Monte Carlos and New Physics

Peter Skands (Monash University)

Pre-SUSY - June 2016

Lecture Notes: P. Skands, arXiv:1207.2389

The Phenomenology Pipeline

Calculations

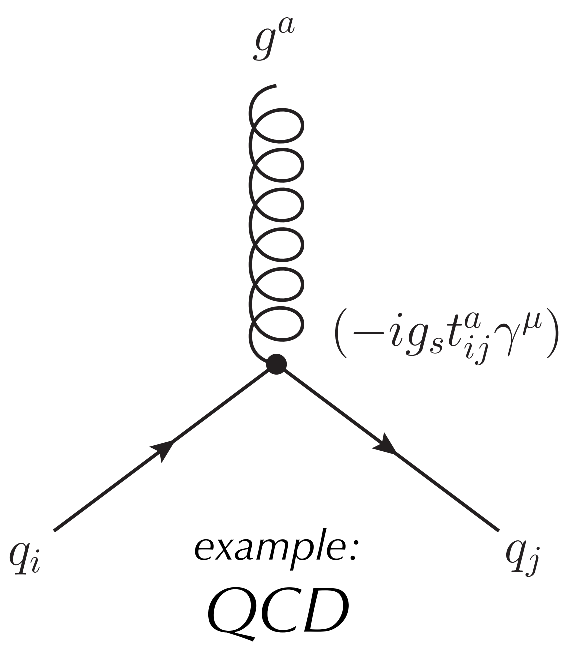
Model

Observables

THEORY → **PHENOMENOLOGY** → **EXPERIMENT**

Planning
Design
R&D
Hardware
Triggers
…

Analyses



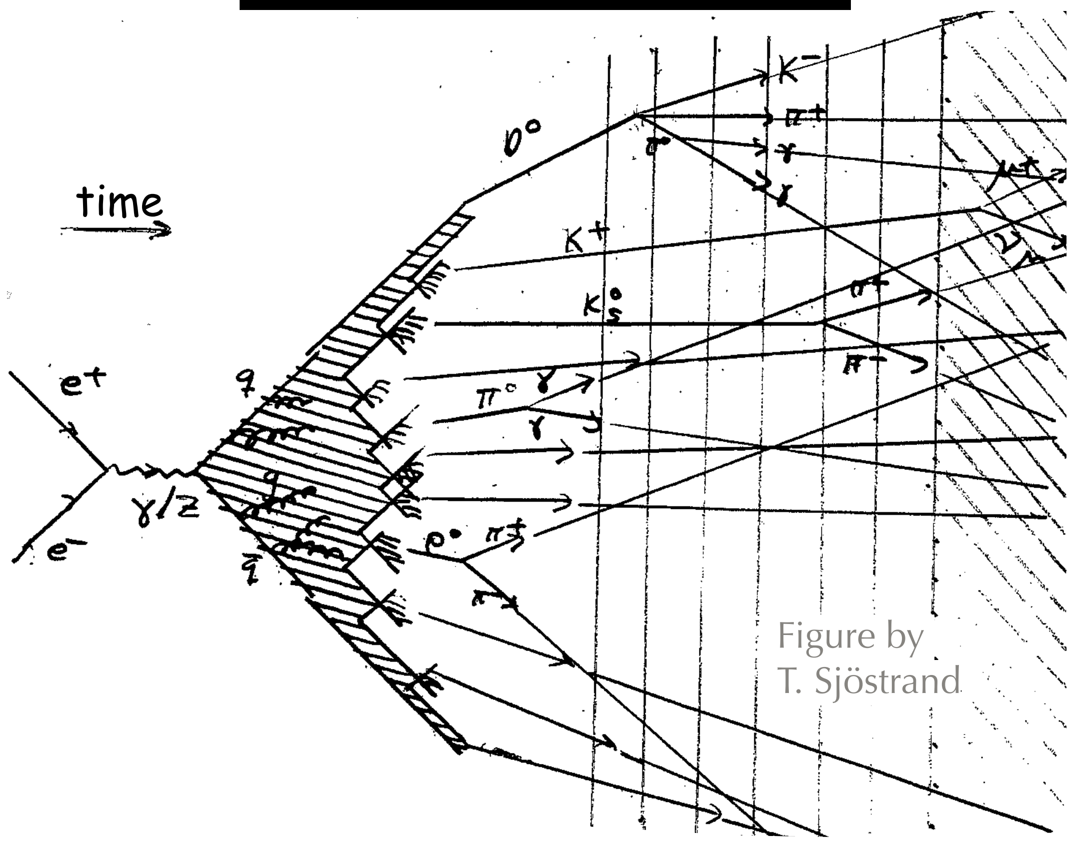
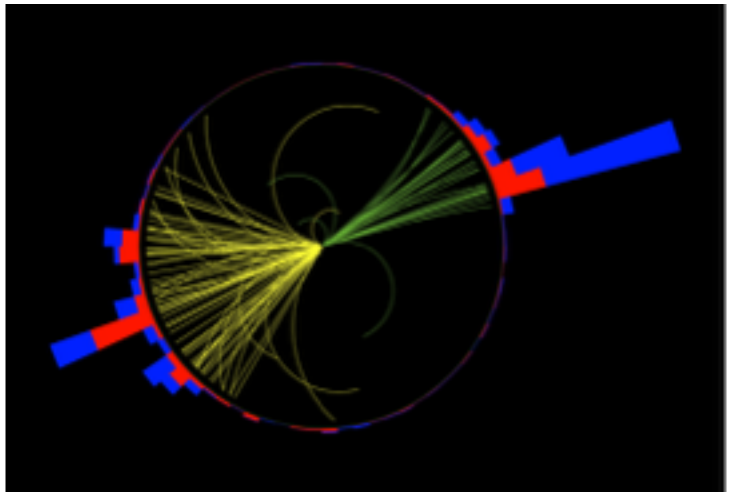


Figure by T. Sjöstrand



"Jets"

Exclusions
Hints
Evidence
Discoveries
Surprises

← **INTERPRETATION** ←

Corrections

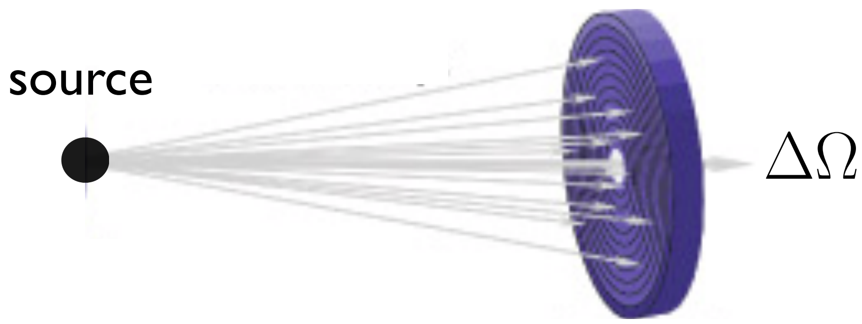
Statistical Tests

Measurements

Validate/Falsify Models
Constrain Free Parameters

Making Predictions

Scattering Experiments:



LHC detector
Cosmic-Ray detector
Neutrino detector
X-ray telescope
…

→ Integrate differential **cross sections** over specific **phase-space** regions

Predicted number of counts = integral over solid angle

$$N_{\mathrm{count}}(\Delta\Omega) \propto \int_{\Delta\Omega} \mathrm{d}\Omega \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$$

$$\mathrm{d}\Omega = \mathrm{d}\cos\theta\,\mathrm{d}\phi$$

In nature, σ is all-orders S-matrix element, integrated over 3 dimensions per particle (with resonances, singularities, loops, non-perturbative dynamics, …)

In particle physics:
Integrate over all quantum histories (+ interferences)

→ Monte Carlo

What is Monte Carlo?

Any technique that makes use of random sampling

Recap Convergence:

Calculus: {A} converges to B
if n exists for which $|A_{i>n} - B| < \varepsilon$, for any $\varepsilon > 0$

Monte Carlo: {A} converges to B
if n exists for which
the probability for $|A_{i>n} - B| < \varepsilon$,
is > P, for any P[0<P<1] for any $\varepsilon > 0$

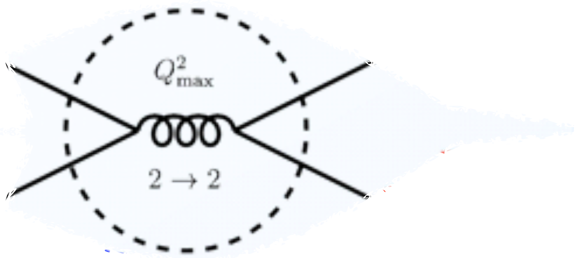
MC: prescribed for cases of **complicated / coupled integrands** in **high dimensions**

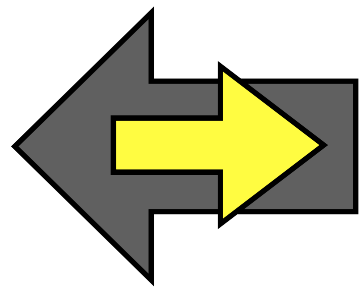
Numerical Uncertainty (after n function evaluations)	n_{eval} / bin	Conv. Rate (in 1D)	Conv. Rate (in D dim)
Trapezoidal Rule (2-point)	2^D	$1/n^2$	$1/n^{2/D}$
Simpson's Rule (3-point)	3^D	$1/n^4$	$1/n^{4/D}$
Monte Carlo	1	$1/n^{1/2}$	$1/n^{1/2}$

+ optimisations (stratification, adaptation), coupled/iterative solutions (Markov-Chain Monte Carlo)

The Role of MC Generators

THEORY





EXPERIMENT

Calculate Everything ≈ solve QFT* → requires compromise!

