Effects of a **Time-Varying String Tension** & **String Repulsion** in Momentum Space

Tau-dependent string tension

*with **N. Hunt-Smith***
Publication in preparation

Physics motivations?

A primitive model

Results: Strangeness-pT correlations

String repulsion in momentum space

*with **C. Duncan***
arXiv:1912.09639

Why momentum space?

Two long, straight, parallel strings

Work in progress: towards more complicated topologies

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January, 2020, Lund

Tension and the Lund String Model

Cornell potential

Potential V(r) between **static** (lattice) and/or **steady-state** (hadron spectroscopy) colour-anticolour charges:

$$V(r) = -\frac{a}{r} + \kappa r$$

Coulomb part

String part
Dominates for $r \gtrsim 0.2\,\mathrm{fm}$

Lund model built on the asymptotic large-*r* linear behaviour

But intrinsically only a statement about the late-time / long-distance / steady-state situation. **Deviations at early times?**

Coulomb effects in the grey area between shower and hadronization?
Low-*r* slope > κ favours "early" production of quark-antiquark pairs?

+ Pre-steady-state effects from a (rapidly) **expanding string?**

Pre-Equilibrium Effects?

In a recent paper (JHEP 04(2018)145), Berges, Floerchinger, and Venugopalan developed a framework for

"computing the entanglement between spatial regions for Gaussian states in quantum field theory"

which they

"… applied to explore an **expanding light cone geometry** in the […] Schwinger model for QED in 1+1 space-time dimensions. "

➤ Entanglement entropy is **extensive in rapidity at early times**

➤ "a thermal density matrix for excitations around a coherent field with a **time dependent temperature**": $T \propto 1/\tau$

What does this mean in Lund Model context?

Implications for Lund Model?

I asked an honours student (N. Hunt-Smith) to take our 4th year quantum information course to see if we could parse the entanglement arguments

He learned a lot but we still didn't have a **dictionary**

We imagine it means the steady state captured by the lattice gets to have thermal excitations characterised by $T \propto 1/\tau$

But what does ***that*** mean?

Additional (virtual) quark-antiquark pairs with thermal distribution, which decay away with time?

Allow some of these to become real ➤ new mechanism for string breaks?

First step **poor man's model**: to explore effects of a higher effective energy scale and/or steeper potential well being relevant at early times.

Tau-Dependent String Tension

As a minimal modification to the existing string model, we studied the consequences of allowing an effective string tension

$$\kappa_{\rm eff}(\tau) = \kappa_0 + \Delta\kappa_{\rm therm}(\tau)$$

where $\kappa_0 \sim 1\,\mathrm{GeV/fm}$ and $\Delta\kappa_{\rm therm}(\tau) \propto 1/(\tau+\tau_0)$ with τ_0 a regularisation parameter that keeps the effective string tension finite and physically reflects that the string model itself is anyway not appropriate for very early (perturbative) times.

Some Questions:

To model Coulomb effect, study $\Delta\kappa \sim d/dr\,(-1/r) = 1/r^2$?

(and does $1/r^2$ really map to $\Delta\kappa \sim 1/\tau^2$?)

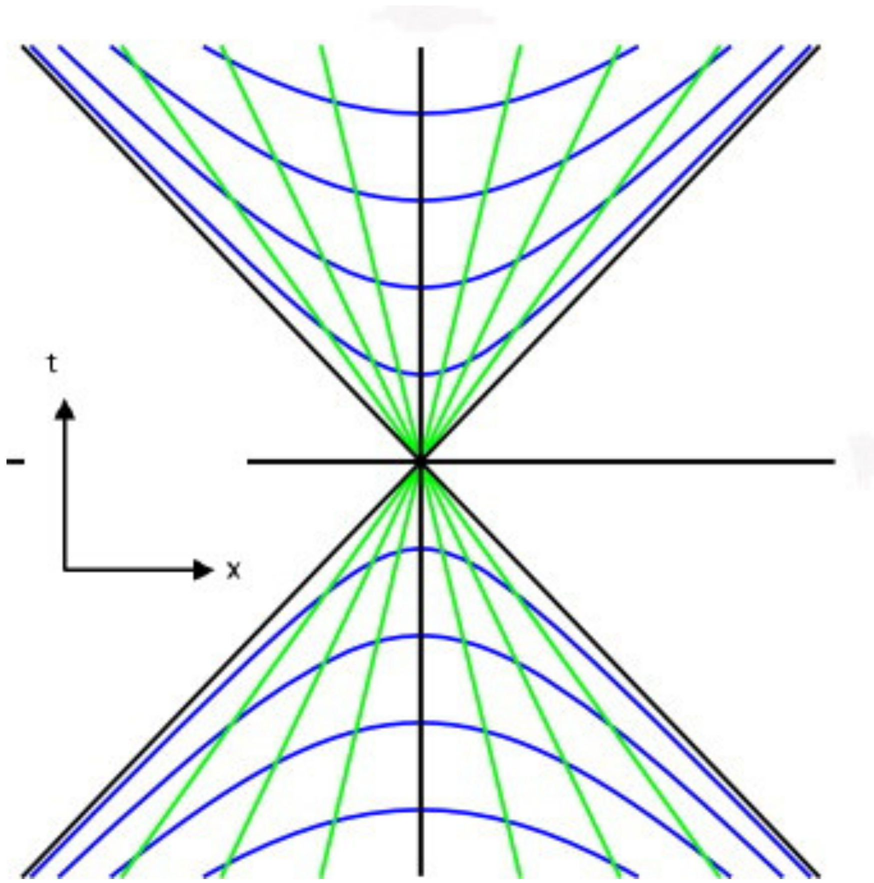
To model thermal effect, does $T \sim 1/\tau$ really map to $\Delta\kappa \sim 1/\tau$?

(Nuts & bolts not strongly tied to any particular form)

Calculating Tau

To use our modified κ(τ), need to know the τ value of each vertex

In UserHooks, we have access to the $\Gamma = \kappa^2 x_+ x_- = \kappa^2 \tau^2$ hyperbolic coordinate (via StringEnd)



Solve for **τ** but now using a non-linear relationship (with $<\tau>=1.2$ GeV^{-1})

$$\Gamma = \left(\kappa_0 + \Delta\kappa_{max} \frac{k < \tau >}{\tau + k < \tau >} \right)^2 \tau^2$$

with $\Delta\kappa_{max}$ and k as free parameters governing the shape of **κ**(**τ**).

(Solution is rather unattractive though.)

$$\tau = \frac{1}{2}\left(\frac{\sqrt{\Gamma}}{\kappa_0} - k < \tau > - \frac{\Delta\kappa_{max} k < \tau >}{\kappa_0} \right) + \frac{1}{2}\sqrt{\frac{\Gamma}{\kappa_0^2} - \frac{2\Delta\kappa_{max}\sqrt{\Gamma} k < \tau >}{\kappa_0^2} + \frac{\Delta\kappa_{max}^2 k^2 < \tau >^2}{\kappa_0^2} + \frac{2\Delta\kappa_{max} k^2 < \tau >^2}{\kappa_0}}$$

UserHooks implementation in Pythia

Want to generate string breaks with modifiable strangeness ratios and p_T broadening values.

Problem: no easy way to modify the trial probabilities; doChangeFragPar() appears to require constant reinitialisation (and changes are not re-set after use).

Solution for strangeness enhancement: no change of trial probabilities; implement instead as **up/down suppression** using doVetoFragmentation().

Generate trial breakups as usual, using nominal $P_{s:ud}$

Always accept a strange quark

Accept u,d with probability $P_{\mathrm{accept},ud}(\tau) = (P_{s:ud})^{1-\kappa_0/\kappa(\tau)}$

- In limit $\kappa \gg \kappa_0$: same probability to accept ud as was already generated for s
- In limit $\kappa \sim \kappa_0$: probability to accept ud → 1 ➤ effective $P_{s:ud}$ unchanged

Transverse Momentum Broadening

Want to generate higher effective p_T broadening values

Again we have the problem that we could not see how to change the trial generation parameters without constant reinitialisation, and such changes do not appear to be re-set after use.

Use the same strategy as for strangeness? (I.e. veto low-p_T hadrons as equivalent to enhancing high-p_T ones)?

