2 - q_{ai}^2)^2} \times \left[(v - \lambda a) \left(\frac{m_a^2}{m_j} \sqrt{x_A} - \frac{m_A^2}{m_j} \frac{1}{\sqrt{x_A}} - 2 m_j \frac{\sqrt{x_A}}{x_j} \right) + (v + \lambda a) \frac{m_a m_A}{m_j} \frac{x_j}{\sqrt{x_A}} \right]^2$$

$$a^{II}_{f_\lambda \mapsto f_{-\lambda} V_0} = \frac{((v - \lambda a) m_A - (v + \lambda a) m_a)^2}{m_j^2} \frac{\tilde{q}_{aj}^2}{(m_A^2 - q_{ai}^2)^2} \frac{x_j}{x_A}$$

$$a^{FF}_{H \mapsto f_\lambda \bar{f}_\lambda} = \frac{e^2}{4 s_w^2} \frac{m_i^2}{s_w^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2}$$

$$a^{FF}_{H \mapsto f_\lambda \bar{f}_{-\lambda}} = \frac{e^2}{4 s_w^2} \frac{m_i^4}{s_w^2} \frac{1}{(m_{ij}^2 - m_I^2)^2} \left(\sqrt{\frac{x_i}{x_j}} - \sqrt{\frac{x_j}{x_i}} \right)^2 .$$

$$a^{FF}_{H \mapsto V_\lambda V_{-\lambda}} = \frac{e^2}{s_w^2} \frac{m_v^4}{m_w^2} \frac{1}{(m_{ij}^2 - m_I^2)^2}$$

$$a^{FF}_{H \mapsto V_\lambda V_0} = \frac{e^2}{2 s_w^2} \frac{m_v^2}{m_w^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{x_j}{x_i}$$

$$a^{FF}_{H \mapsto V_0 V_\lambda} = \frac{e^2}{2 s_w^2} \frac{m_v^2}{m_w^2} \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{x_i}{x_j}$$

$$a^{FF}_{H \mapsto V_0 V_0} = \frac{e^2}{4 s_w^2} \frac{1}{m_w^2} \frac{1}{(m_{ij}^2 - m_I^2)^2} \left(m_I^2 - 2 m_v^2 \left(\frac{1}{x_i x_j} - 1 \right) \right)^2 .$$

$$a^{II}_{f_\lambda f_\lambda H} = \frac{e^2}{4 s_w^2} \frac{m_a^4}{s_w^2} \frac{1}{(m_A^2 - q_{ai}^2)^2} \frac{1}{x_A} \left(\sqrt{x_A} + \frac{1}{\sqrt{x_A}} \right)^2$$

$$a^{II}_{f_\lambda f_{-\lambda} H} = \frac{e^2}{4 s_w^2} \frac{m_a^2}{s_w^2} \frac{\tilde{q}_{aj}^2}{(m_A^2 - q_{ai}^2)^2} \frac{1}{x_A} x_j .$$

Collinear Limits

$$\tilde{m}_{ij}^2 = m_{ij}^2 - \frac{m_i^2}{z^2} - \frac{m_j^2}{(1-z)^2}$$

λ_I	λ_i	λ_j	$f \to f'V$		
λ	λ	λ	$2(v-\lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2} \frac{1}{1-z}$	$P(z) \propto \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2} \frac{1+z^2}{1-z}$	Pure vector
λ	λ	$-\lambda$	$2(v-\lambda a)^2 \frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2} \frac{z^2}{1-z}$		
λ	$-\lambda$	λ	$2\frac{1}{(m_{ij}^2-m_I^2)^2}\left(m_I(v-\lambda a)\sqrt{z} - m_i(v+\lambda a)\frac{1}{\sqrt{z}}\right)^2$		Pure vector
λ	$-\lambda$	$-\lambda$	0	$P(z) \propto \frac{m^2}{(m_{ij}^2 - m_I^2)^2}$	
λ	λ	0	$\frac{1}{(m_{ij}^2-m_I^2)^2}\left[(v-\lambda a)\left(\frac{m_I^2}{m_j}\sqrt{z} - \frac{m_i^2}{m_j}\frac{1}{\sqrt{z}} - 2m_j\frac{\sqrt{z}}{1-z}\right) + (v+\lambda a)\frac{m_i m_I}{m_j}\frac{1-z}{\sqrt{z}}\right]^2$		Vector + Scalar
λ	$-\lambda$	0	$\frac{\tilde{m}_{ij}^2}{(m_{ij}^2-m_I^2)^2}(1-z)\left(\frac{m_i}{m_j}(v-\lambda a) - \frac{m_I}{m_j}(v+\lambda a)\right)^2$	$P(z) \propto \frac{\tilde{m}_{ij}^2}{(m_{ij}^2 - m_I^2)^2}(1-z)$	Pure scalar

Overestimate Determination

$\mathcal{O}(1000)$ types of branchings (all FSR + ffV ISR)