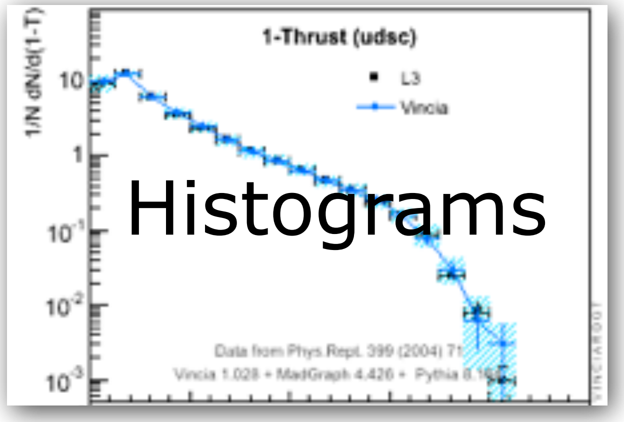
Event Generators : start from elementary scattering process

Include the '*most significant*' corrections: higher-order matrix elements, bremsstrahlung, resonance decays, hadronization, underlying event, beam remnants, …

A detailed picture that connects directly with the observable world

↔

of hadrons, photons, and leptons



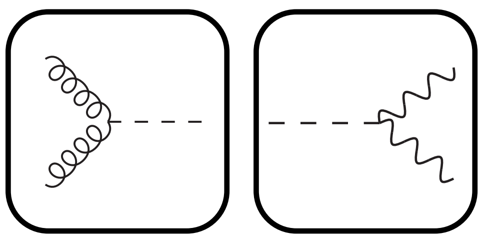
*QFT = Quantum Field Theory

Organising the Calculation

Divide and Conquer → Split the problem into many (nested) pieces

+ Quantum mechanics → Probabilities → Random Numbers

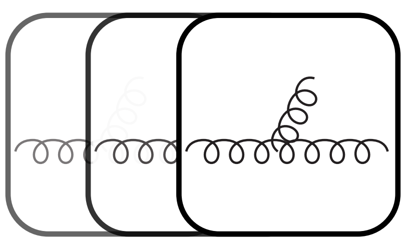
$$\mathcal{P}_{\mathrm{event}} = \mathcal{P}_{\mathrm{hard}} \otimes \mathcal{P}_{\mathrm{dec}} \otimes \mathcal{P}_{\mathrm{ISR}} \otimes \mathcal{P}_{\mathrm{FSR}} \otimes \mathcal{P}_{\mathrm{MPI}} \otimes \mathcal{P}_{\mathrm{Had}} \otimes \ldots$$



Hard Process & Decays:

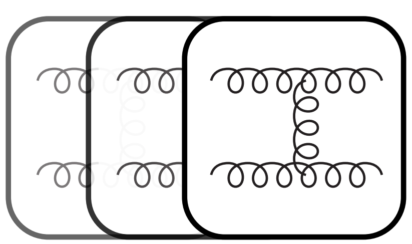
Use process-specific (N)LO matrix elements (e.g., $gg \to H^0 \to \gamma\gamma$)

→ Sets "hard" resolution scale for process: Q_{MAX}



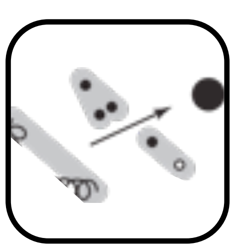
ISR & FSR (Initial- & Final-State Radiation):

Bremsstrahlung, driven by differential (DGLAP) evolution equations, dP/dQ^2, as function of resolution scale; rom Q_{MAX} to Q_{HAD} ~ 1 GeV



MPI (Multi-Parton Interactions)

Protons contain lots of partons → can have additional (soft) parton-parton interactions → Additional (soft) "Underlying-Event" activity



Hadronization

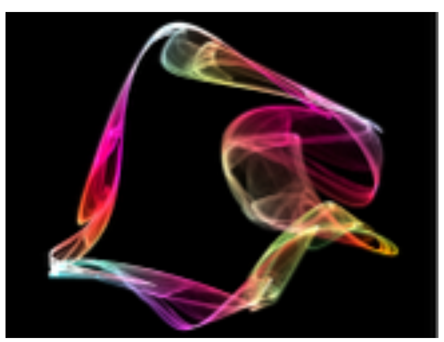
Non-perturbative modeling of partons → hadrons transition

Challenges Beyond Fixed Order

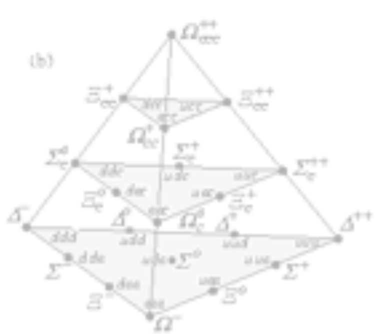
QCD is more than just a perturbative expansion in α_s

The relation between α_s, Feynman diagrams, and the full QCD dynamics involves spectacular "emergent" phenomena:

Jets (perturbative QCD, initial- and final-state radiation) ⟷ amplitude structures in quantum field theory ⟷ factorisation & unitarity. Precision jet (structure) studies.

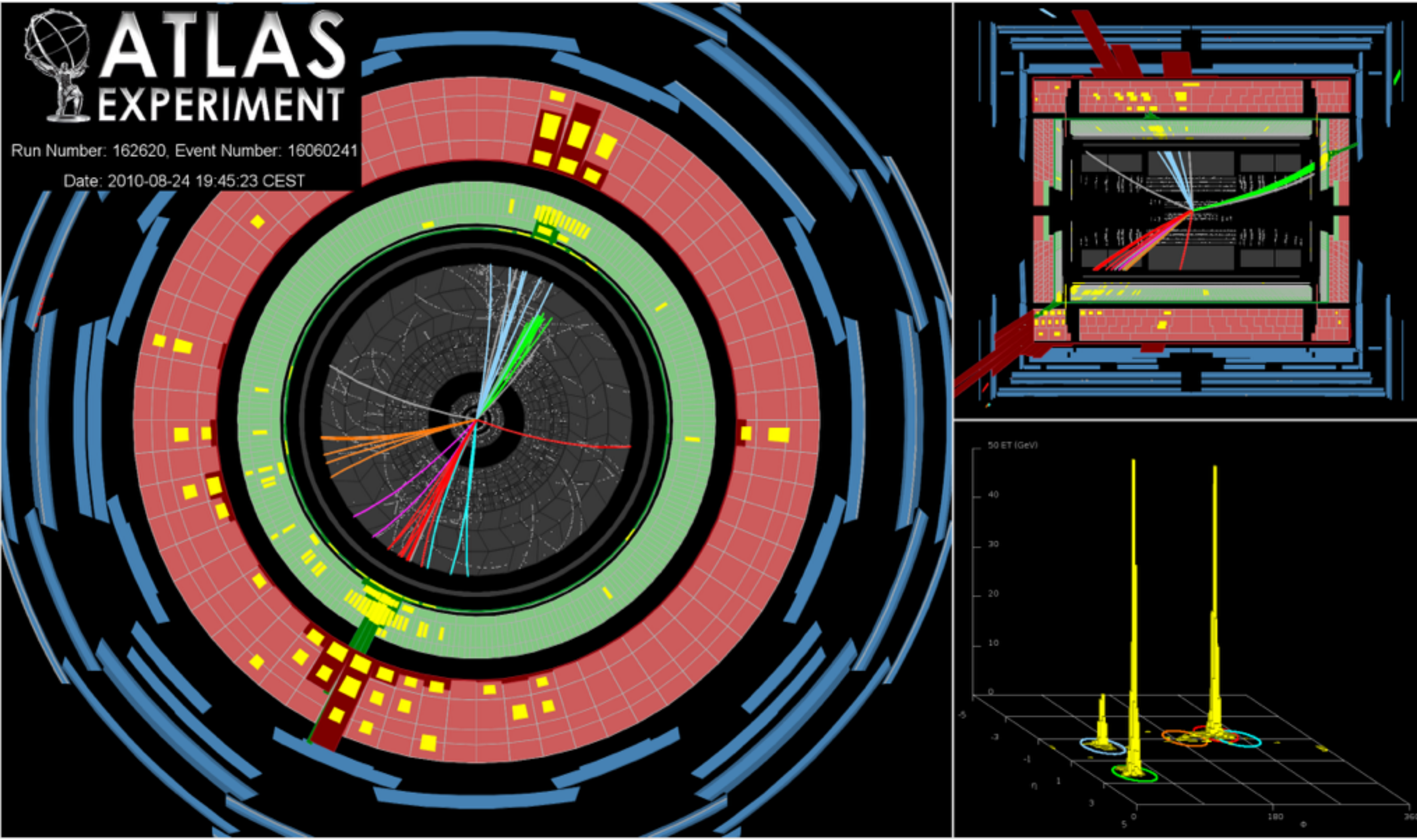


Strings (strong gluon fields) ⟷ quantum-classical correspondence. String physics. String breaks. Dynamics of hadronization phase transition.



Hadrons ⟷ Spectroscopy (incl excited and exotic states), lattice QCD, (rare) decays, mixing, light nuclei. Hadron beams → multiparton interactions, diffraction, …

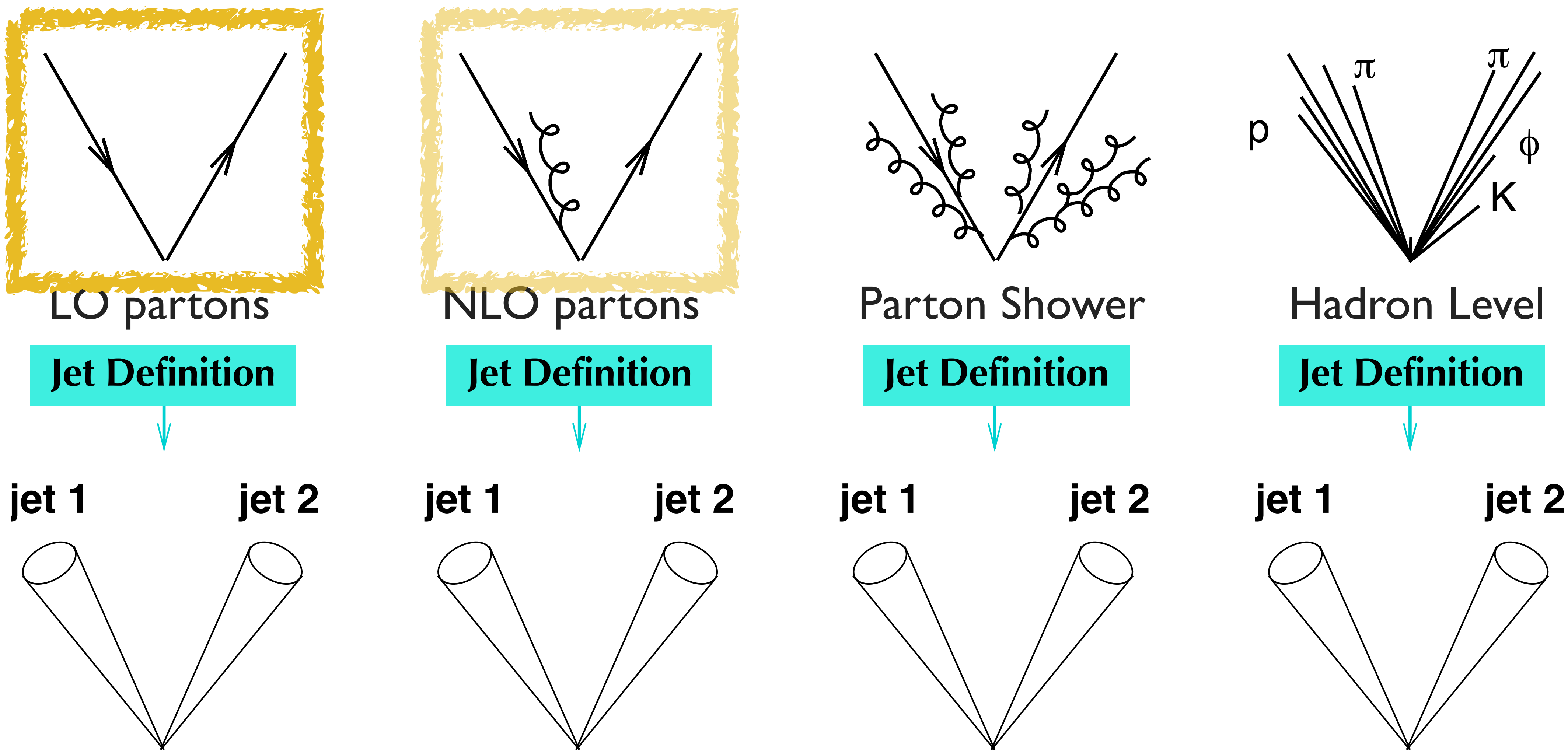
"The emergent is unlike its components insofar as … it cannot be reduced to their sum or their difference." *G. Lewes (1875)*