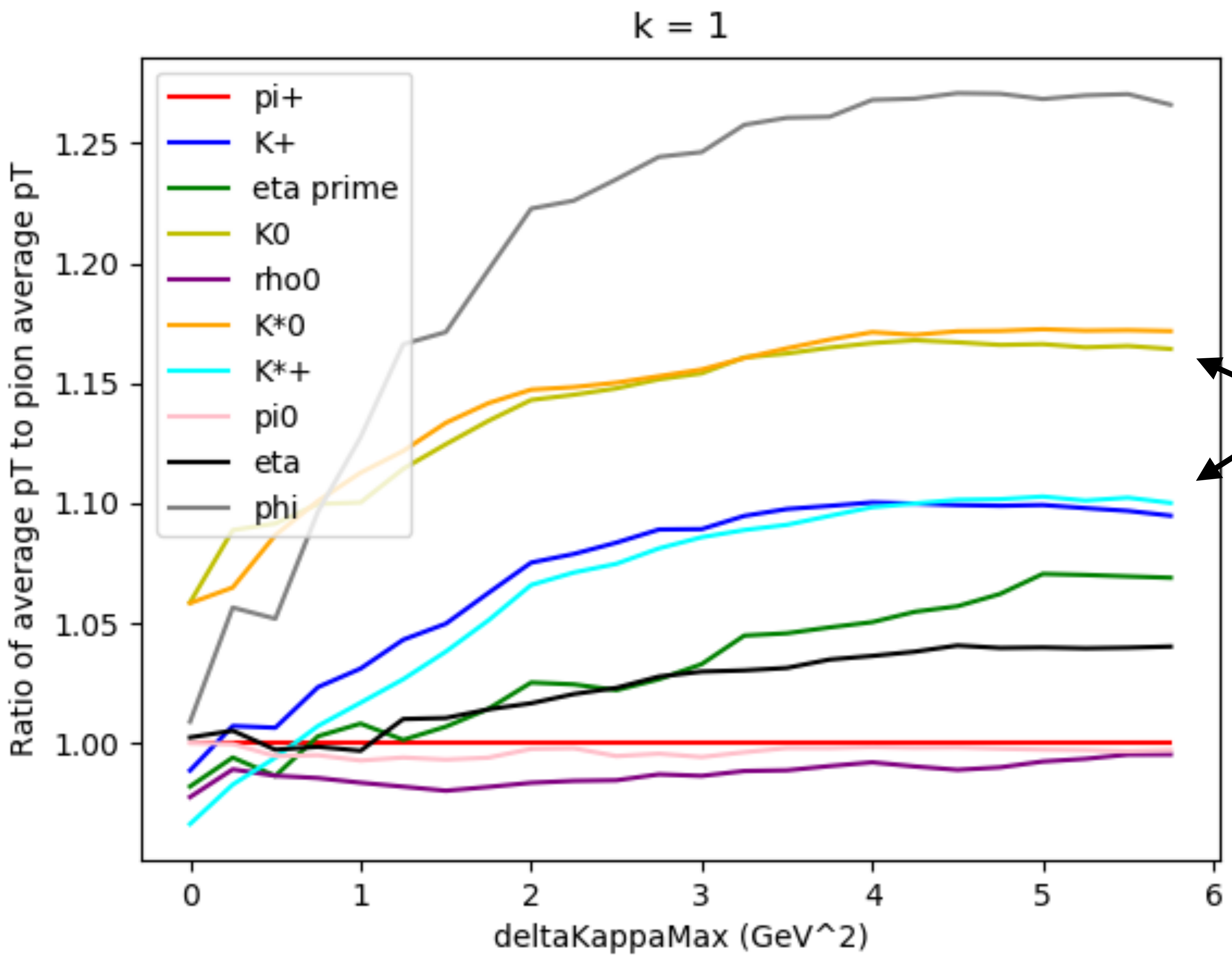
StringEnd provides pxHad,pyHad. But bad idea. Using a narrow Gaussian to sample a wider one very quickly becomes **extremely inefficient**.

Instead: use doChangeFragPar

Re-initialise with a larger StringPT:sigma value + implemented additional method to reset our modifications afterwards.

(Seems overkill / inefficient. To discuss?)

Some Results



Difference between K^0 and K^+ already present at $\Delta\kappa_{max}$=0.

Caused by leading hadrons having lower <p_T> and Z→quarks branching fractions give asymmetry in type of leading hadrons

Note: this is without retuning to same <N_{ch}>, <p_T>, or <strangeness>. Work to be done.

Comments

Regardless of technical implementation

Changes to the effective tension (τ dependence, thermal excitations, or fluctuating string tension - Bialas 99) ➤ mechanism to correlate strangeness and <p_T> without collective effects.

May affect interpretation of data for collective models too?

In perturbative stage, we are generating ss pairs (and others) which do not have a Gaussian p_T spectrum. Then we stop the shower and everything after that is Schwinger. Reasonable (?) that there should be **some sort** of intermediate/interpolating behaviour?

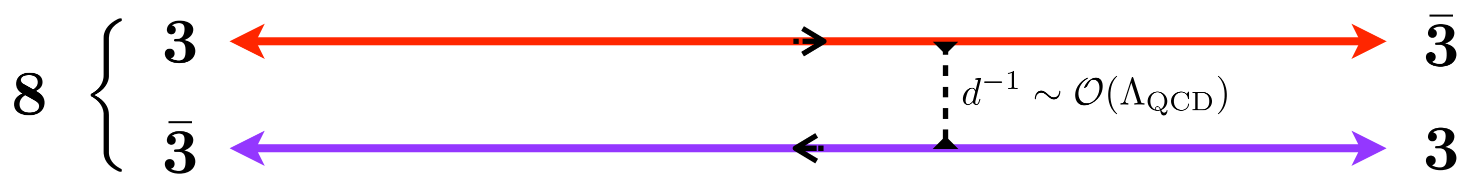
In general, when looking at departures from Gaussian, the mass and p_T dependence no longer factorises.

What masses to use? Conventional constituent masses probably a good starting point, but much too large for pions?

String-String Interactions

Consider a pp collision with a single soft gluon exchange

➤ Two parallel straight strings. Idealised picture:



If $d \ll r_{\mathrm{string}}$ and/or in a Type I SC analogy:

Model as a single (coherent) string, with an initial tension $\kappa_8 = 2.25\ \kappa_3$ (assuming Casimir scaling) ➤ **Rope Model** (no shoving)

If $d \gg r_{\mathrm{string}}$ and/or in a Type II SC analogy:

Model as separate strings, with interaction energy proportional to $1/d$.

Shoving model *(my understanding)*: starting from initial *d*, do explicit time steps for space-time evolution with repulsive* force (currently modelled as a number of gluons each carrying a small amount of p_T)

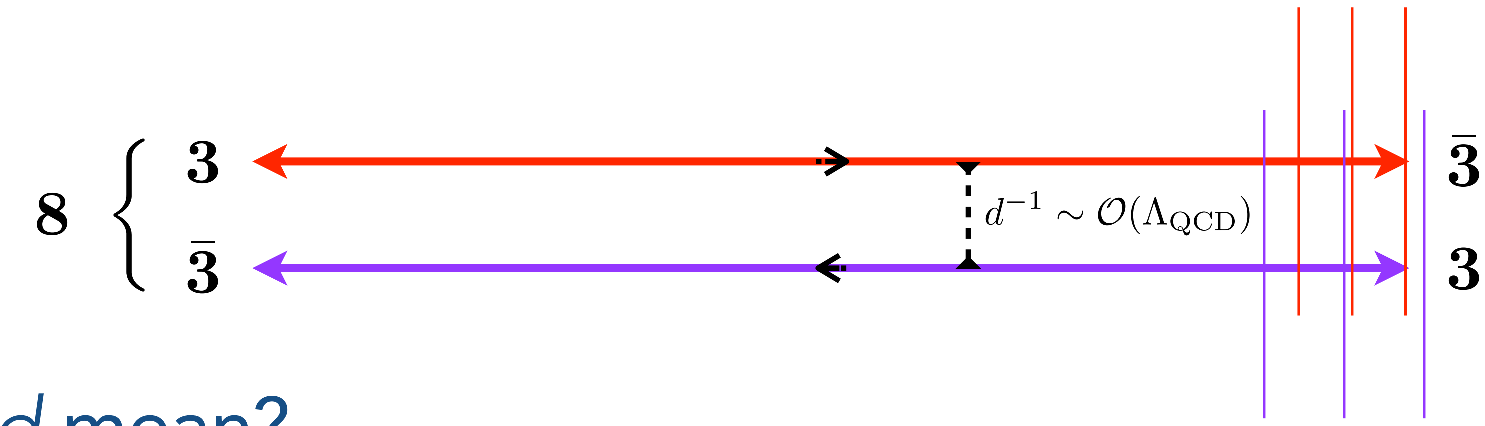
*Repulsive: assumes CR modeling effectively accounts for attractive configurations, at least to a first approximation. We shall make the same ansatz.

Coordinate vs Momentum Space

In Pythia, MPI model is based on perturbative scattering matrix elements (with p_{T0} screening regulator of couplings and propagators)

Strictly speaking, in- and outgoing states are plane waves.

Well-defined momenta ➤ completely delocalised in space:



What does *d* mean?

Can't puff and have meal in the mouth …

Fortunately, the momentum is not infinitely resolved. In a calculation with a factorisation scale Q_F the momentum is only defined up to $\mathbf{\Delta Q = O(Q_F)}$.

Shower cutoff Q_{HAD} ➤ outgoing shower states localised within $\mathbf{O(1/Q_{HAD})}$.

Distances $d > 1/Q_{HAD}$ are meaningful. Distances $d < 1/Q_{HAD}$ not meaningful.

Scales

What is d? (Or at least <d>, to start with)?

Considering **only pp**: related to r_{proton} convoluted with mass distribution

In pp, $1/\langle d \rangle$ is somewhat smaller than $1/r_{proton}$, somewhere in **$[\Lambda_{QCD}, 1\ \mathrm{GeV}]$**

What is Q_{HAD}?

Nominally IR cutoff of shower ~ **1 GeV**: same order of magnitude as $1/\langle d \rangle$

Another relevant quantity is $\mathrm{sqrt}(\kappa/\pi) \sim$ **$O(\Lambda_{QCD})$**

What is r_{string}?

A fraction of r_{proton} , $r^2 \propto 1/\kappa$? ➤ same **order of magnitude** as the other numbers

(PS: are we talking about coherence length or penetration depth? I don't know.)

Option 1: careful modelling dependent on relative O(1) sizes

Option 2: everything $O(\Lambda_{QCD})$ ➤ put all of it in the same (smeared-out) point

Dynamics determined by time evol. of **dofs** $\gg$ **Λ_{QCD}** (p_z & perturbative p_T values)

➤ **Stay in momentum space** ➤ Simpler modeling. (Some caveats here, ignored.)