Parameterized overestimate

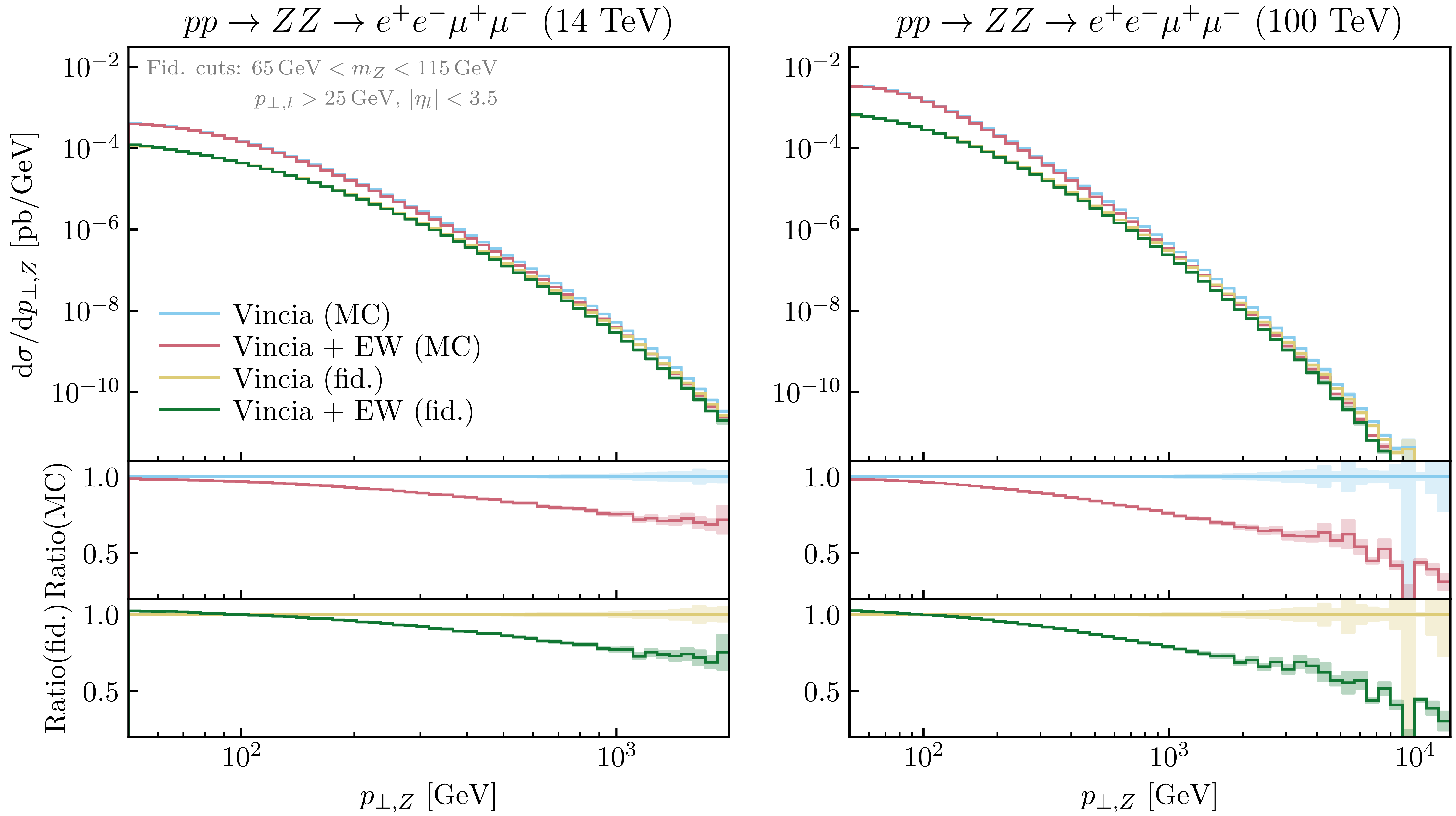
$$a^{\mathrm{FF}}_{\mathrm{trial}} = \frac{1}{m^2_{ij} - m^2_I}\left[c^{\mathrm{FF}}_1 + c^{\mathrm{FF}}_2 \frac{1}{z} + c^{\mathrm{FF}}_3 \frac{1}{1-z} + c^{\mathrm{FF}}_4 \frac{m^2_I}{m^2_{ij} - m^2_I}\right] \longrightarrow$$ Efficient veto algorithm

For every branching:

- Generate random branchings in random antennae
- Set up *linear programming* system
- Solve numerically

$$\text{Minimize } a^{\mathrm{FF}}_{\mathrm{trial},i} - a^{\mathrm{FF}}_i$$
$$\text{While } \forall i:\ a^{\mathrm{FF}}_{\mathrm{trial},i} > a^{\mathrm{FF}}_i$$