- 1st jet: p_T = 520 GeV, η = -1.4, ϕ = -2.0
- 2nd jet: p_T = 460 GeV, η = 2.2, ϕ = 1.0
- 3rd jet: p_T = 130 GeV, η = -0.3, ϕ = 1.2
- 4th jet: p_T = 50 GeV, η = -1.0, ϕ = -2.9

What are Jets?

Think of jets as **projections** that provide a universal view of events



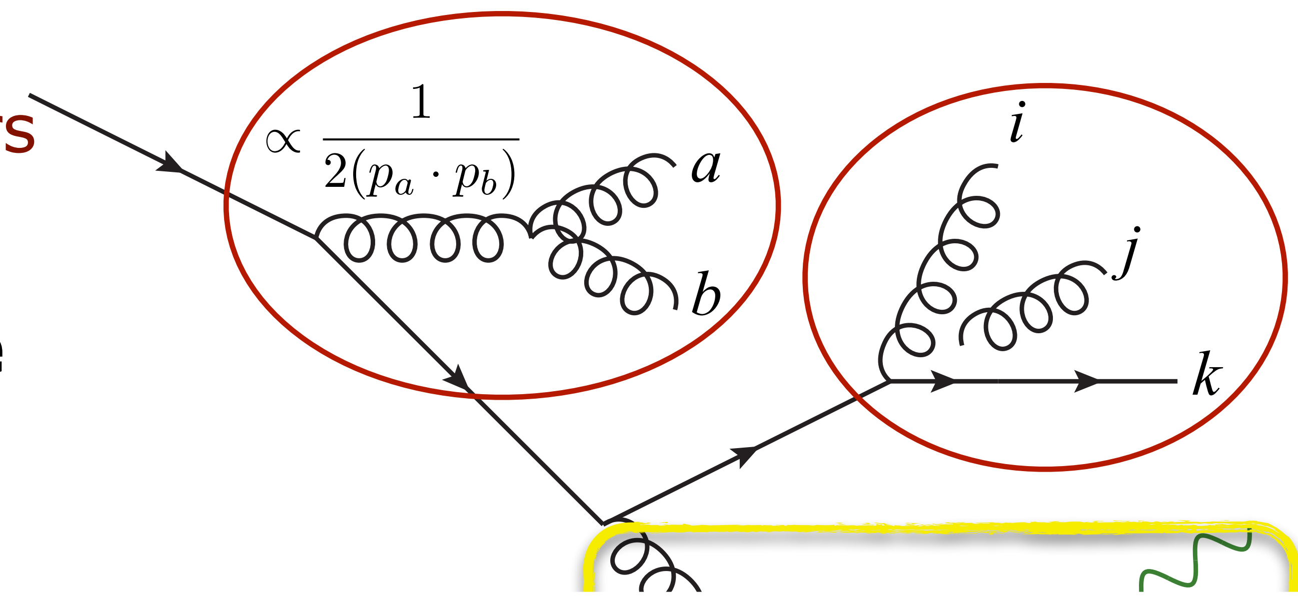
Illustrations by G. Salam

I'm not going to cover the many different types of jet clustering algorithms (k_T, anti-k_T, C/A, cones, ...) – see e.g., lectures & notes by G. Salam.

➤ Focus instead on the physical origin and MC modeling of jets

The Structure of Jets

Most bremsstrahlung is driven by divergent propagators → simple structure



Gauge amplitudes factorize in singular limits (→ universal *"conformal"* or *"fractal"* structure)

Partons ab → "collinear":

P(z) = "DGLAP" splitting kernels, with z = energy fraction = $E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \overset{a||b}{\to} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

Gluon j → "soft":

Coherence → Parton j really emitted by (i,k) "colour antenna"

$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \overset{j_g\to 0}{\to} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

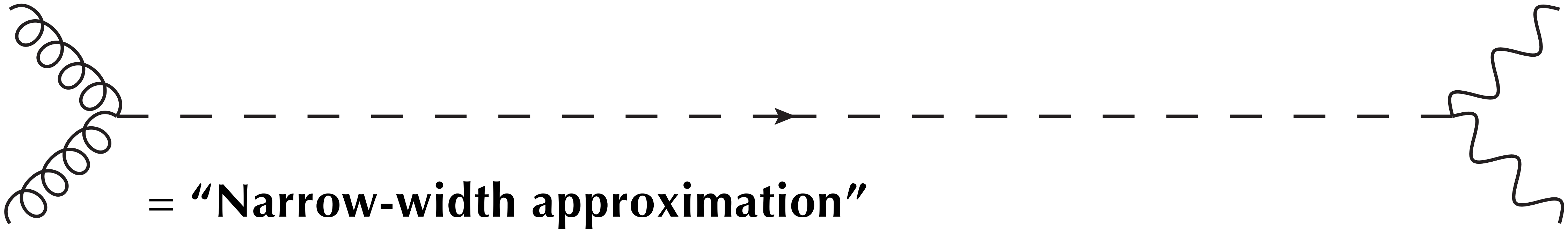
\+ scaling ***violation***: $g_s^2 \to 4\pi\alpha_s(Q^2)$

Can apply this many times → nested factorizations

See e.g.: PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389

Other Examples of Factorisation

Factorization of Production and Decay:



= **“Narrow-width approximation”**

Valid up to corrections Γ/m → breaks down for large Γ

(More subtle when particle is coloured/charged/polarised)

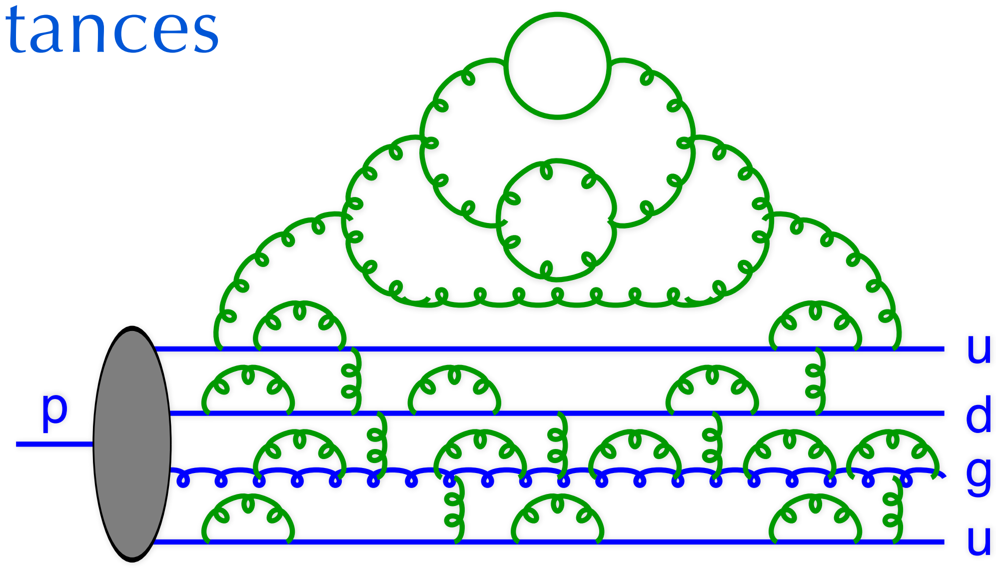
Factorization of Long and Short Distances

Scale of fluctuations inside a hadron

~ Λ_{QCD} ~ 200 MeV

Scale of hard process » Λ_{QCD}

→ proton looks “frozen”