Starting Point

Massless quark-antiquark string with invariant mass *W*:

Invariant measure of string length ~ multiplicity of hadrons (with mass m_0)

$$\propto \Delta y(m_0) = \ln(W^2/m_0^2)$$

Note: we take $m_0 \sim m_\rho \sim 0.77$ GeV ~ 2 * $m_{\text{constituent-quark}}$. Regulates rapidity-span calculation so that we get ~ same results for massless endpoints as when using PYTHIA's constituent-quark masses.

(Assumes all of the invariant mass is available for particle production)

If another string is nearby: assume some of the initial endpoint energy is converted to transverse motion instead

- ➤ **some fraction of the energy is not available for particle production**
- ➤ Two-step model. **"Compression"** (reduce W^2) + **"Repulsion"** (add p_T^2)

Idea: preserve string "transverse mass" $W_\perp^2 = W^2 + p_\perp^2 = W_+ W_-$

Momentum space ➤ assume total effect of repulsion is proportional to rapidity overlap Δy_{ov} (= Δy_{string} for identical strings)

In principle, could incorporate (physically consistent) knowledge about *d* via a "form factor"? with $F \to 0$ for $d \to \infty$ and $F \to 1$ for $d \to 0$.

Would probably need to be F(y,d) for more general configurations.

For now, we "hide" <F> in a constant of proportionality.

Repulsion p_T (total): $$p_{\perp R} = \pm c_R \cdot \Delta y_{\rm ov}$$

c_R: Effective amount of repulsion p_T per unit of overlapping rapidity

Compression: $$W^2 \to W'^2 = \left(1 - \frac{p^2_{\perp,R}}{W^2}\right) W^2 \le W^2$$

Right-moving (massless) endpoint scaled by: $W_+ \to W'_+ = f_+ W_+$

Left-moving (massless) endpoint scaled by: $W_- \to W'_- = f_- W_-$

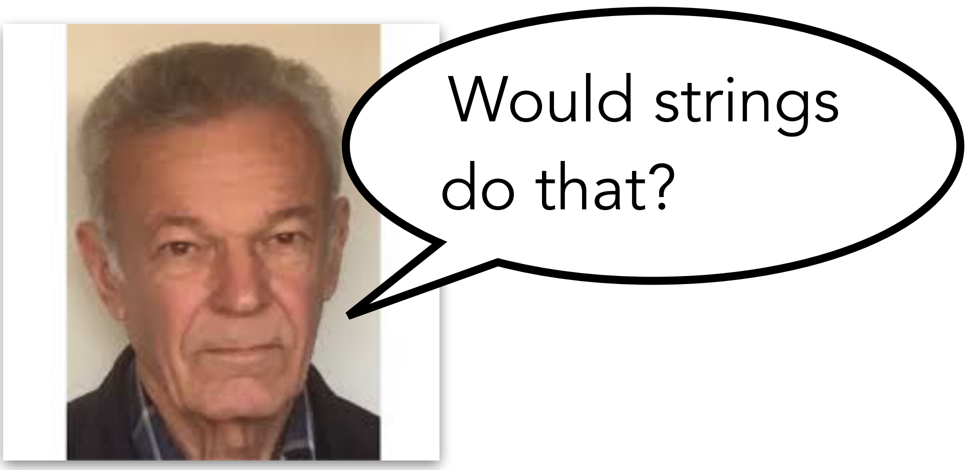
with $f_+ f_- = 1 - \frac{p^2_{\perp,R}}{W^2}$

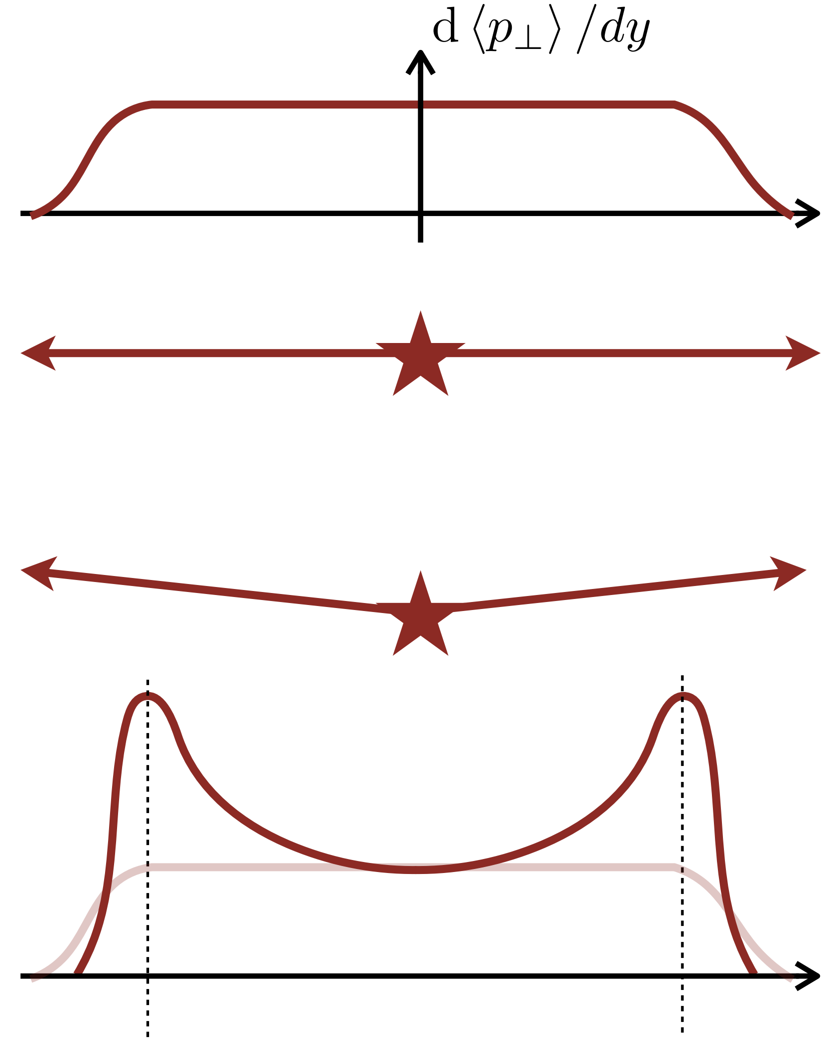
and $f_+ = f_- = f$ for now (by symmetry, for identical strings)

Step 2. Repulsion

A particularly simple way of representing the repulsion effect would be to boost the W′ system by a factor $\boldsymbol{\beta}_T = \mathbf{p}_{TR}/W'$

Happy that we had found a very simple way to do the whole thing. But …





Transverse boost:

Creates two (forward) jets.
Hadrons at large rapidities get more of the p_T
Hadrons at mid-rapidities get no additional p_T

What we want: a longitudinally boost-invariant uniform push

Repulsion at the Fragmentation Level

Add repulsion p_T as we fragment off the individual hadrons

How much p_T to give to each hadron?

Should be proportional to the (overlapping portion of the) **rapidity span taken by that hadron**

$$\Delta y_i = \ln\left(\frac{W_{i-1}^2}{m_0^2}\right) - \ln\left(\frac{W_i}{m_0^2}\right) = \ln\left(\frac{W_{i-1}^2}{W_i^2}\right)$$

Rapidity span before hadron i was fragmented off

Rapidity span after hadron i was fragmented off

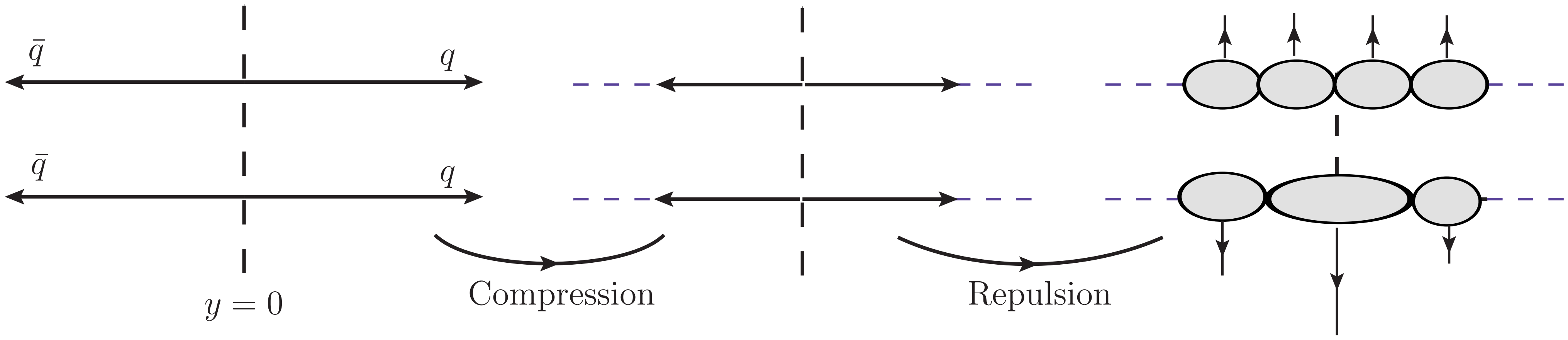
Rapidity span of hadron i independent of m_0 parameter

➤ **pT from repulsion given to hadron *i*:**

$$p_{\perp,i} = c_R \Delta y_i f_{\mathrm{ov},i} = p_{\perp R} \frac{\Delta y_i f_{\mathrm{ov},i}}{\Delta y_{\mathrm{string}}}$$

with $\sum_i f_{\mathrm{ov},i} = \frac{\Delta y_{\mathrm{ov}}}{\Delta y_{\mathrm{string}}}$ to account for if we step into / out of a region of string overlap.