Virtual Sudakov logs

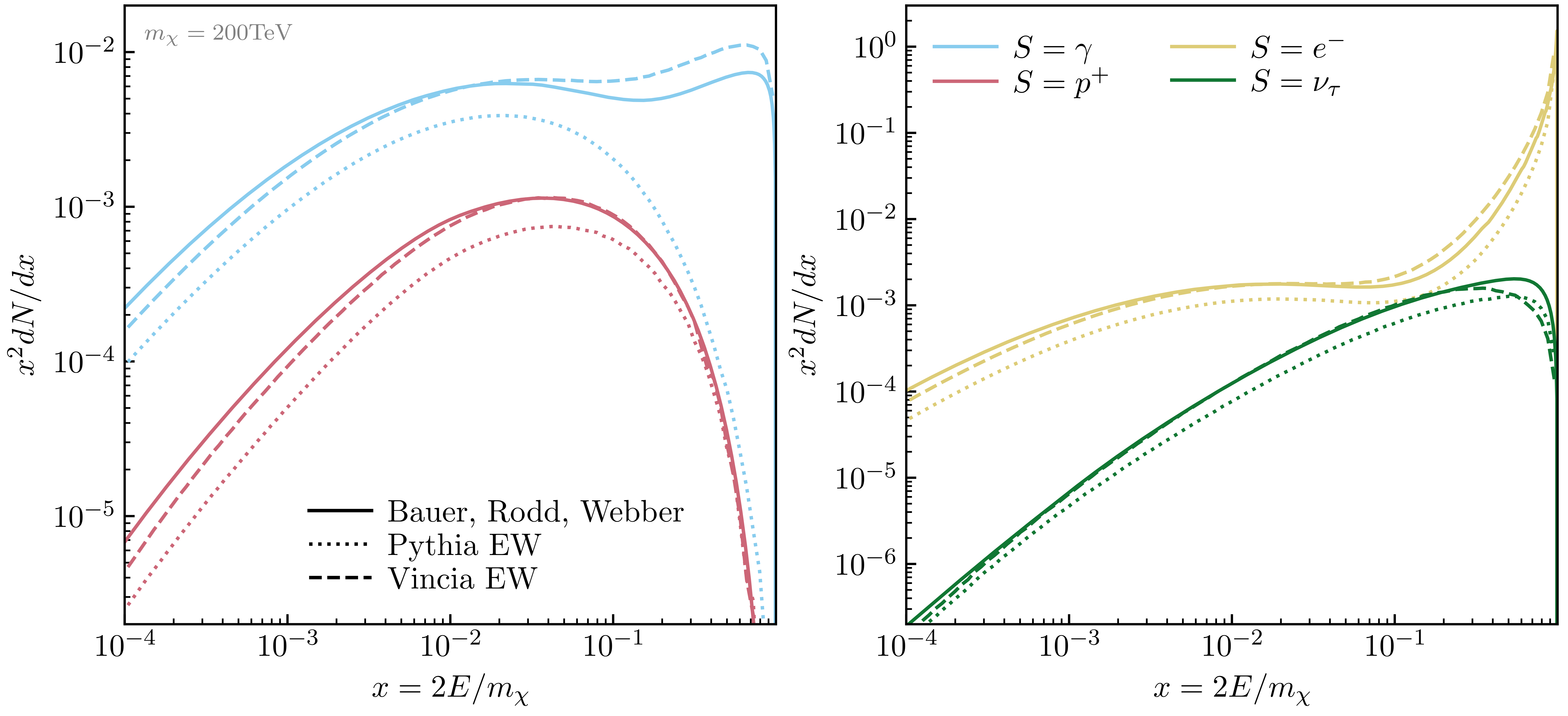


Dark Matter Decay Spectra

Comparison with analytic results

Bauer, Rodd, Webber 2007.15001

$\chi \to \nu_e \bar{\nu}_e \to S$

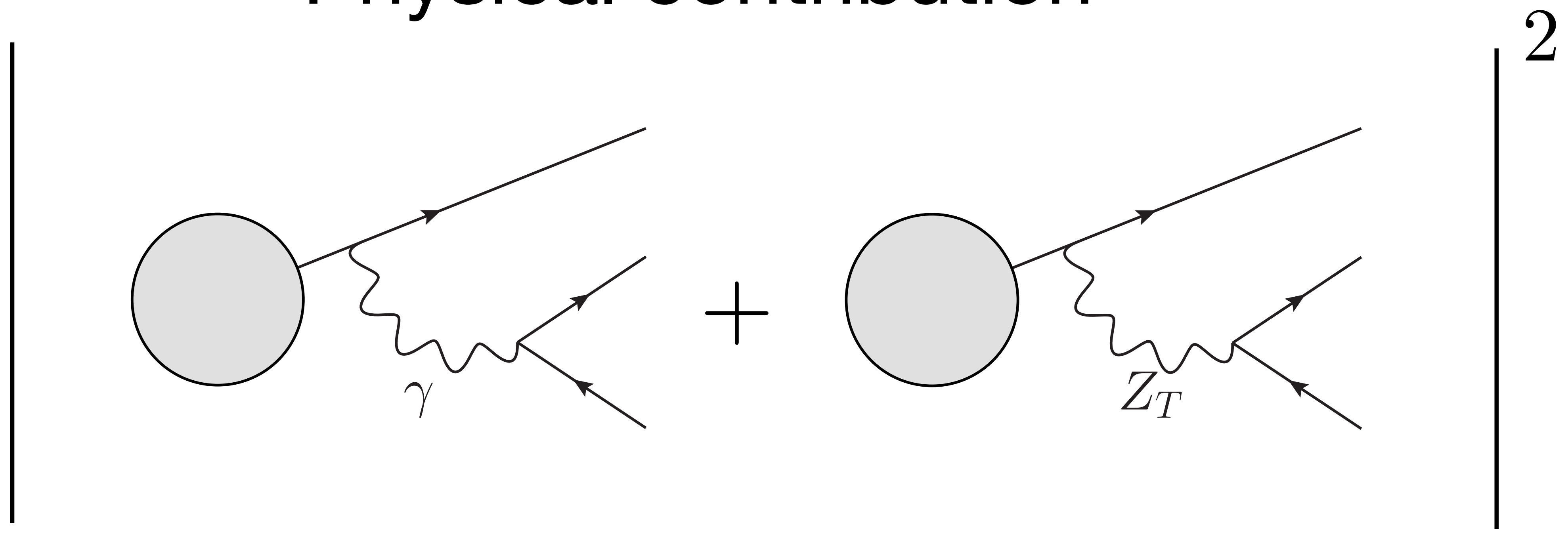


Novel features in the Electroweak Sector

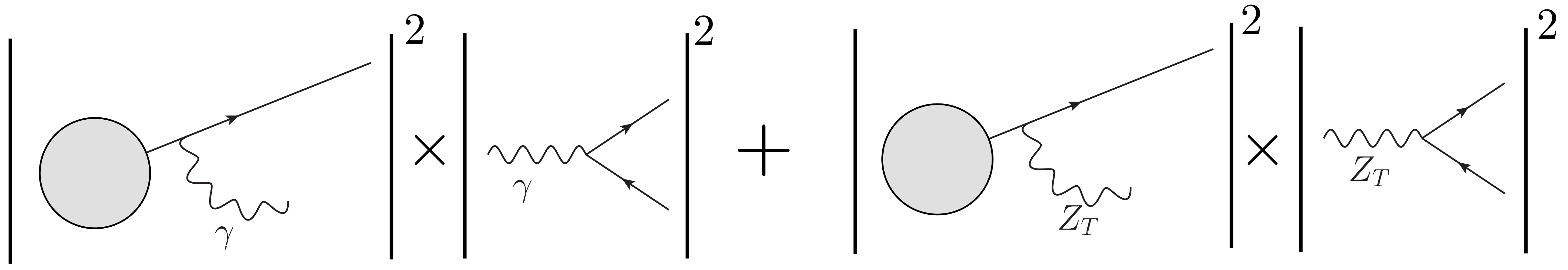
Neutral Boson Interference