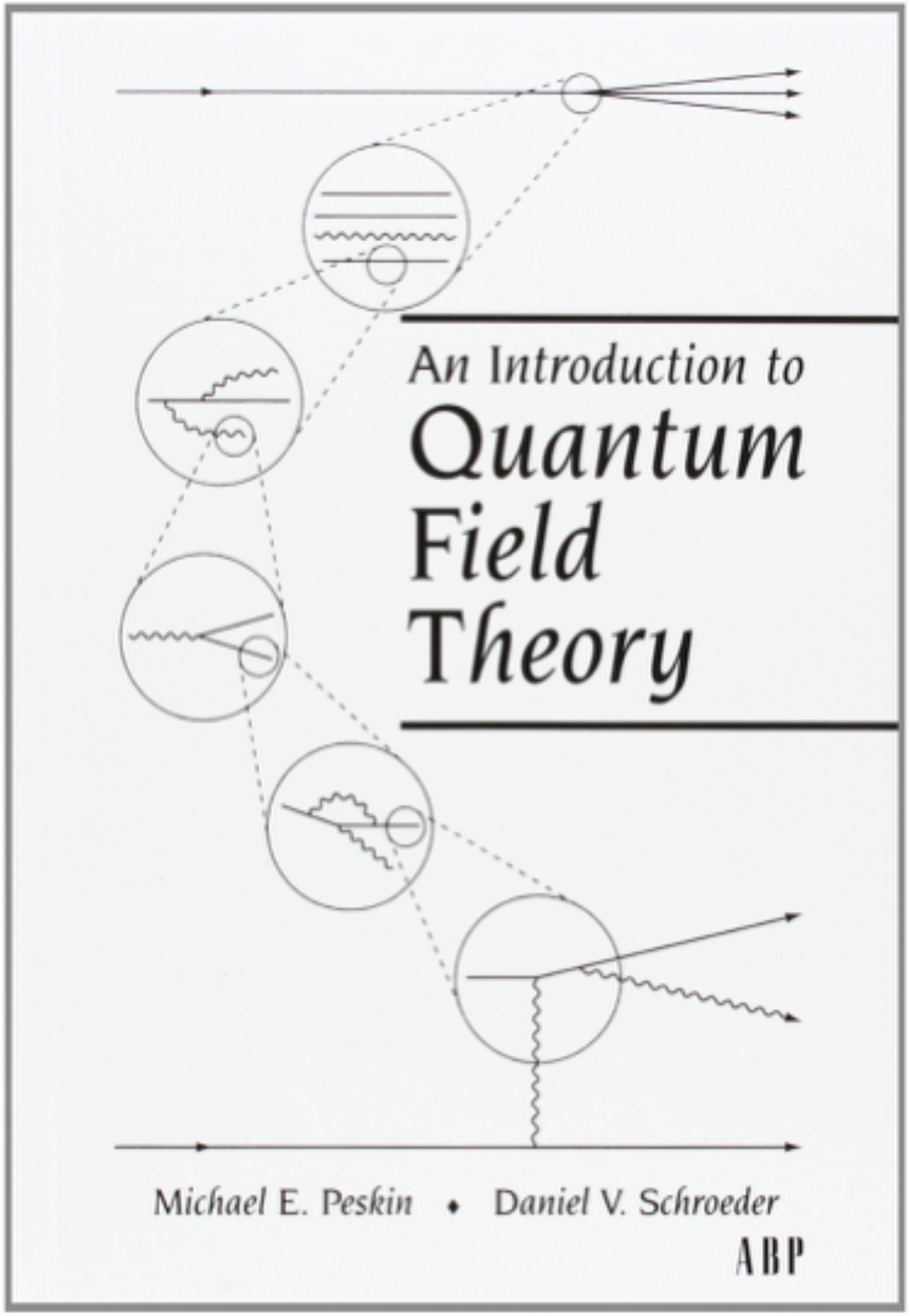


Instantaneous snapshot of long-wavelength structure, independent of nature of hard process → **PDFs**

The Structure of Quantum Fields

What we actually see when we look at a “jet”, or inside a proton

- An ever-repeating self-similar pattern of quantum fluctuations
- At increasingly smaller energies or distances : ***scaling*** (modulo $\alpha(Q)$ scaling violation)
- To our best knowledge, this is what a fundamental (‘elementary’) particle really looks like



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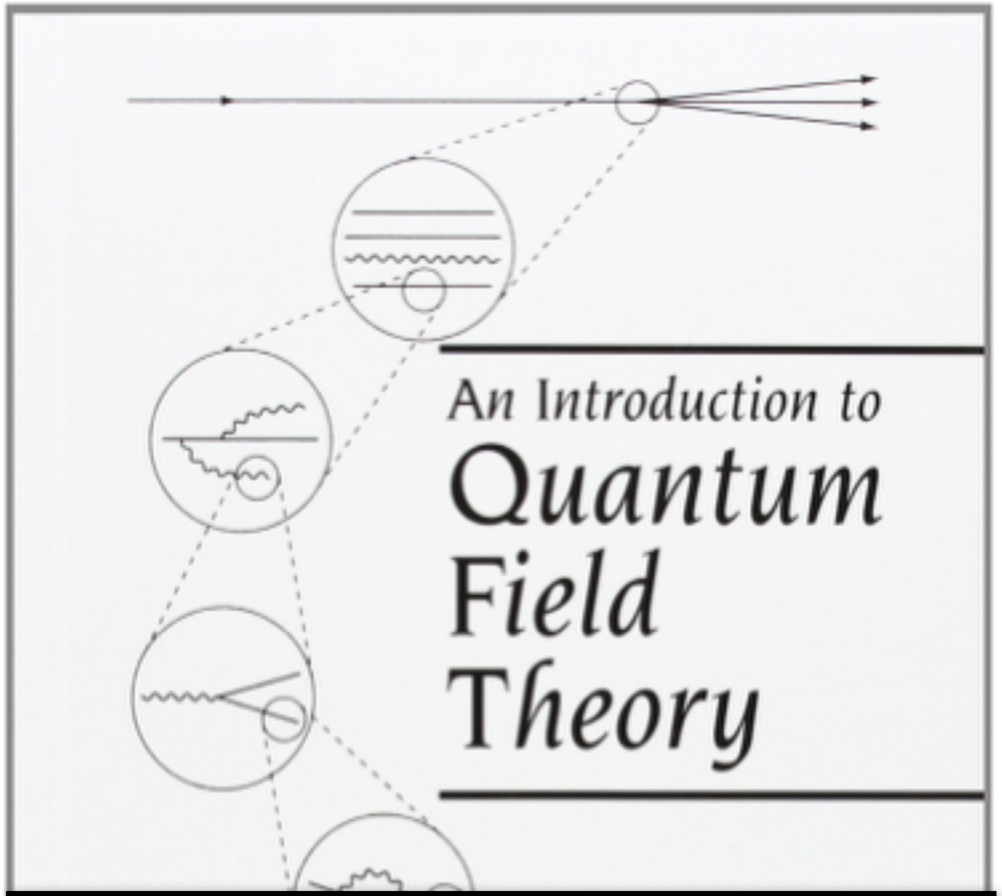
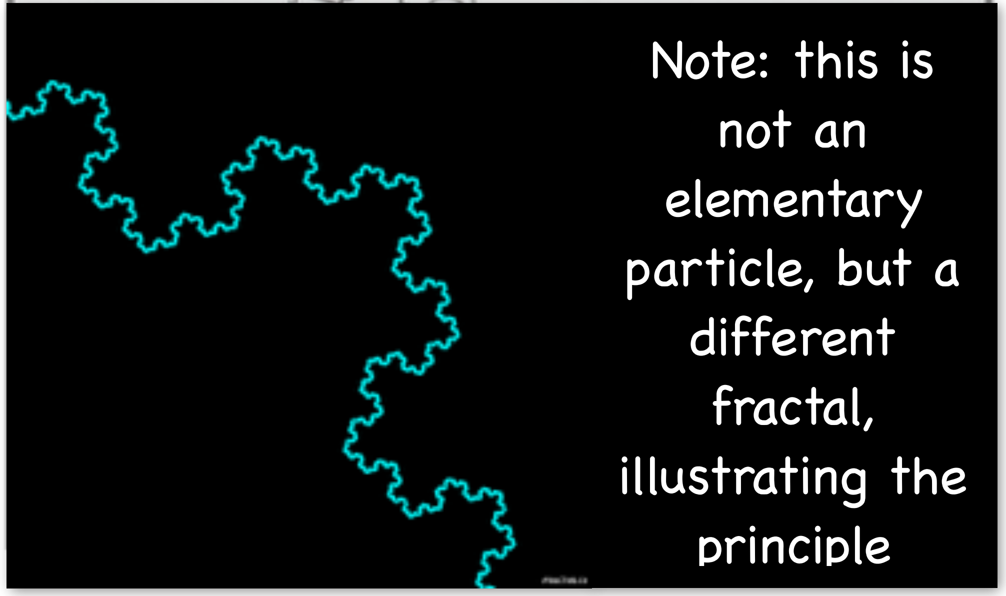
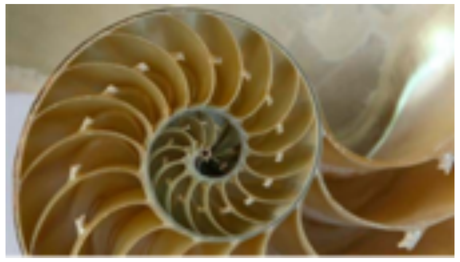
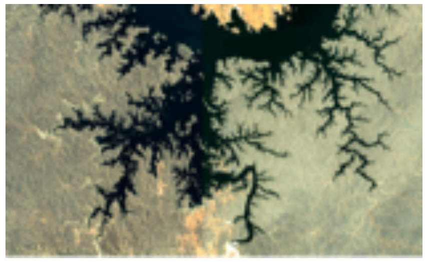




Nature makes copious use of such structures - **Fractals**

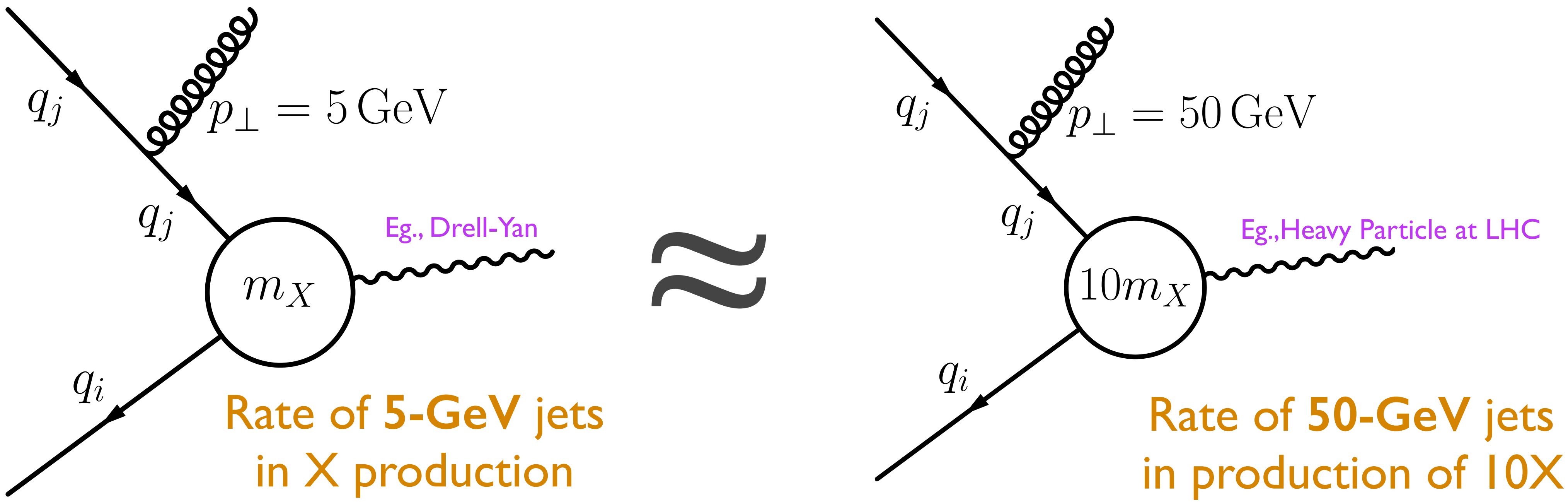




Fractal QFT

Bremsstrahlung

Rate of bremsstrahlung jets mainly depends on the RATIO of the jet p_T to the "hard scale"