Parallel Identical Strings: Results



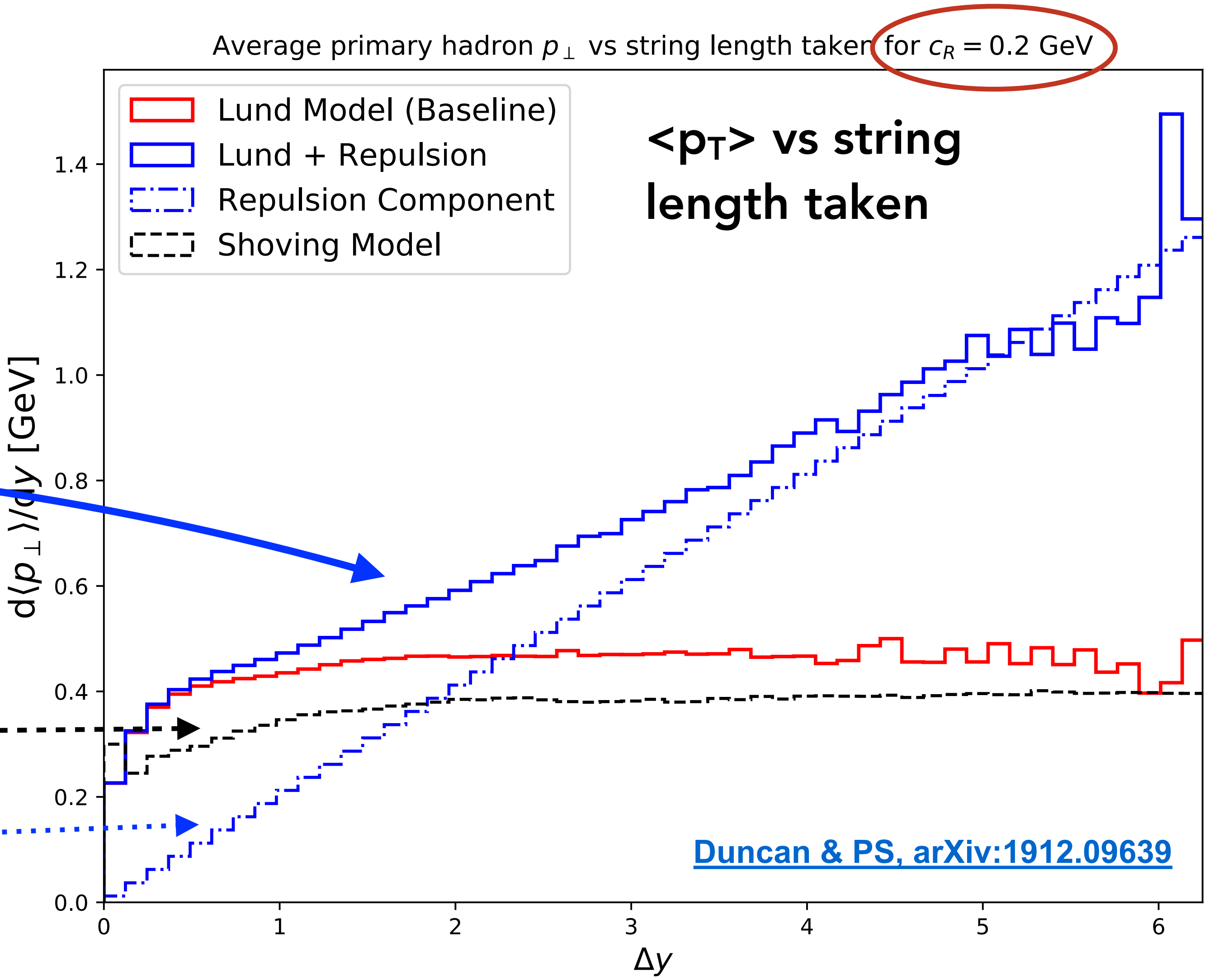
$$p_{+1} = p_{+2} = 400\left(1, 0, \vec{0}_\perp\right)\ \text{GeV}$$

$$p_{-1} = p_{-2} = 400\left(0, 1, \vec{0}_\perp\right)\ \text{GeV}$$

Default (random) fragmentation p_T + repulsion p_T

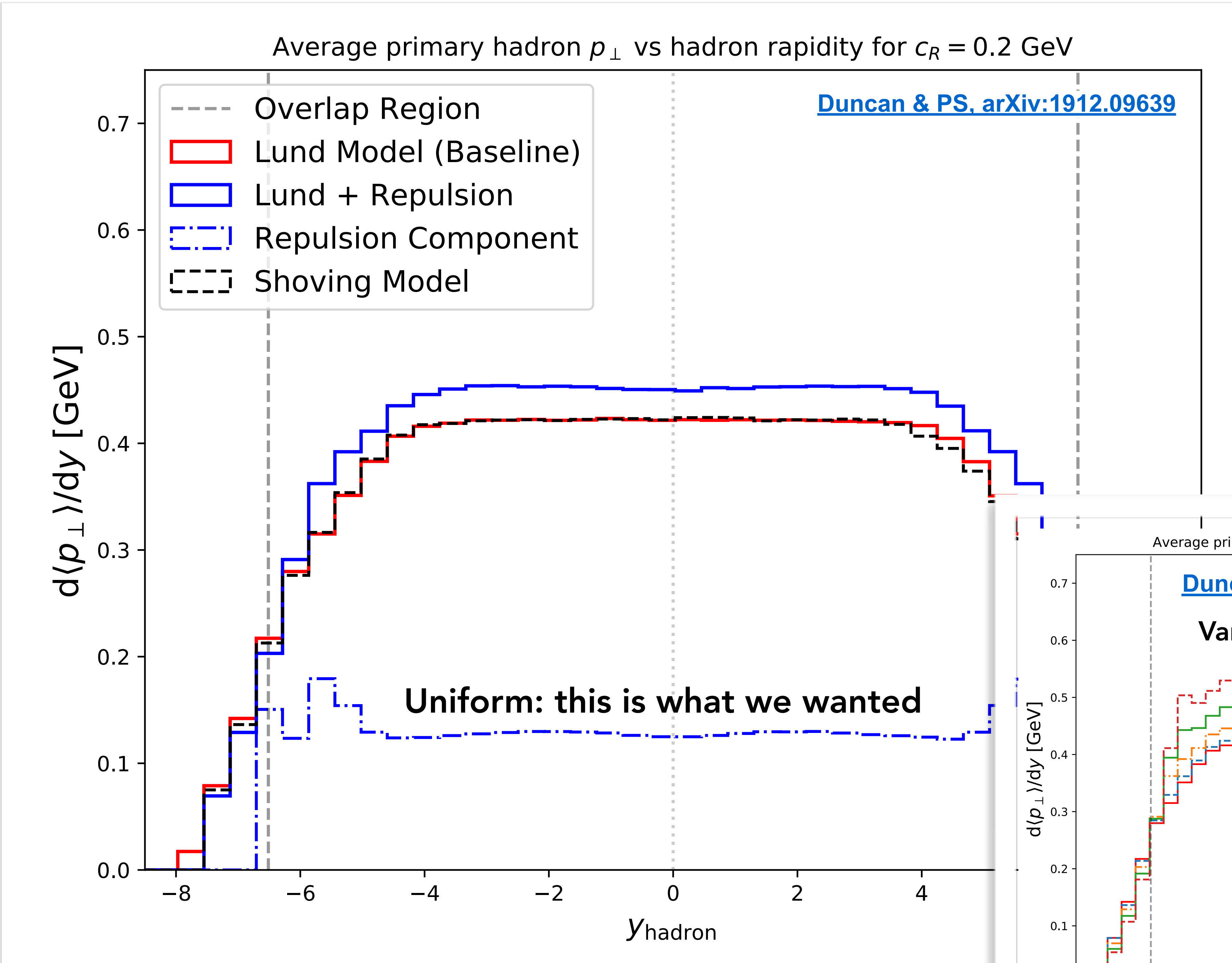
Lower $\langle p_T \rangle$ for shoving model: soft gluons increase multiplicity faster than total p_T?

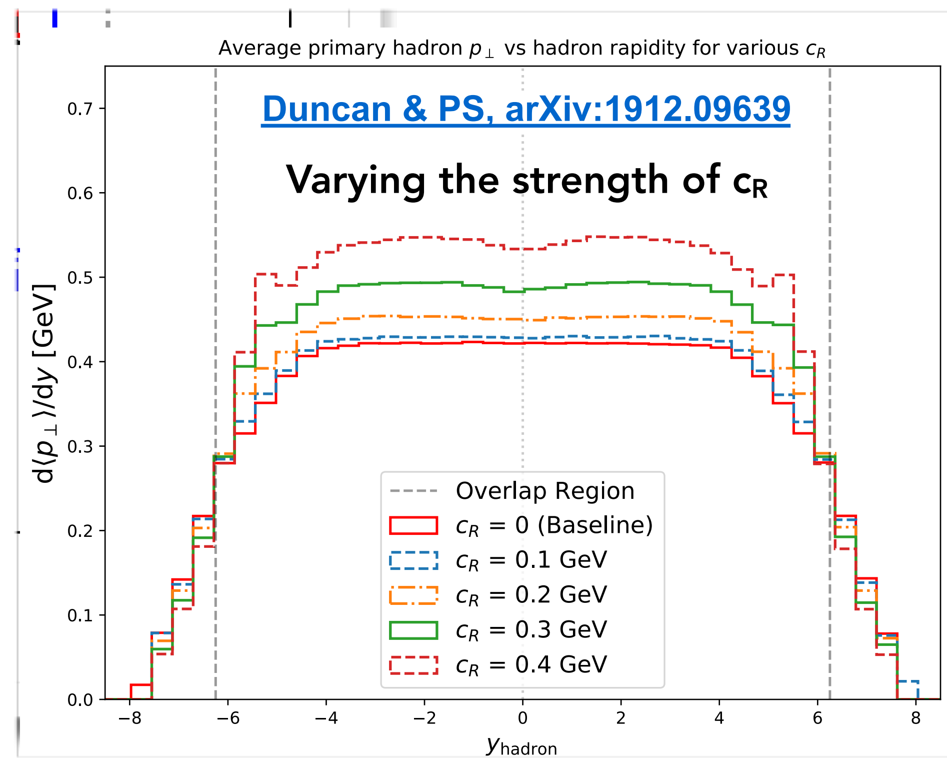
Repulsion component only (obtained by setting StringPT:sigma=0)