Interference between γ, Z_T and h, Z_L

Physical contribution



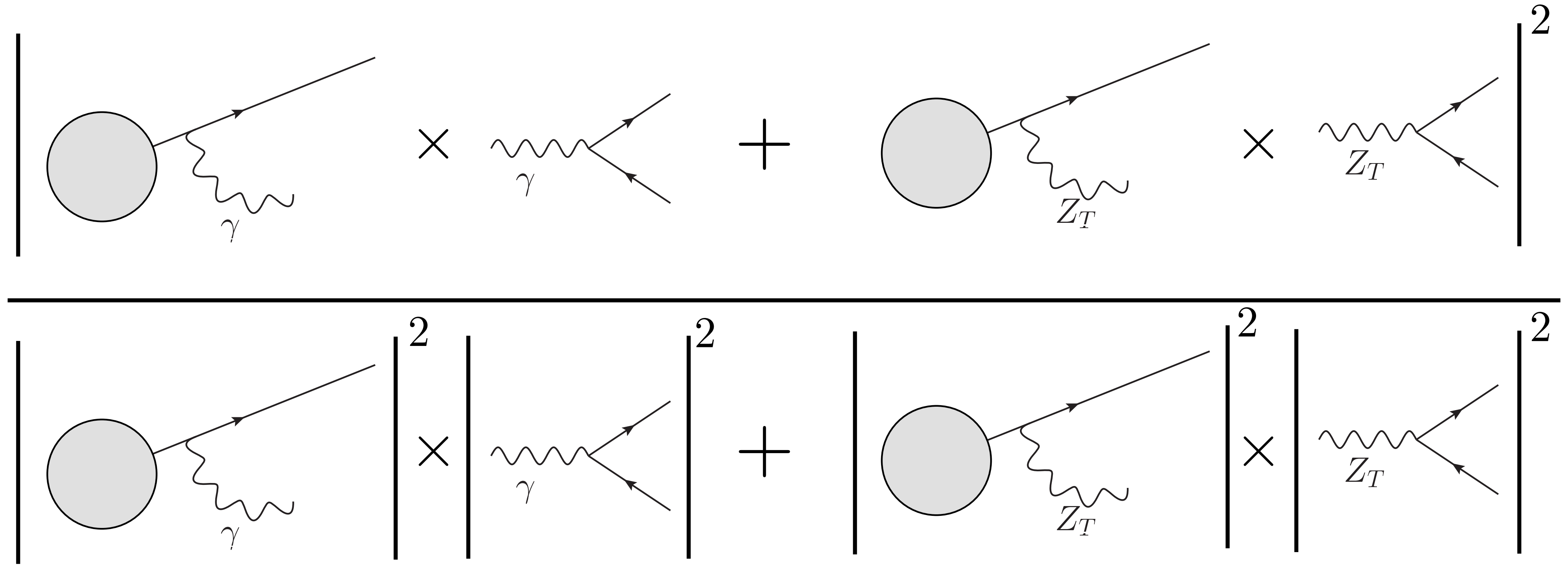
Shower approximation



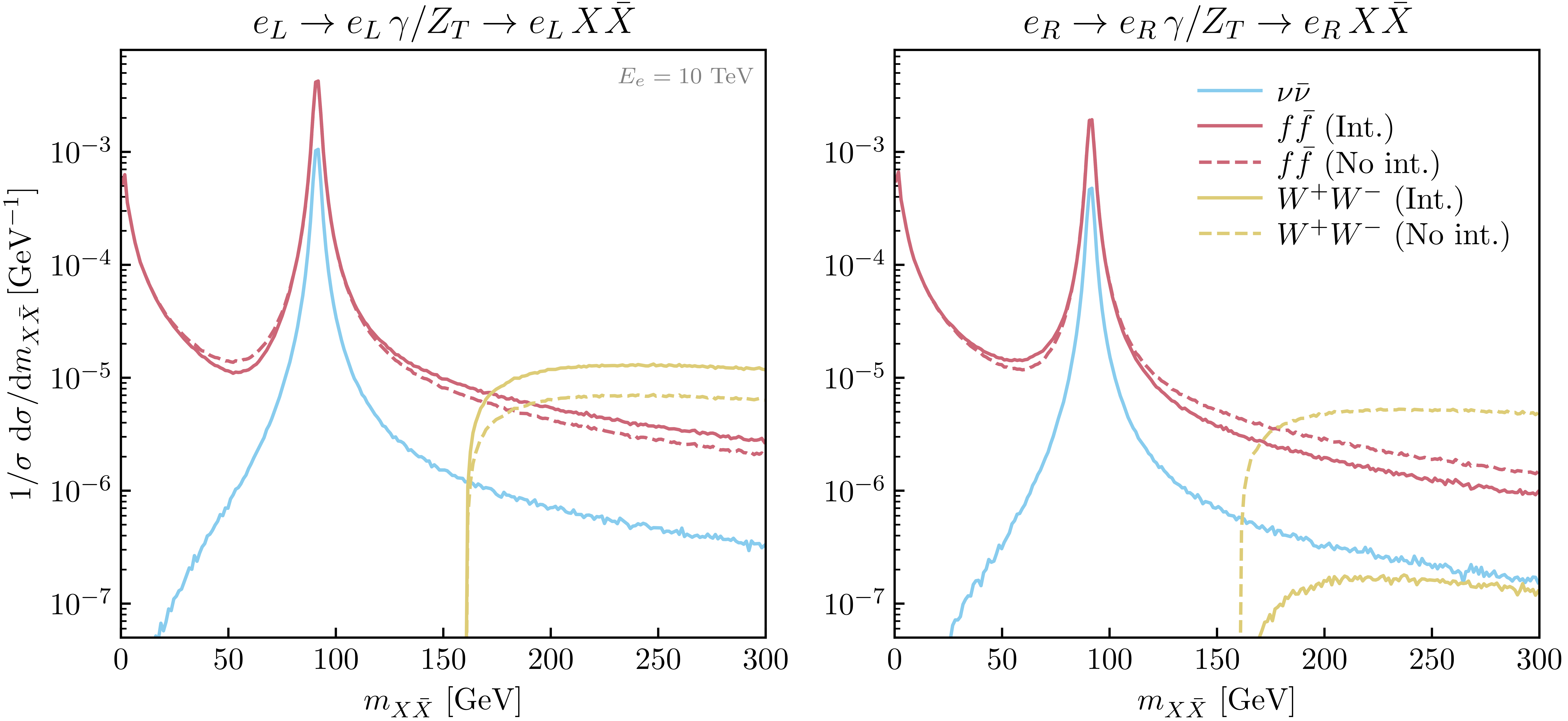
- Complicated solution: Evolve density matrices
  $\longrightarrow$ Very computationally expensive

- Simple solution: Apply event weight
  $\longrightarrow$ Does not get Sudakov right