Harder Processes are Accompanied by Harder Jets

Conformal QCD in Action

Naively, QCD radiation suppressed by $\alpha_s \approx 0.1$

→ Truncate at fixed order = LO, NLO, …

But beware the jet-within-a-jet-within-a-jet …

Example: 100 GeV can be "soft" at the LHC

SUSY pair production at LHC_{14}, with $M_{SUSY} \approx 600$ GeV

LHC - sps1a - m~600 GeV — Plehn, Rainwater, PS PLB645(2007)217

FIXED ORDER pQCD	σ_{tot}[pb]	$\tilde{g}\tilde{g}$	$\tilde{u}_L\tilde{g}$	$\tilde{u}_L\tilde{u}_L^*$	$\tilde{u}_L\tilde{u}_L$	TT
$p_{T,j} >$ 100 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
inclusive X + 1 "jet" →	σ_{1j}	2.89	2.74	0.136	0.145	0.73
inclusive X + 2 "jets" →	σ_{2j}	1.09	0.85	0.049	0.039	0.26
$p_{T,j} >$ 50 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
	σ_{1j}	5.90	5.37	0.283	0.285	1.50
	σ_{2j}	4.17	3.18	0.179	0.117	1.21

(Computed with SUSY-MadGraph)

σ for X + jets much larger than naive estimate

σ for 50 GeV jets ≈ larger than total cross section → not under (fixed-order) control

All the scales are high, Q >> 1 GeV, so perturbation theory **should** be OK

Apropos Factorisation

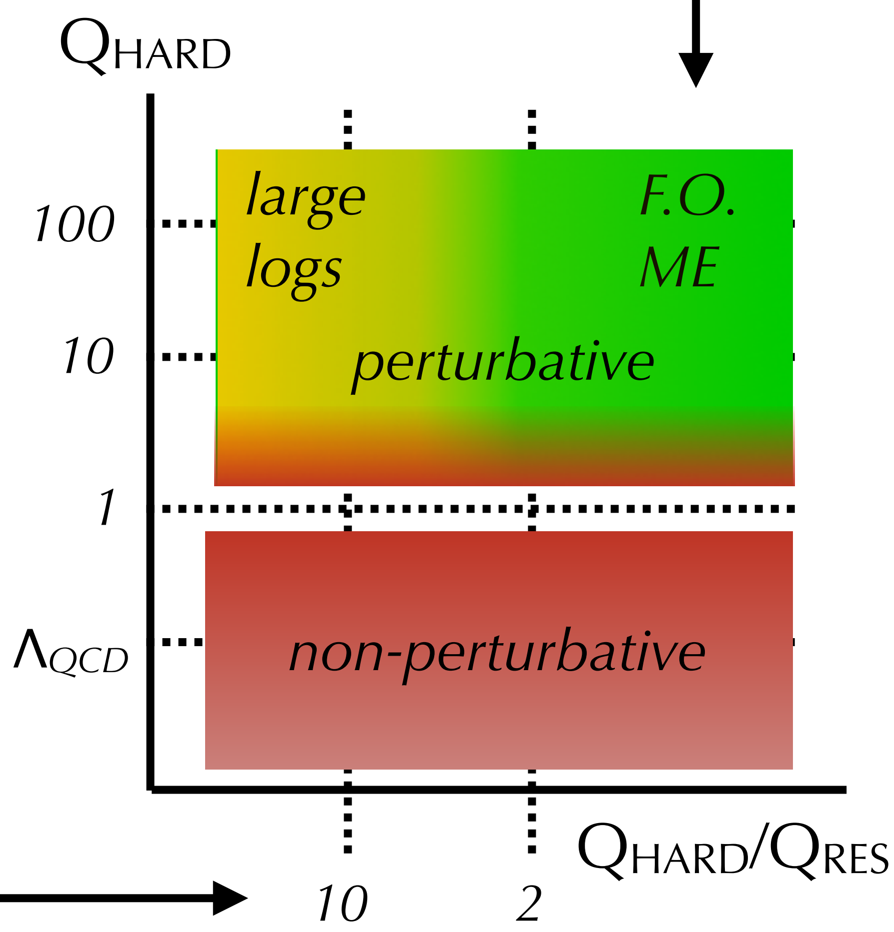
Why are Fixed-Order QCD matrix elements not enough?

F.O. QCD requires **Large scales** (α_s small enough to be perturbative - not too bad, since we anyway *often* want to consider large-scale processes)

F.O. QCD also requires **No hierarchies**

Conformal jets-within-jets structure: integrated over phase space, bremsstrahlung poles → logarithms

→ large if upper and lower integration limits are hierarchically different



Resummation to the Rescue

PDFs: connect incoming hadrons with the high-scale process
PDF evolution: sums the (leading, next-to-leading, …) logarithms to all orders, between the high scale and the initial-state proton scale ↔ **initial-state radiation**

Fragmentation Functions: connect high-scale process with final-state hadrons
FF evolution: sums the logarithms to all orders, between the high scale and the final-state hadronic (or more general observable) scale ↔ **final-state radiation**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = \sum_{a,b} \sum_{f} \int_{\hat{X}_f} f_a(x_a, Q_i^2) f_b(x_b, Q_i^2) \frac{\mathrm{d}\hat{\sigma}_{ab\to f}(x_a, x_b, f, Q_i^2, Q_f^2)}{\mathrm{d}\hat{X}_f} D(\hat{X}_f \to X, Q_i^2, Q_f^2)$$

PDFs: needed to compute inclusive cross sections

FFs: needed to compute (semi-)exclusive cross sections

Resummed pQCD: All resolved scales >> Λ_{QCD} AND X Infrared Safe

*)pQCD = perturbative QCD

But can now include hierarchies

Interpretation

Naively, QCD radiation suppressed by $\alpha_s \approx 0.1$

→ Truncate at fixed order = LO, NLO, …

But beware the jet-within-a-jet-within-a-jet …

Example: 100 GeV can be "soft" at the LHC

SUSY pair production at 14 TeV, with $M_{SUSY} \approx 600$ GeV

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σ for 50 GeV jets ≈ larger than total cross section → not under (fixed-order) control

Interpretation : Most of these events will have **more than one 50-GeV jet !**

Parton Showers

So it's not like you can put a cut at X (e.g., 50, or even 100) GeV and say: "ok, now fixed-order matrix elements will be OK"

Harder Processes are Accompanied by Harder Jets

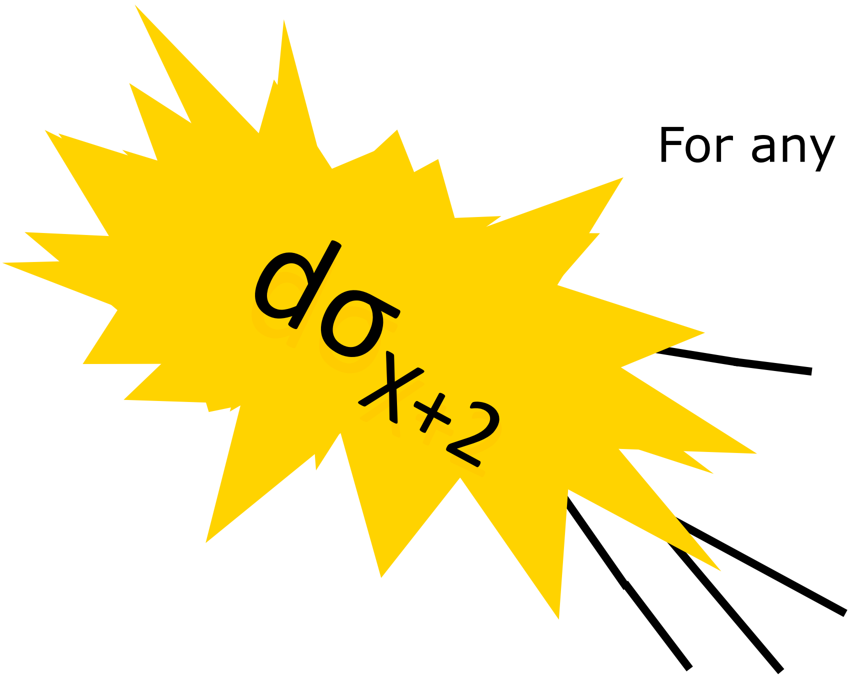
The hard scale Q_{HARD} of your process will start off the fractal

- Sooner or later you **will** resolve bremsstrahlung structure, for Q_{JET}/Q_{HARD} (or more generally $Q_{RESOLVED}/Q_{HARD}$) $<< 1$
- Will generate corrections to your kinematics,
- Can be important combinatorial background if you are looking for decay jets of similar p_T scales (often, $\Delta M << M$)

This is what parton showers are made for

- (as well as resolving the fractal structure inside each of the jets)

Bremsstrahlung



For any basic process $d\sigma_X$ = ✓ (calculated process by process)

$$d\sigma_{X+1} \sim N_C 2 g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad ✓$$

$$d\sigma_{X+2} \sim N_C 2 g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad ✓$$

$$d\sigma_{X+3} \sim N_C 2 g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad ...$$