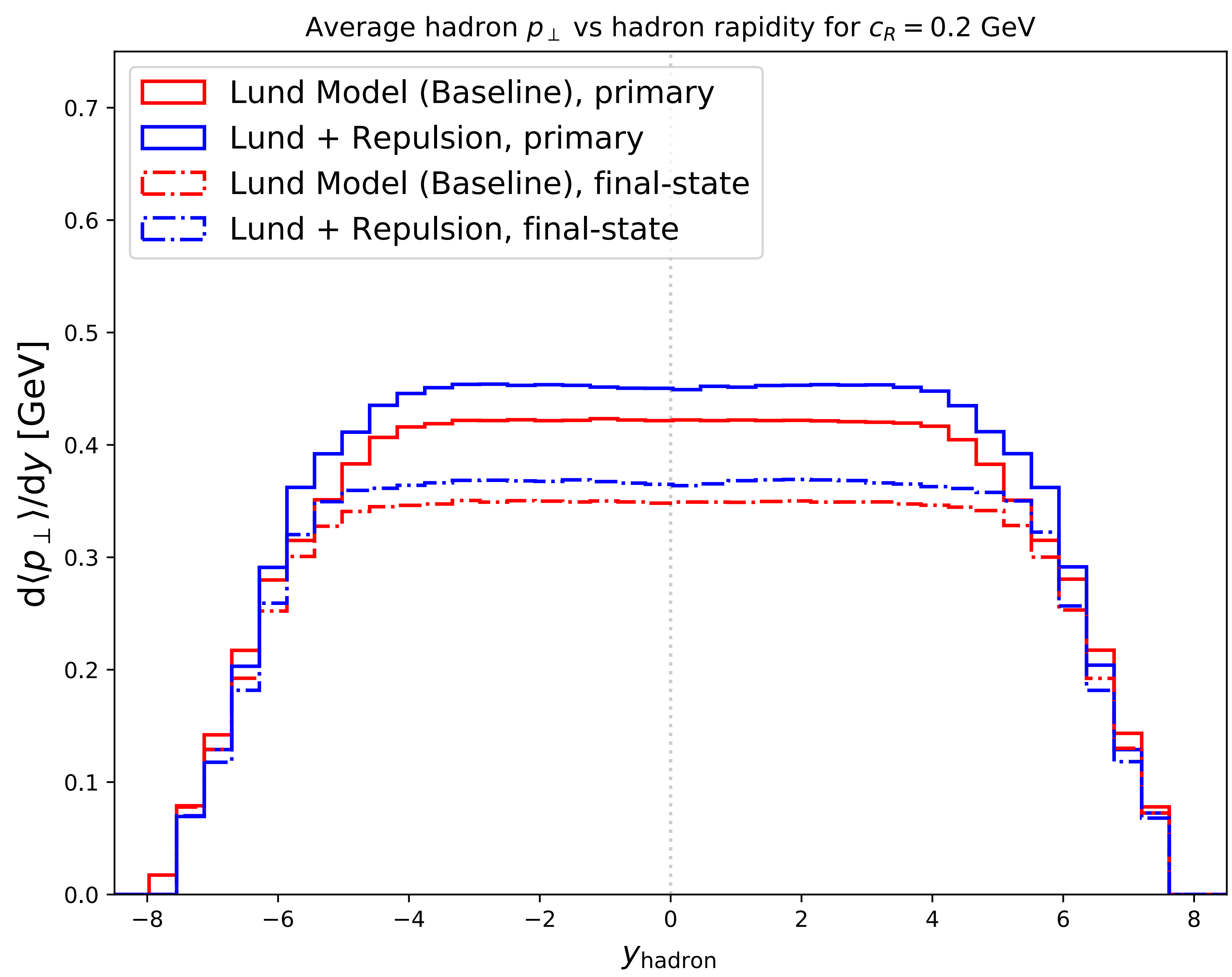
Amount of string length taken

$\langle p_T \rangle$ vs y_{hadron}





(Effect of Hadron Decays)



Towards more general topologies

It is rare that nature hands you two identical straight strings

- Asymmetric straight parallel strings
- Strings with a relative boost
- Strings with a relative rotation
- Strings with heavy endpoints
- More than 2 strings
- Strings with gluon kinks
- Junction strings
- Finite-distance effects

Towards more general topologies

It is rare that nature hands you two identical straight strings

- Asymmetric straight parallel strings
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- Strings with a relative rotation
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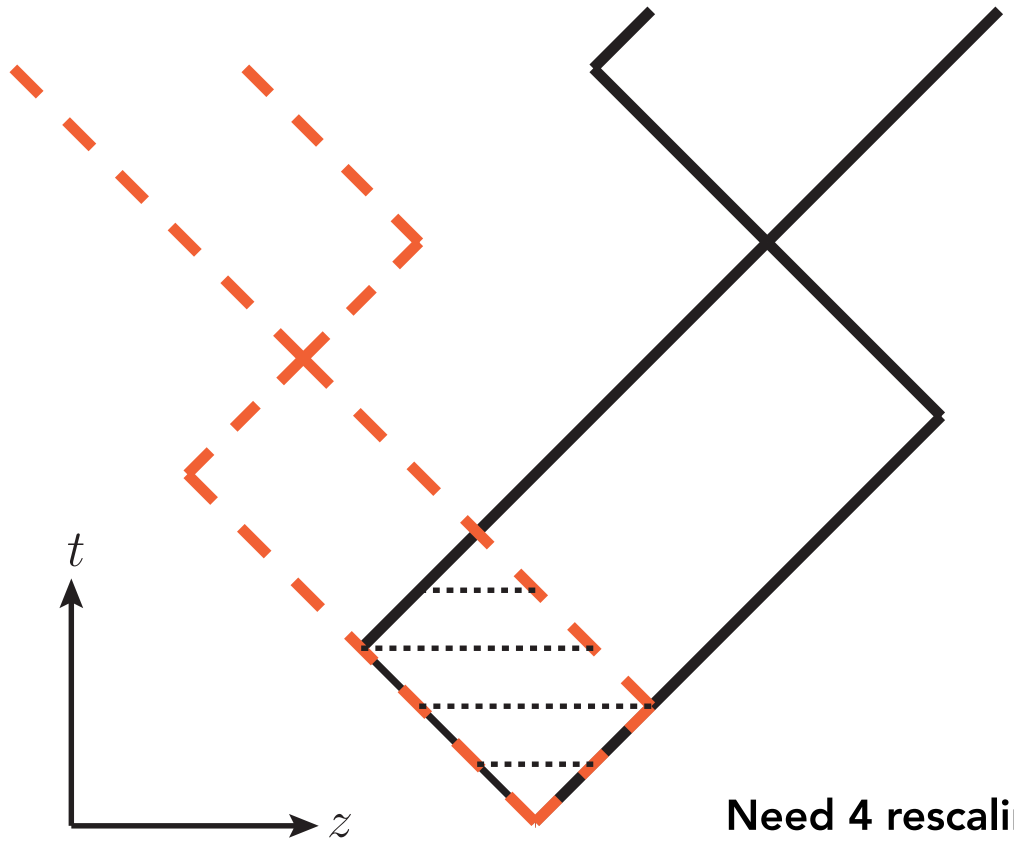
These are the cases we managed to consider in Duncan & PS, arXiv:1912.09639

- More than 2 strings
- Strings with gluon kinks
- Junction strings
- Finite-distance effects

Asymmetric Strings



Computation of rapidity overlap (and hence p_{TR}) still straightforward

Main new question is whether to allow p_z exchange: "longitudinal recoil" ?

Regardless of p_z strategy, the rescaling factors must satisfy:

$$f_{+1}f_{-1} = f_1^2 = 1 - \frac{p_{\perp,R}^2}{W_1^2}$$

Need 4 rescaling factors

$$f_{+2}f_{-2} = f_2^2 = 1 - \frac{p_{\perp,R}^2}{W_2^2}$$

Longitudinal momentum conservation, $\Delta p_{z1} = -\Delta p_{z2}$:

$$(1 - f_{+1})W_{+1} - (1 - f_{-1})\,W_{-1} \;= (1 - f_{-2})\,W_{-2} - (1 - f_{+2})\,W_{+2}$$

Need one more constraint. For now, we impose **no p_z exchange** (for simplicity; not convinced it is consistent with Lorentz invariance: p_z frame dependent. Reasonable starting point(?): no Δp_z in frame with centre of overlap at y=0).