$w =$

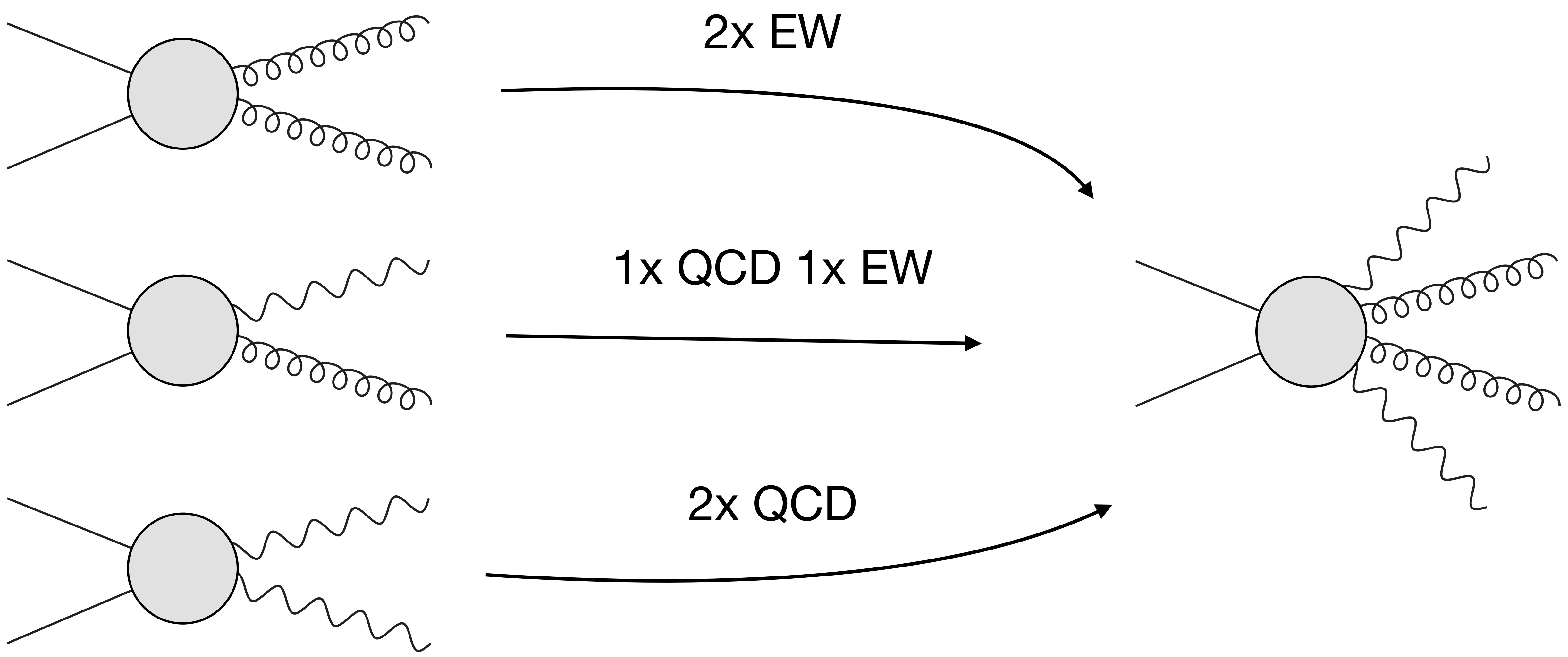


Bosonic Interference

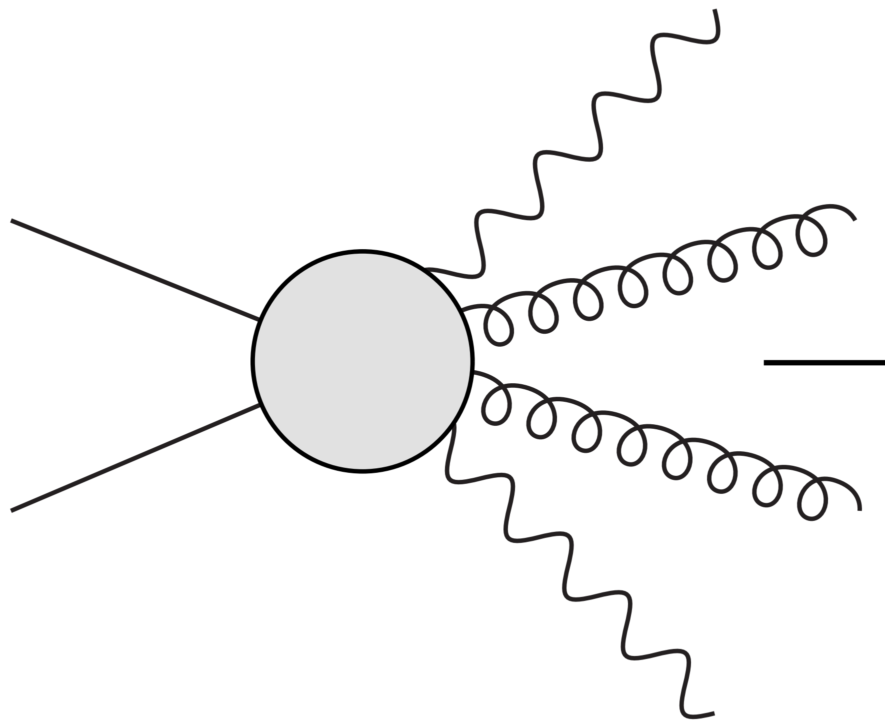


Overlap Veto

Double counting problem



Veto procedure



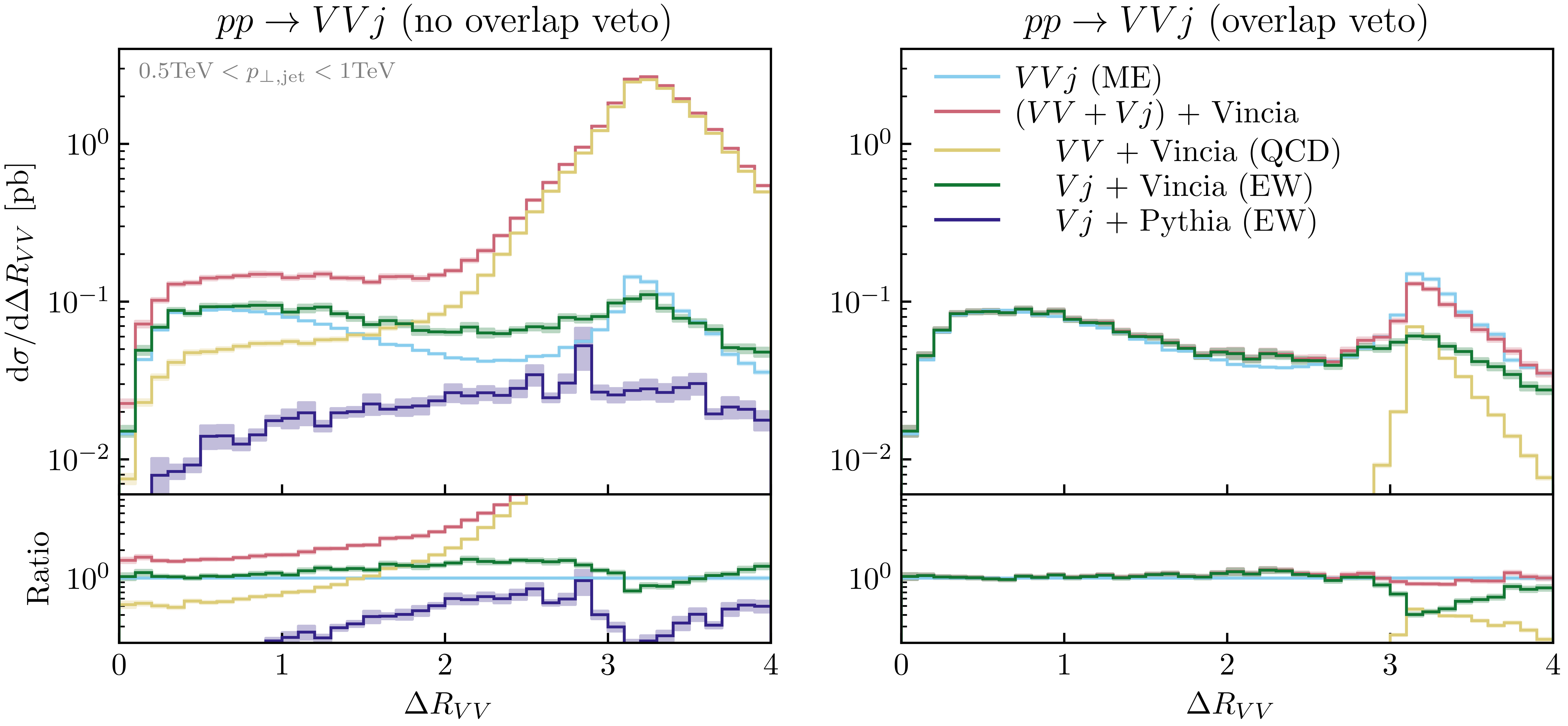
Last emission QCD? $\longrightarrow$ $d_{ij}^{\text{Last}} < \min\left(d_{ij}^{\text{EW}}\right)$

Last emission EW? $\longrightarrow$ $d_{ij}^{\text{Last}} < \min\left(d_{ij}^{\text{QCD}}\right)$