Factorization in Soft and Collinear Limits

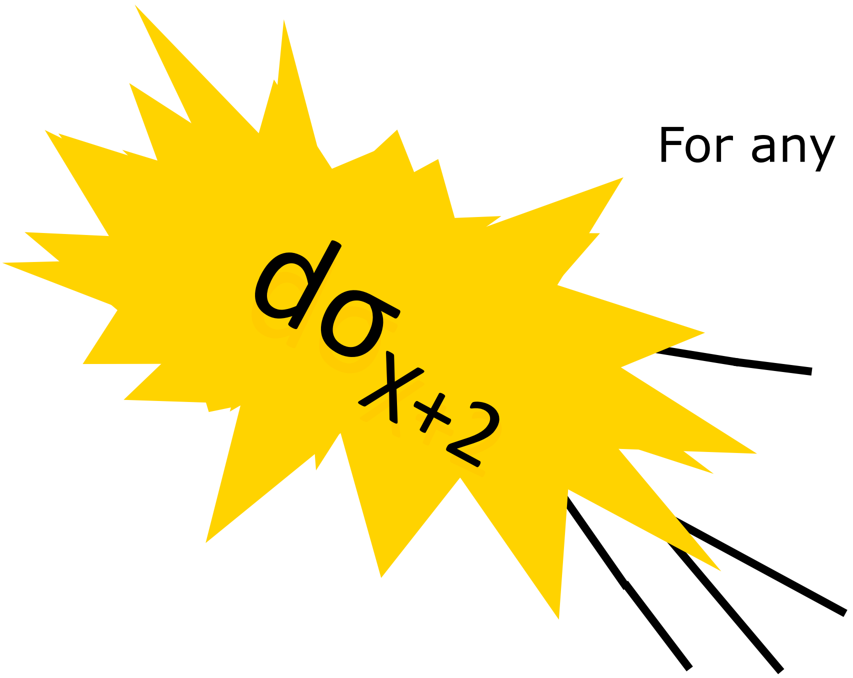
$P(z)$: "DGLAP Splitting Functions"

$$|M(\ldots,p_i,p_j\ldots)|^2 \stackrel{i||j}{\to} g_s^2 \mathcal{C} \frac{P(z)}{s_{ij}} |M(\ldots,p_i+p_j,\ldots)|^2$$

$$|M(\ldots,p_i,p_j,p_k\ldots)|^2 \stackrel{j_g\to 0}{\to} g_s^2 \mathcal{C} \frac{2s_{ik}}{s_{ij}s_{jk}} |M(\ldots,p_i,p_k,\ldots)|^2$$

"Soft Eikonal" : generalizes to Dipole/Antenna Functions

Bremsstrahlung



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$$d\sigma_{X+3} \sim N_C 2g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \ldots$$

Singularities: mandated by gauge theory
Non-singular terms: process-dependent

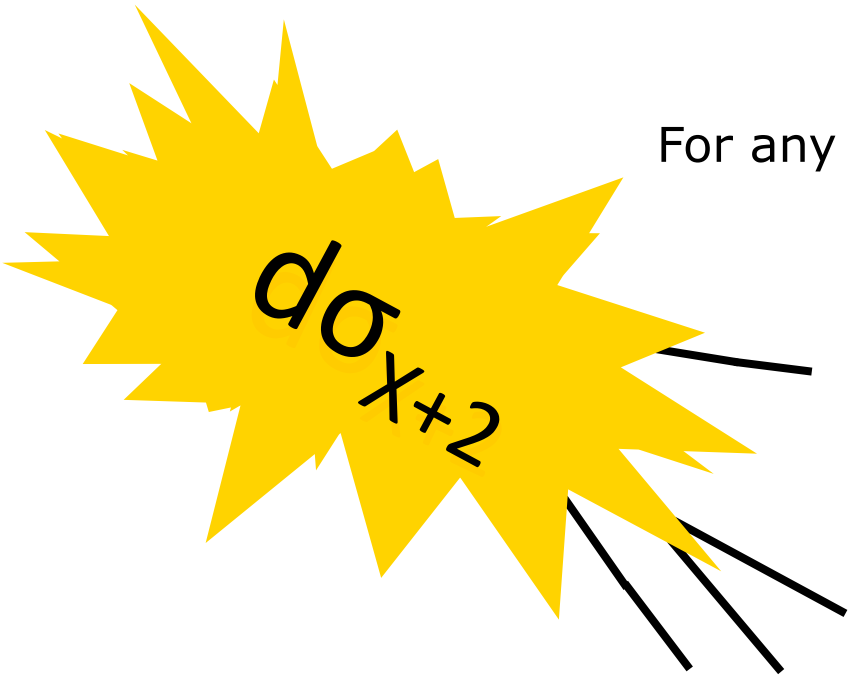
SOFT COLLINEAR

$$\frac{|\mathcal{M}(Z^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(Z^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right]$$

$$\frac{|\mathcal{M}(H^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(H^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right]$$

SOFT COLLINEAR+F

Bremsstrahlung



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$$d\sigma_{X+3} \sim N_C 2 g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad ...$$

Iterated factorization

Gives us a universal approximation to ∞-order tree-level cross sections.

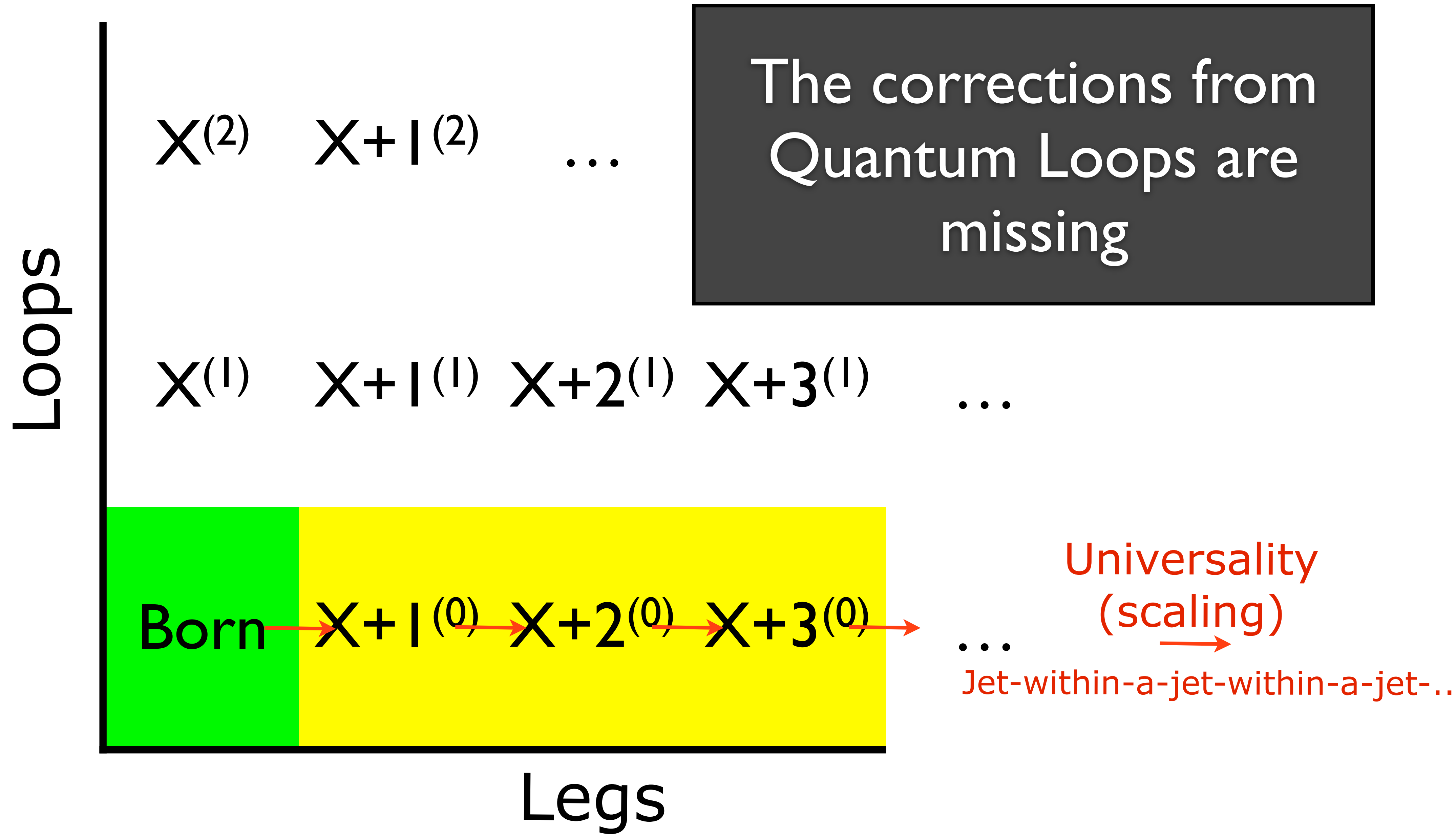
Exact in singular (strongly ordered) limit.

Finite terms (non-universal) → Uncertainties for non-singular (hard) radiation

But something is not right … Total σ would be infinite …

Loops and Legs

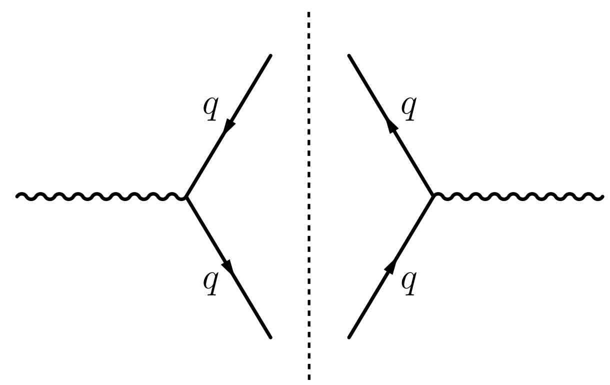
Coefficients of the Perturbative Series

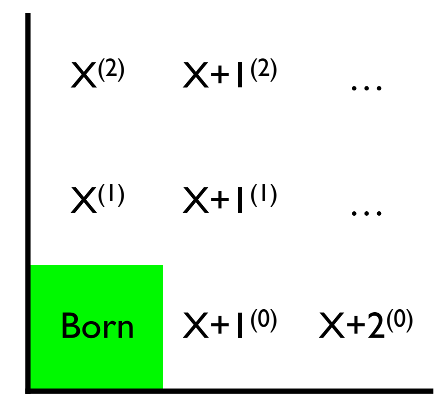


Cross sections at LO

Born @ LO

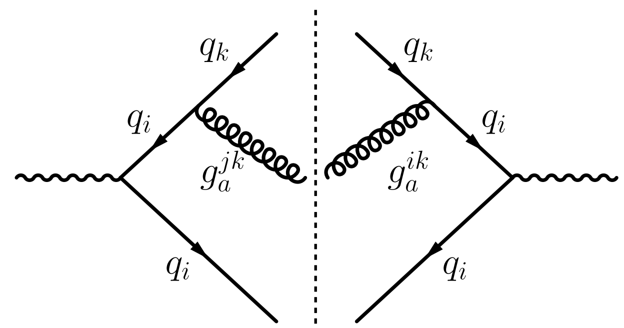
$$\sigma_{\text{Born}} = \int |M_X^{(0)}|^2$$