Asymmetric Strings: Solutions

Assuming no p_z exchange:

$$W'_{-i} = f_{-i} W_{-i} = \sqrt{W_{Li}^2 + W_i^2 f_i^2} - W_{Li} \ ,$$

$$W'_{+i} = f_{+i} W_{+i} = \sqrt{W_{Li}^2 + W_i^2 f_i^2} + W_{Li} \ .$$

(reproduces the symmetric case in the limit $W_{+i} = W_{-i}$ ie $W_{Li} = 0$)

By construction longitudinal momentum is conserved:

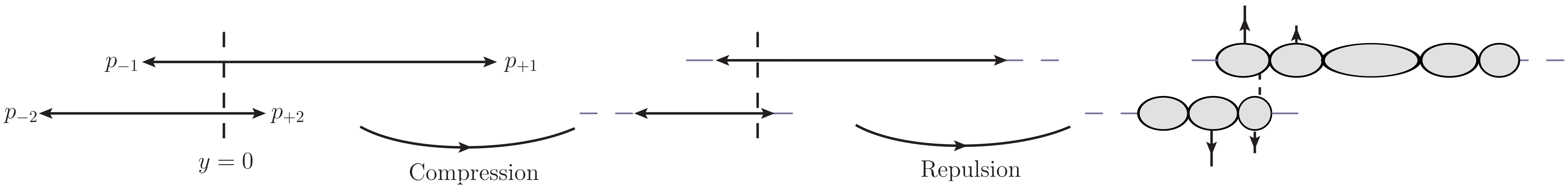
$$W'_{+i} - W'_{-i} = W_{+i} - W_{-i} .$$

Energy, however, is reduced (compression):

$$E'_i = \frac{W'_{+i} + W'_{-i}}{2} = E_i \sqrt{1 - \frac{p_{\perp,R}^2}{E_i^2}}$$

We regain the "lost" energy by giving the primary hadrons the repulsion $p_\perp$ and putting them on-shell again, with the string remnant absorbing the remaining energy.

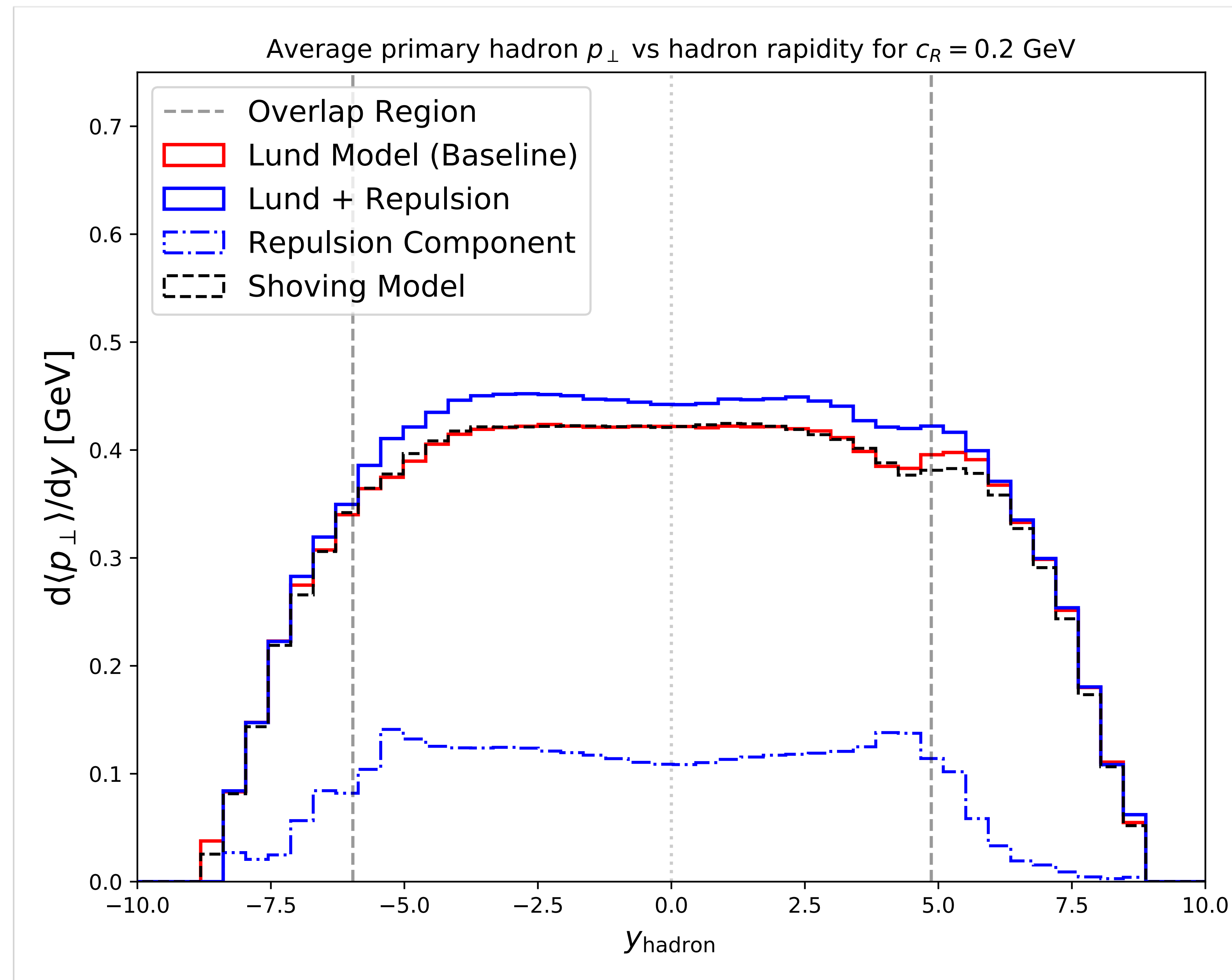
Asymmetric parallel strings: Results



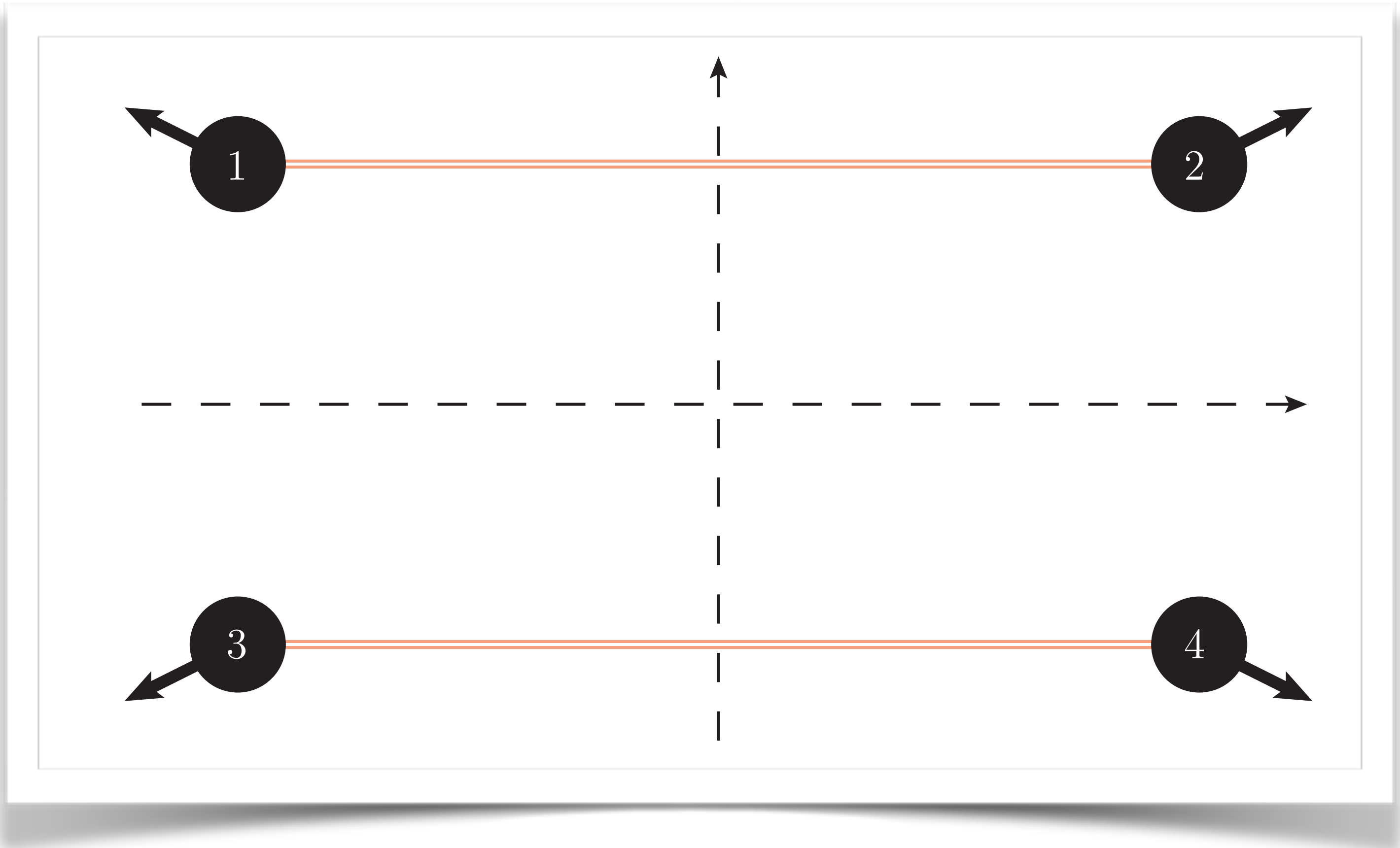
+ , - , T

$$p_{+1} = 1200\left(1, 0, \vec{0}_\perp\right)\ \text{GeV},$$
$$p_{-1} = 300\left(0, 1, \vec{0}_\perp\right)\ \text{GeV},$$
$$p_{+2} = 100\left(1, 0, \vec{0}_\perp\right)\ \text{GeV},$$
$$p_{-2} = 1000\left(0, 1, \vec{0}_\perp\right)\ \text{GeV},$$

Although we used pretty long strings (we thought), effects of partial overlaps still somewhat obscured by endpoint falloffs.