Accept Branching

$$d_{ij} = \min\left(k_{T,i}^2, k_{T,j}^2\right)\frac{\Delta_{ij}}{R} + m_i^2 + m_j^2 - m^2$$

Overlap Veto



Resonance Matching

Branchings like $t \to bW,\ Z \to q\bar{q}$ etc.

- Large scales:
  EW shower offers best description
- Small scales:
  Breit-Wigner distribution

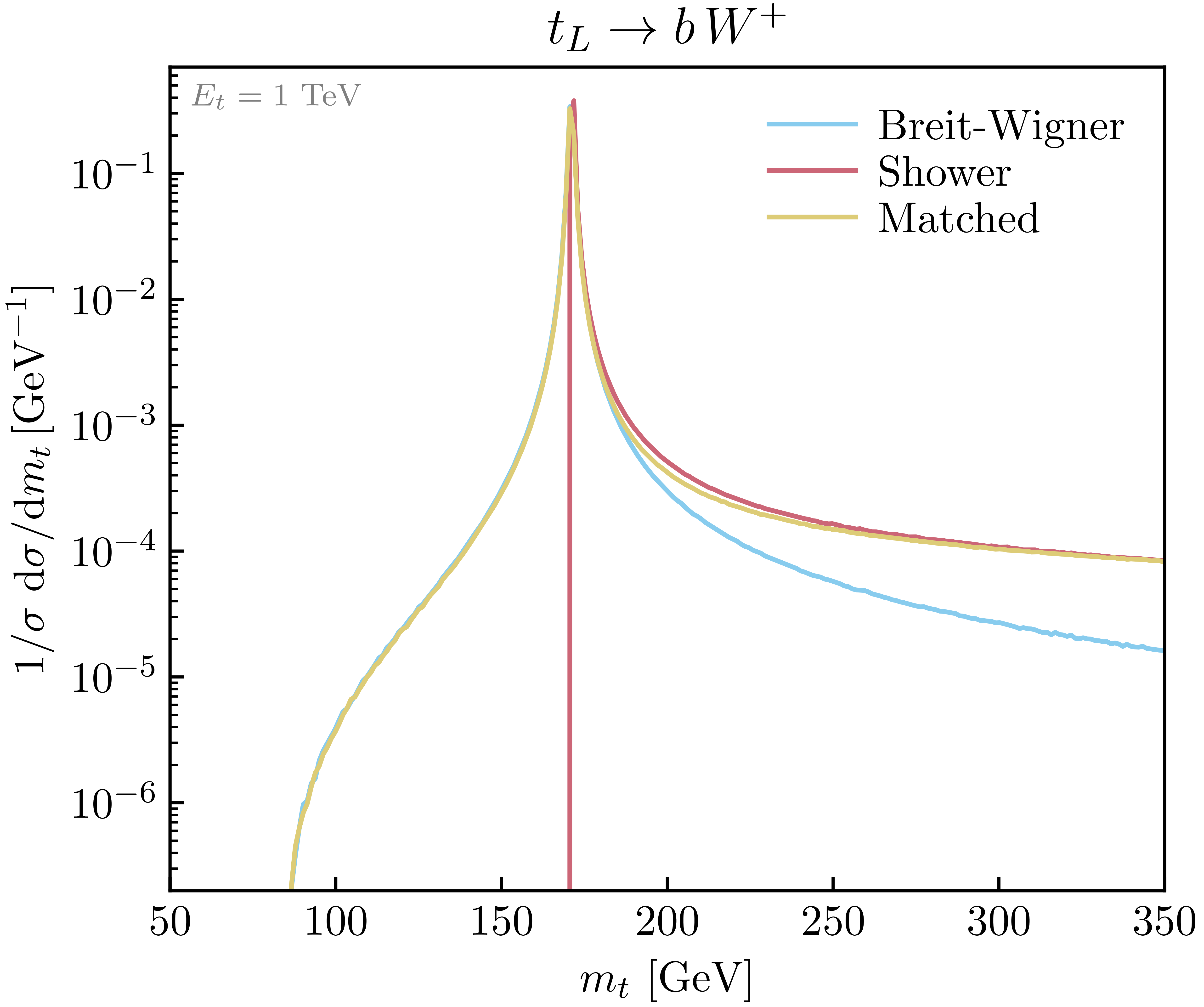
$$\mathrm{BW}(Q^2) \propto \frac{m_0\Gamma(m)}{Q^4 + m_0^2\Gamma(m)^2}$$