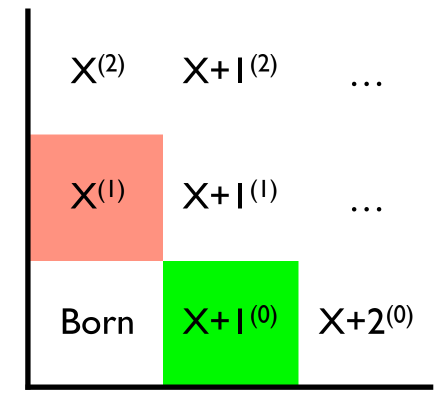




Born + n @ LO

$$\sigma^{\text{LO}}_{\text{X+1}}(R) = \int_R |M^{(0)}_{X+1}|^2$$





R = some "Infrared Safe" phase space region (Often a cut on $p_\perp > X$ GeV)

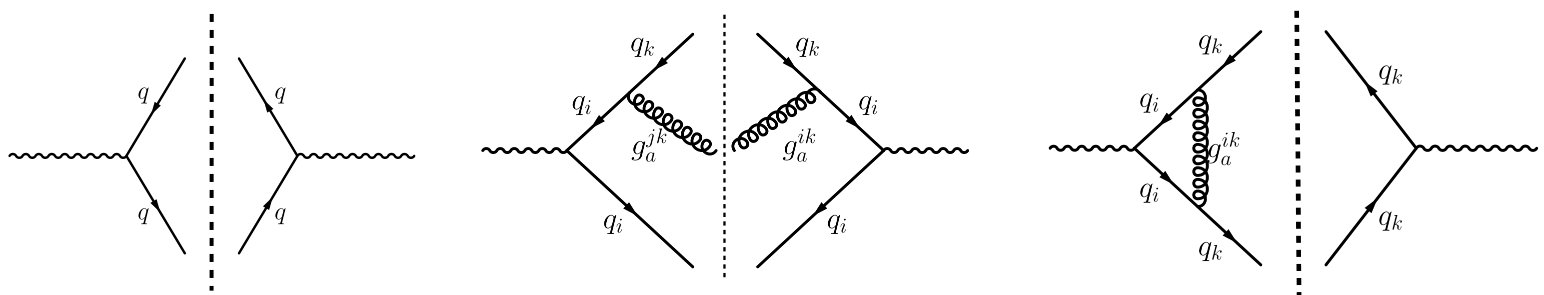
Careful not to take it too low!

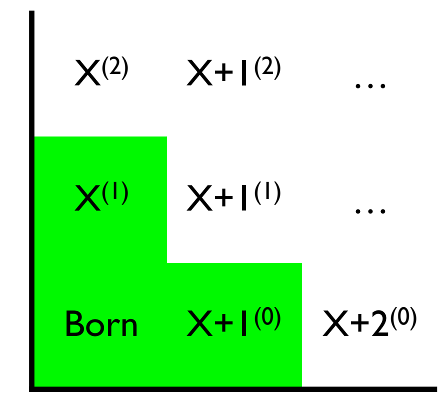
$$\frac{|M_{X+1}|^2}{|M_X|^2} \propto g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right]$$

Infrared divergent (when s_{ij} and/or $s_{jk} \to 0$): Integral → **Logarithms**

UNITARITY (at NLO)

NLO:





$$\sigma_{\mathrm{X}}^{\mathrm{NLO}} = \int |M_X^{(0)}|^2 + \int |M_{X+1}^{(0)}|^2 + \int 2\mathrm{Re}[M_X^{(1)} M_X^{(0)*}]$$

(note: this is *not* the 1-loop diagram squared)

KLN Theorem (Kinoshita-Lee-Nauenberg)

Sum over 'degenerate quantum states' :

Singularities cancel at complete order (only finite terms left over)

$$= \sigma_{\mathrm{Born}} + \mathrm{Finite}\left\{\int |M_{X+1}^{(0)}|^2\right\} + \mathrm{Finite}\left\{\int 2\mathrm{Re}[M_X^{(1)} M_X^{(0)*}]\right\}$$

$$\sigma_{\mathrm{NLO}}(e^+e^- \to q\bar{q}) = \sigma_{\mathrm{LO}}(e^+e^- \to q\bar{q}) \left(1 + \frac{\alpha_s(E_{\mathrm{CM}})}{\pi} + \mathcal{O}(\alpha_s^2)\right)$$

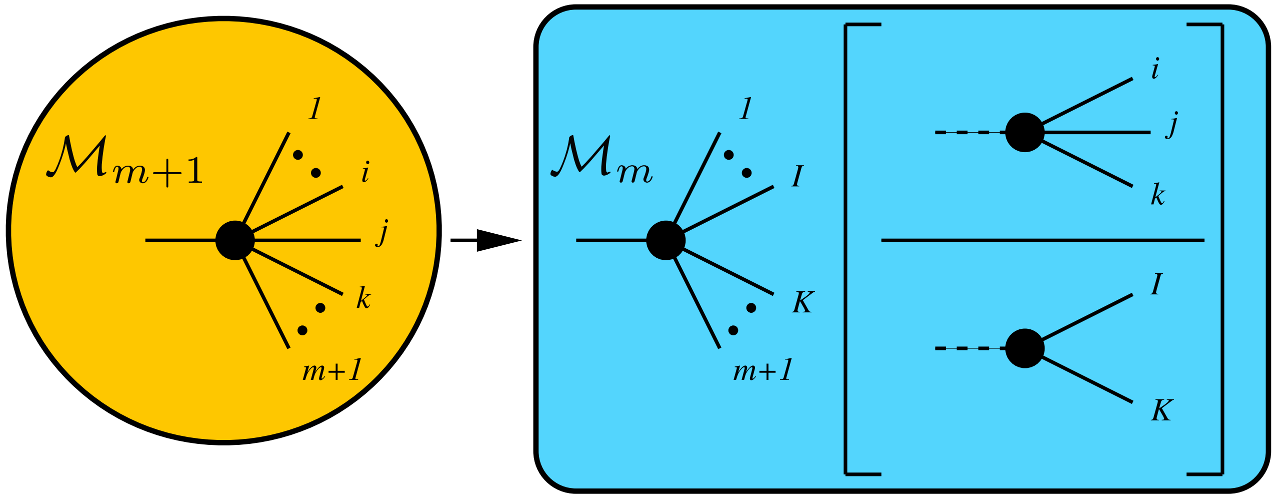
(The Subtraction Idea)

How do I get finite{Real} and finite{Virtual} ?

First step: classify IR singularities using universal functions

EXAMPLE: factorization of amplitudes in the **soft** limit

Soft Limit
($E_j \to 0$):



$$|\mathcal{M}_{n+1}(1,\cdots,i,j,k,\cdots,n+1)|^2 \xrightarrow{j_g \to 0} g_s^2 \mathcal{C}_{ijk}\, S_{ijk}\, |\mathcal{M}_n(1,\cdots,i,k,\cdots,n+1)|^2$$

Universal
"Soft Eikonal"

$$S_{ijk}(m_I,m_K) = \frac{2s_{ik}}{s_{ij}s_{jk}} - \frac{2m_I^2}{s_{ij}^2} - \frac{2m_K^2}{s_{jk}^2} \qquad s_{ij} \equiv 2p_i \cdot p_j$$

(The Subtraction Idea)

How do I get finite{Real} and finite{Virtual} ?

First step: classify IR singularities using universal functions

Add and subtract IR limits (SOFT and COLLINEAR)

$$\mathrm{d}\sigma_{NLO} = \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^R_{NLO} - \mathrm{d}\sigma^S_{NLO} \right) + \left[\int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^S_{NLO} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^V_{NLO} \right]$$

Finite by Universality

Finite by KLN

Dipoles (Catani-Seymour)

Global Antennae (Gehrmann, Gehrmann-de Ridder, Glover)

Sector Antennae (Kosower)

…

Choice of subtraction terms:

Singularities mandated by gauge theory

Non-singular terms: up to you (added and subtracted, so vanish)

$$\frac{|\mathcal{M}(Z^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(Z^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right]$$

$$\frac{|\mathcal{M}(H^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(H^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right]$$

Infrared Safety

Definition: **an observable is infrared safe if it is *insensitive* to**

SOFT radiation:

Adding any number of infinitely *soft* particles (zero-energy) should not change the value of the observable

COLLINEAR radiation:

Splitting an existing particle up into two *comoving* ones (conserving the total momentum and energy) should not change the value of the observable

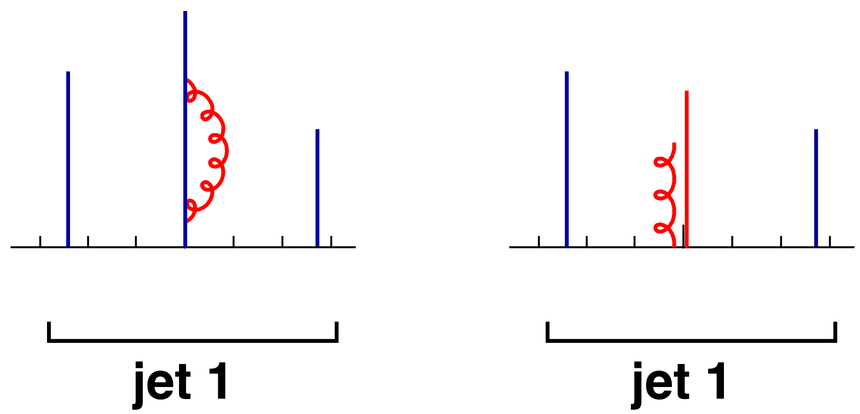
Note: some people use the word "infrared" to refer to soft only. Hence you may also hear "infrared and collinear safety". Advice: always be explicit and clear what you mean.

Why do we care?