Topologies with a relative transverse boost



$$\begin{aligned} p_1 &= E(\ 1, \quad \sin\theta, \quad 0, \quad -\cos\theta) \\ p_2 &= E(\ 1, \quad \sin\theta, \quad 0, \quad \cos\theta) \\ p_3 &= E(\ 1, \quad -\sin\theta, \quad 0, \quad -\cos\theta) \\ p_4 &= E(\ 1, \quad -\sin\theta, \quad 0, \quad \cos\theta) \end{aligned}$$

Boost $\beta = \pm \sin(\theta) = 0.1$

1. Evaluate rapidity overlap along common axis (smaller than the individual string CM rapidity spans) ➤ total p_{TR}

2. Rescale string ends similarly to before This causes the ends to lose some p_T. Added to p_T reservoir to be added back during fragmentation.

Alternative: boost compressed strings so they regain their original p_T? Reduce p_z , then E to bring back on shell?

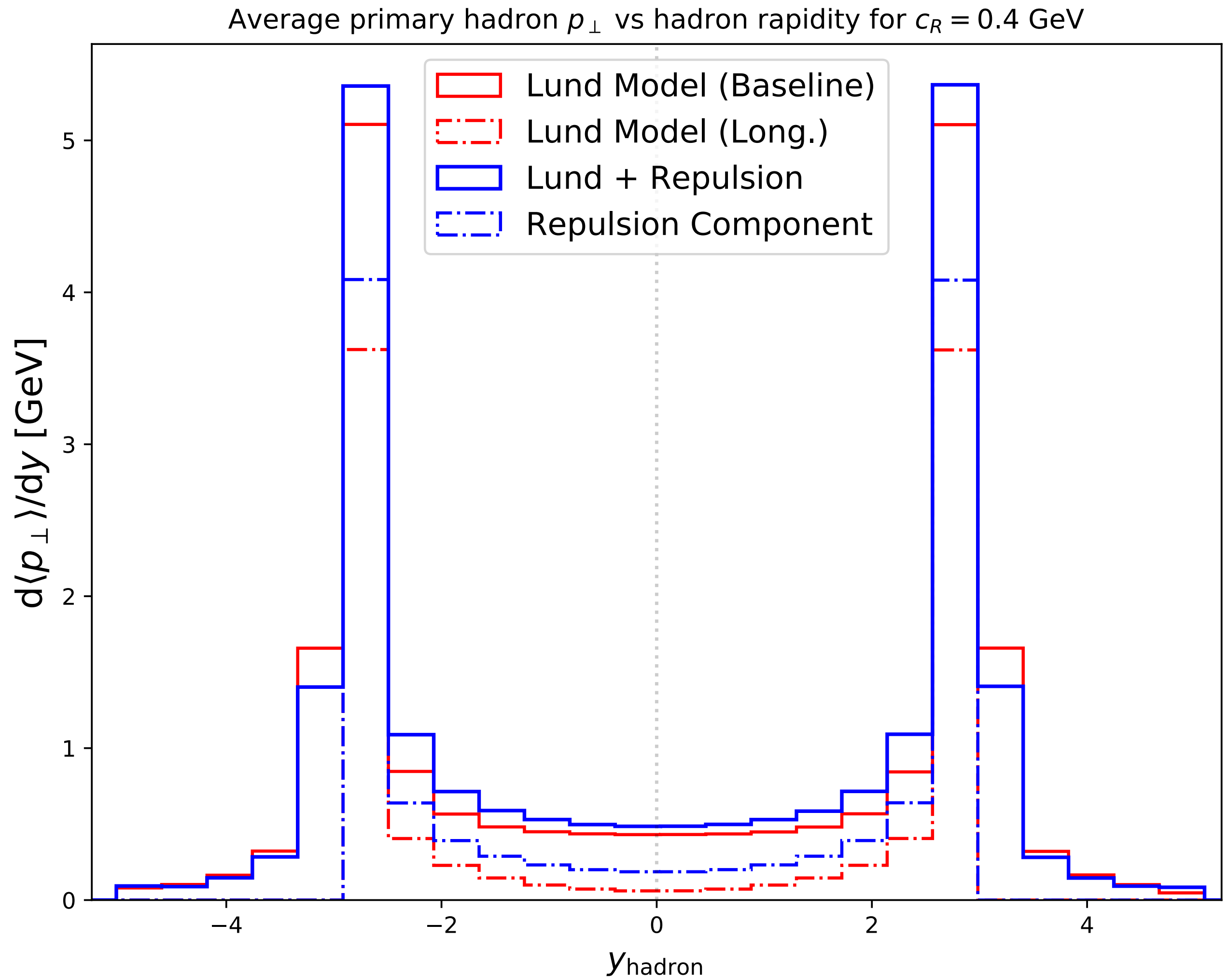
3. Hadron rapidity spans projected onto common axis:

$$\Delta y_{\mathrm{eff}} = \frac{\Delta y_{\mathrm{string}}}{\Delta y^*_{\mathrm{string}}} \Delta y^*_{\mathrm{taken}}$$

In reality, soft hadrons should have f_{ov}~1?

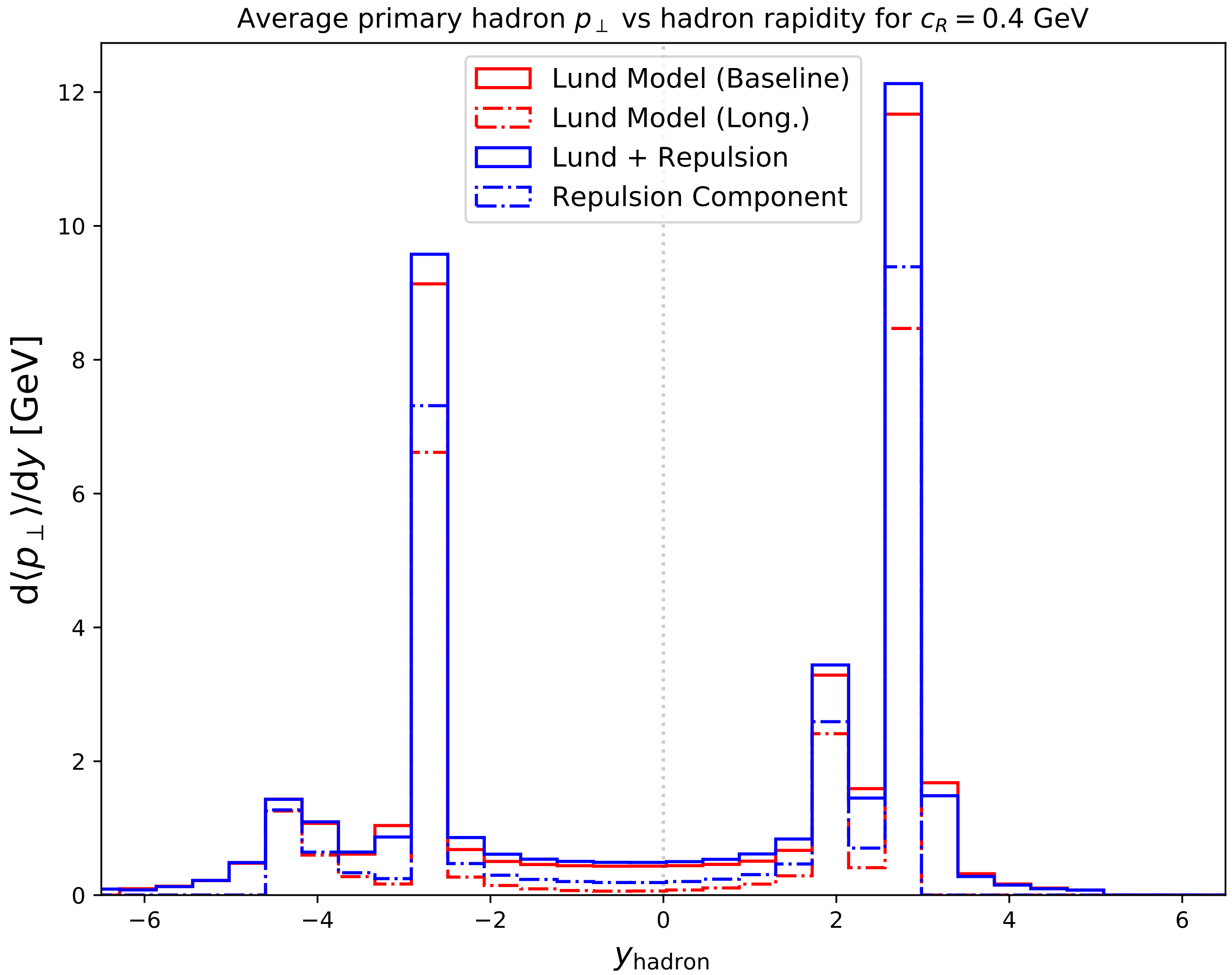
Results: Boosted topologies

Symmetric



Asymmetric

(same as the one used earlier with boost β=0.1 in opposite directions)