Matching:

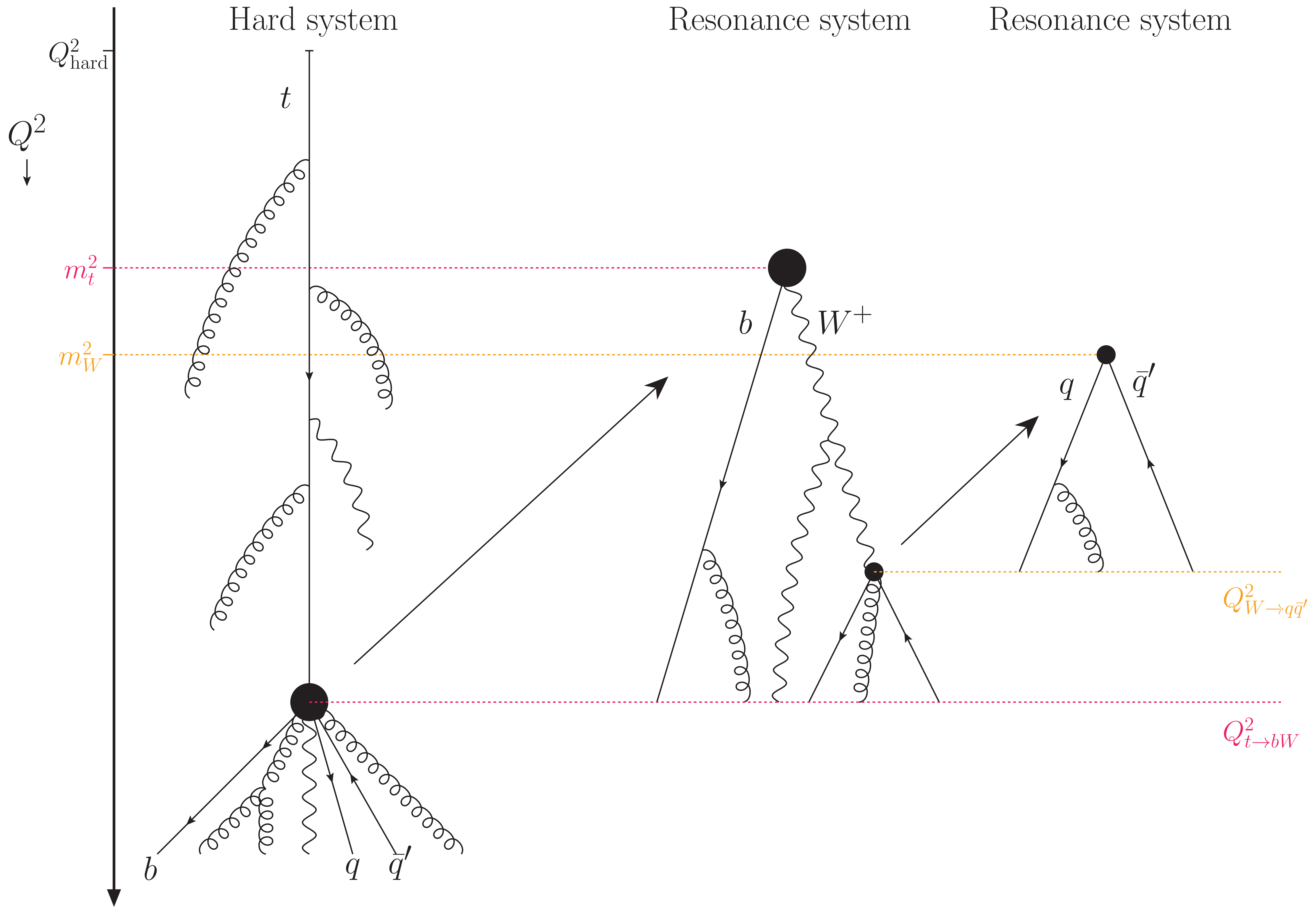
- Sample mass from Breit-Wigner upon production
- Suppress shower by factor