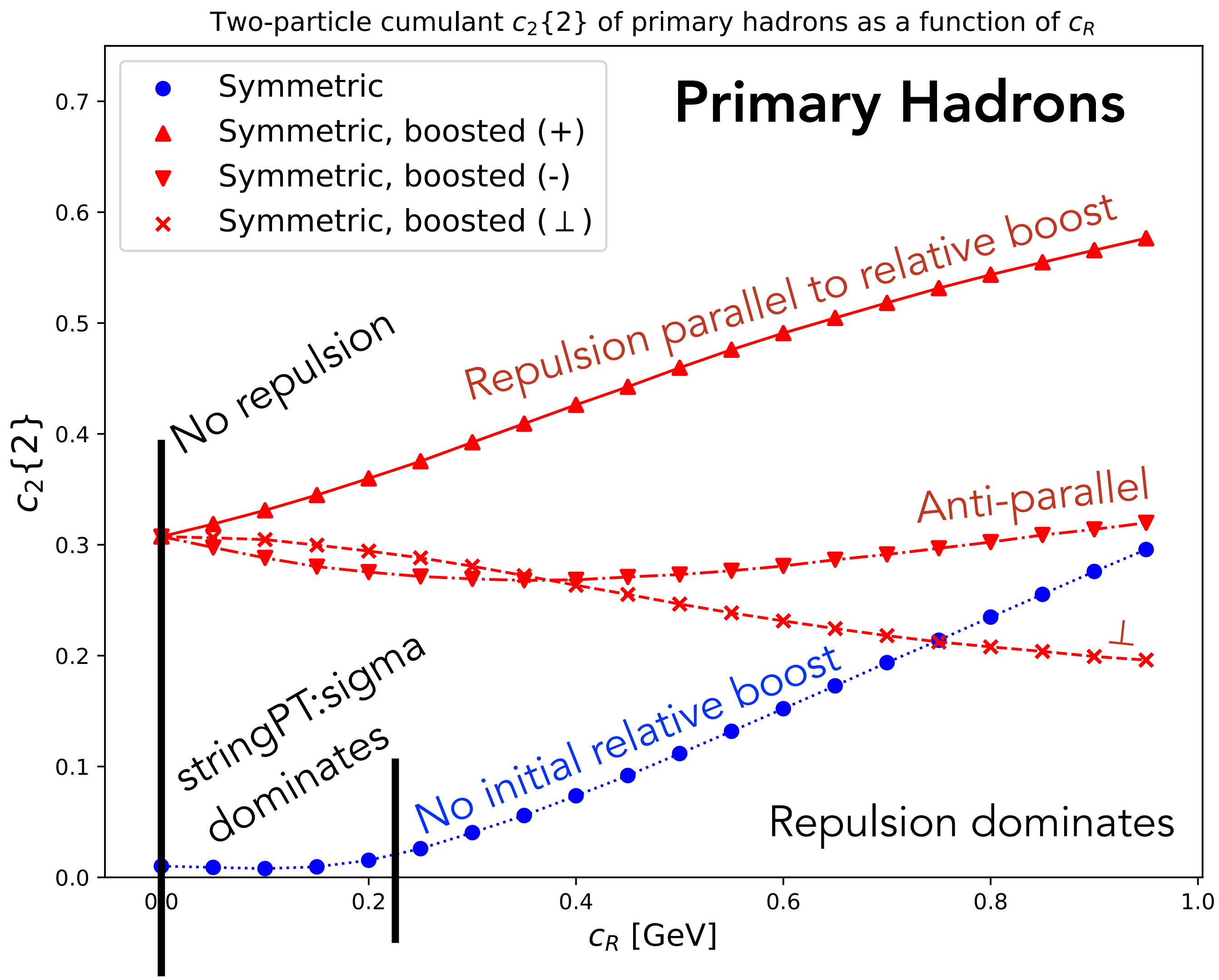


Subtlety: which direction? We assume same direction as relative boost, with random component added to have well-defined behaviour in boost→0 limit

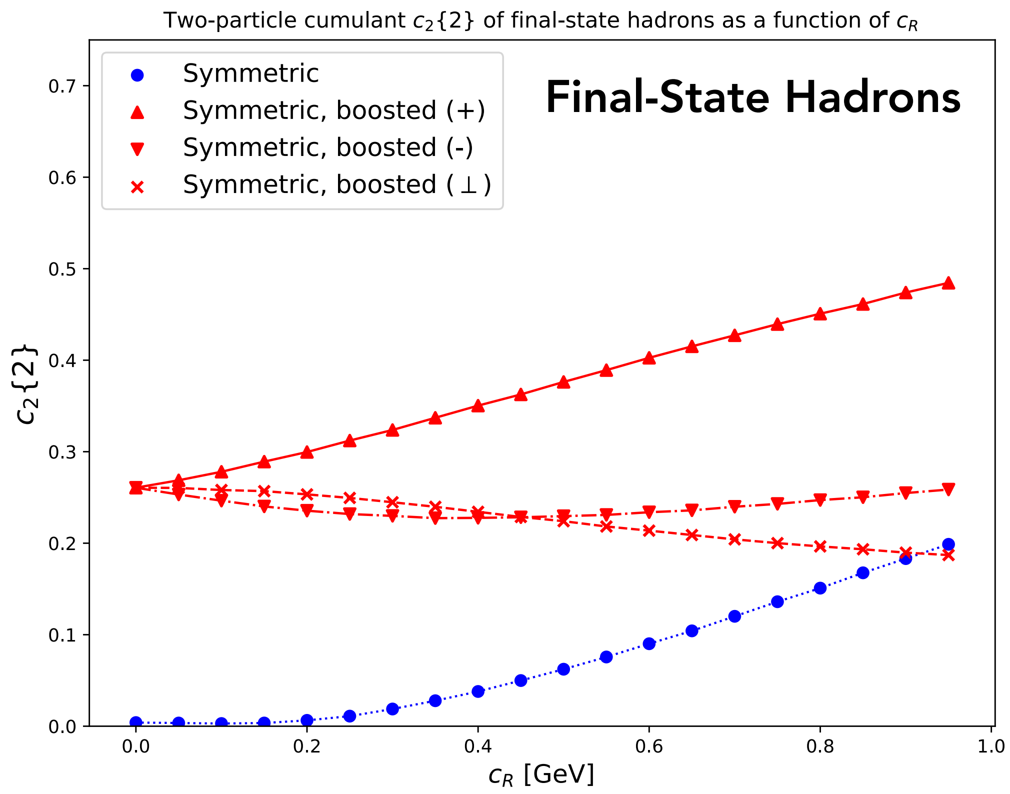
Two-Particle Cumulants

To connect with collective-flow / HI observables, we considered the two-particle cumulant

$$c_2\{2\} = \left\langle \langle e^{2i(\phi_i - \phi_j)} \rangle \right\rangle = \left\langle \frac{2}{n(n-1)} \sum_{i<j}^{n} \cos\left(2(\phi_i - \phi_j)\right) \right\rangle$$



With hadron decays: smaller magnitude but same trends



Summary

Much theoretical activity to understand, model, and disentangle signs of collective effects in pp collisions

Interesting to take a step further back: re-examine the modelling of the fragmentation of a single string.

- Grey zone between shower, V_{Coulomb}, and asymptotic string descriptions.
- Expanding geometry ⟷ entanglement ⟷ effective thermal effects?
- E.g., a **τ-dependent effective string tension** can generate a <p_T> vs strangeness correlation. (Fluctuating string tension likewise?)
- **I have no good LEP measurements on <p_T> vs strangeness**? Only inclusive <$p_{T\mathrm{in}}$>, <$p_{T\mathrm{out}}$> and (limited) PID x spectra dominated by p_z.

First steps towards a simple framework for momentum-space modelling of string-string repulsion effects

- Basic framework: 2-step **"compression"** + **"fragmentation repulsion"**
- So far considered only rather simple / textbook sort of setups. Interested to discuss merits (or showstoppers) to motivate further work.

Shoving Model Parameters

arXiv:1912.09639

Parameter	Value
Ropewalk:rCutOff	10.0
Ropewalk:limitMom	on
Ropewalk:pTcut	2.0
Ropewalk:r0	0.41
Ropewalk:m0	0.2
Ropewalk:gAmplitude	10.0
Ropewalk:gExponent	1.0
Ropewalk:deltat	0.1
Ropewalk:tShove	1.0
Ropewalk:deltay	0.1
Ropewalk:tInit	1.5

Table 1: Input parameters used in Fig. 3 for the shoving model.

(Note on fluctuating string tension)

Following a suggestion by Bialas (hep-ph/9909417), a recent study (Pirner, Kopeliovich, Reygers, arXiv:1810.0473) allowed for a **fluctuating κ**.

Flux tube size $r^2 \propto 1/\kappa$. Allow Gaussian fluctuations with $\kappa^2 = \lambda$ and

$$P(\lambda)\,\mathrm{d}\lambda = \sqrt{\frac{2}{\pi\mu}}e^{-\frac{\lambda^2}{2\mu}}\,\mathrm{d}\lambda. \quad \text{with} \quad \langle\kappa\rangle \equiv \langle\lambda^2\rangle = \mu = \int_0^\infty \lambda^2 P(\lambda)\,\mathrm{d}\lambda.$$

Extremely simplified pion spectrum: $$\frac{\mathrm{d}N}{\mathrm{d}^2p_\perp} = N_0 e^{-\sqrt{\frac{2\pi(m_\mathrm{q}^2+p_\perp^2/2)}{\langle\kappa\rangle}}}$$

They fit <κ> from dN/dp_T in [0.5, 1.4] GeV in 4 multiplicity classes (using a Tsallis function to extrapolate for the total Nch to pT=0)

$(\mathrm{d}N_\mathrm{ch}/\mathrm{d}\eta)_{\eta=0}$	$\langle\kappa\rangle$ in GeV^2	$s\bar{s}/(u\bar{u}+d\bar{d})$
7.92	0.21	0.237
11.87	0.22	0.243
18.8	0.25	0.258
31.7	0.29	0.275

Crude techniques but the idea of extracting an effective average tension from $\langle p_T\rangle(N_{ch})$ and relating that to strangeness enhancement may have merit.