$$\frac{Q^4}{(Q^2 + Q_{\mathrm{EW}}^2)^2}$$

- Decay when shower hits off-shellness scale



Interleaved Resonance Decays



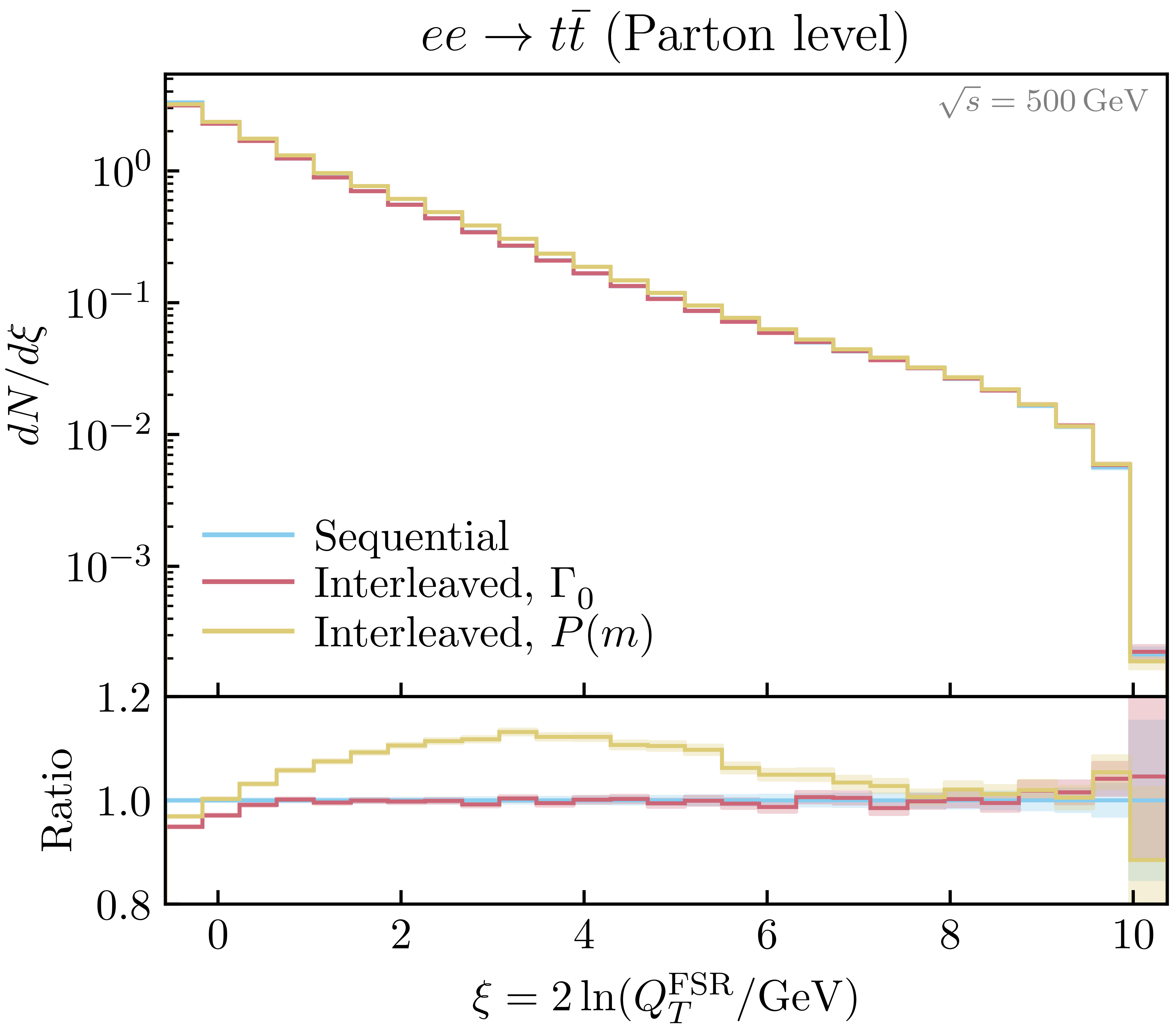
Sequential

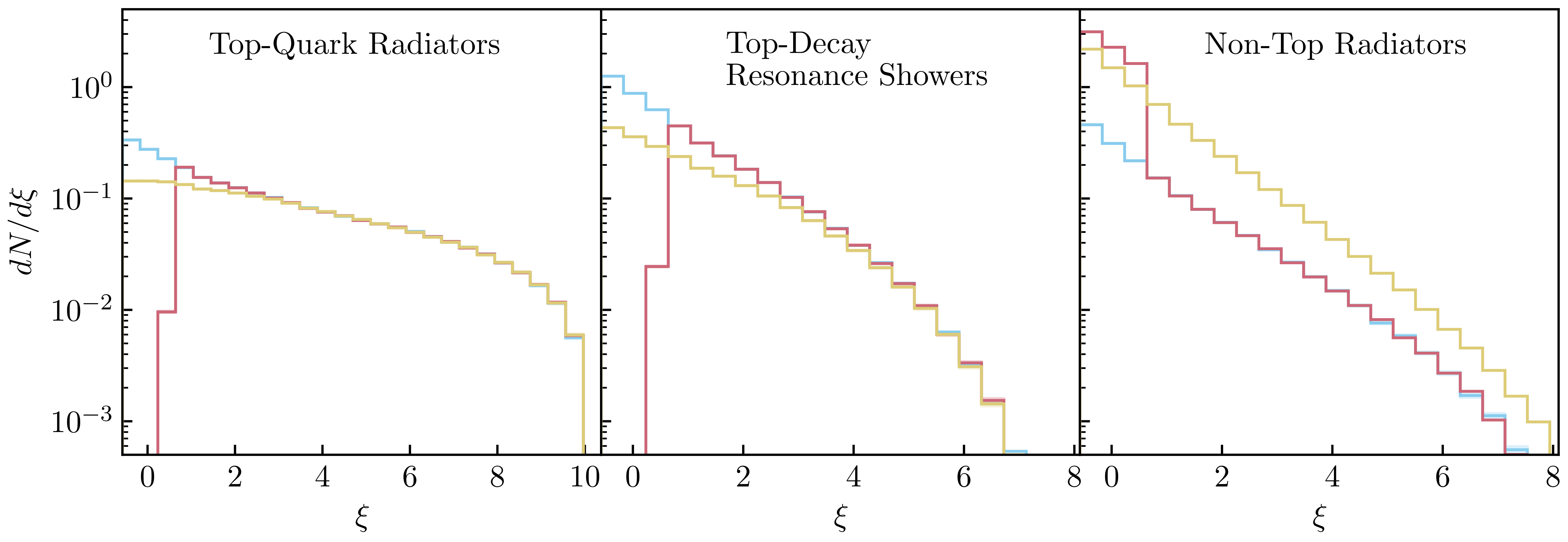
- Complete evolution of the hard system
- Perform resonance shower

Interleaved

- Evolution up to offshellness scale of the resonance
- Perform resonance shower
- Insert showered decay products and continue evolution

Interleaved Resonance Decays





Conclusions

QED Shower

- Includes full soft multipole structure, while interleaved with QCD shower

EW Shower

- Rich physics & many features unique to the EW sector
  - EW symmetry breaking / Goldstone contributions
  - Matching to resonance decays
  - Neutral boson interference
  - Overlap between hard scatterings

- Many other features yet to implement
  - Treatment of soft & spin interference
  - Bloch-Nordsieck violations

Interleaved Resonance Decays

- Physically-intuitive treatment of finite-width / offshell resonances

- More results in 2108.10786

QED & EW shower, and interleaved resonance decays available in Pythia 8.304

Interleaving Results