(example by G. Salam)

Collinear Safe

Virtual and Real go into **same bins!**



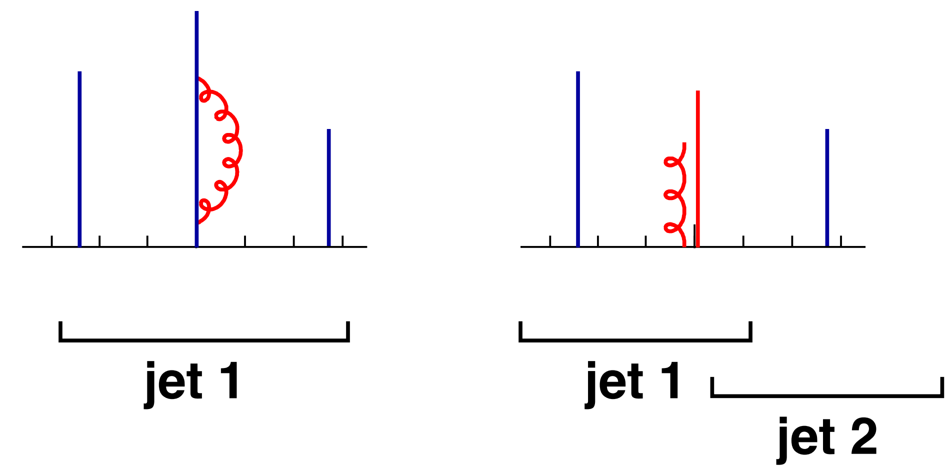
α_s^n x ($-\infty$) α_s^n x ($+\infty$)

Infinities cancel

(KLN: 'degenerate states')

Collinear Unsafe

Virtual and Real go into **different bins!**



α_s^n x ($-\infty$) α_s^n x ($+\infty$)

Infinities do not cancel

Invalidates perturbation theory

Real life does not have infinities, but pert. infinity leaves a real-life trace

$$\alpha_s^2 + \alpha_s^3 + \alpha_s^4 \times \infty \to \alpha_s^2 + \alpha_s^3 + \alpha_s^4 \times \ln p_t/\Lambda \to \alpha_s^2 + \underbrace{\alpha_s^3 + \alpha_s^3}_{\text{BOTH WASTED}}$$

Summary

This Lecture:

Making Predictions: the Role of MC Generators

Jets: Factorisation of QCD amplitudes in soft/collinear limits

Harder Processes are Accompanied by Harder Jets

We collide - and observe - hadrons, with low-scale non-perturbative structure. They participate in hard processes, with Q_{HARD} hierarchically greater than m_{HAD} ~ 1 GeV.

With (IR safe) **jets**, we get to replace m_{HAD} by $\mathbf{p_{TJET}}$

Can be computed perturbatively (using PDFs for initial state)

but hierarchies Q_{HARD}/P_{TJET} can still → need resummation

Next Two Lectures (Tuesday & Friday)

Lecture 2: Parton showers + Matching & Merging

Lecture 3: Hadronisation + BSM Signals and Backgrounds

Extra Slides

Easy to collect millions of events of "high-cross-section-physics"

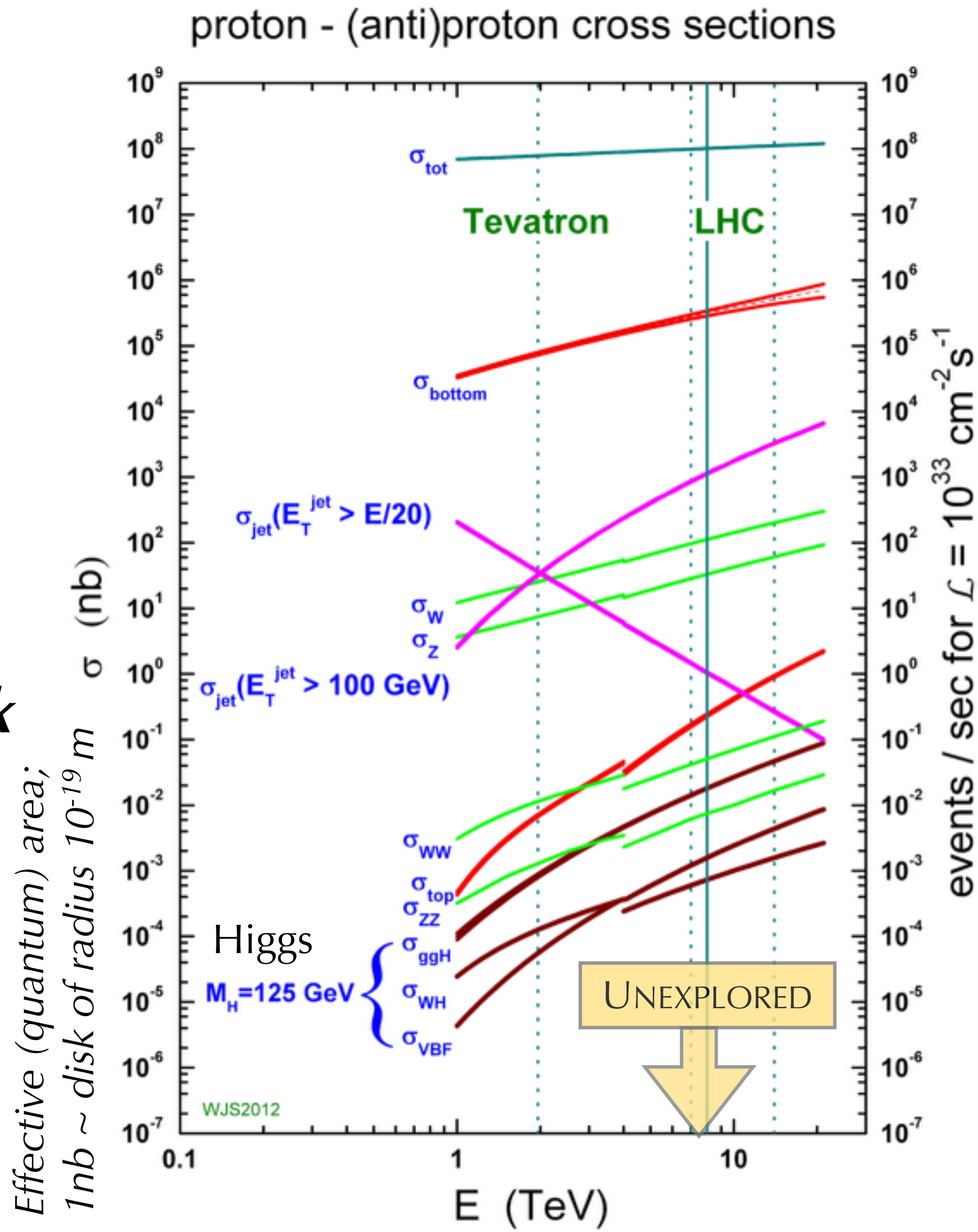
➜ Test models of "known physics" to high precision

Triggers target the ***needles in the haystack***

Trigger on signatures of decays of heavy particles, violent reactions

"Photons" "Leptons"

"Missing Energy" "Jets"

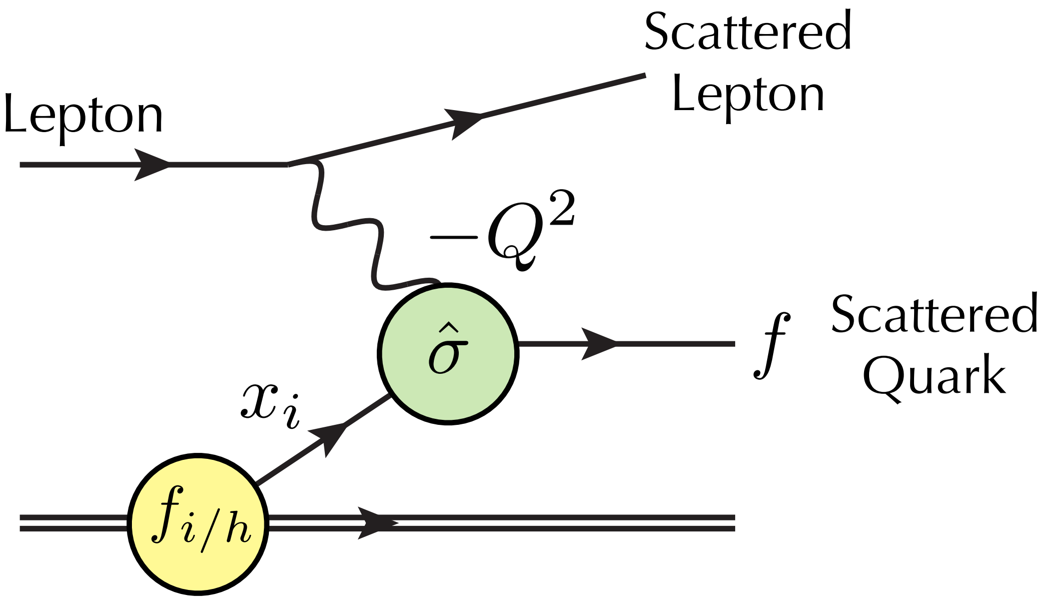


PDFs: Factorisation Theorem

In DIS, there is a formal proof of PDF (collinear) factorisation (Collins, Soper, 1987)

Deep Inelastic Scattering (DIS)

(By "deep", we mean $Q^2 >> M_h^2$)



Note: Beyond LO, *f* can be more than one parton

→ We really can write the cross section in factorized form :

$$\sigma^{\ell h} = \sum_i \sum_f \int dx_i \int d\Phi_f \, f_{i/h}(x_i, Q_F^2) \, \frac{d\hat{\sigma}^{\ell i \to f}(x_i, \Phi_f, Q_F^2)}{dx_i \, d\Phi_f}$$

Sum over Initial (i) and final (f) parton flavors

Φ_f = Final-state phase space

$f_{i/h}$ = PDFs Assumption: $Q^2 = Q_F^2$

Differential partonic Hard-scattering Matrix Element(s)

There is no unique or "best" jet definition

YOU decide how to look at event

The construction of jets is inherently ambiguous

1. Which particles get grouped together?
   JET ALGORITHM (+ parameters)
2. How will you combine their momenta?
   RECOMBINATION SCHEME
   (e.g., 'E' scheme: add 4-momenta)

⇨ **Jet Definition**

Ambiguity complicates life, but gives flexibility in one's view of events → Jets non-trivial!

Types of Algorithms

1. Sequential Recombination

Take your 4-vectors. Combine the ones that have the lowest 'distance measure'

Different names for different distance measures

Durham k_T : $\Delta R_{ij}^2 \times \min(k_{Ti}^2, k_{Tj}^2)$

Cambridge/Aachen : ΔR_{ij}^2

Anti-k_T : $\Delta R_{ij}^2 / \max(k_{Ti}^2, k_{Tj}^2)$

ArClus $(3\to2)$: $p_\perp^2 = s_{ij} s_{jk} / s_{ijk}$

$$k_{Ti}^2 = E_i^2 (1 - \cos\theta_{ij})$$

$$\Delta R_{ij}^2 = (\eta_i - \eta_j)^2 + \Delta\phi_{ij}^2$$

+ Prescription for how to combine 2 momenta into 1

(or 3 momenta into 2)

→ New set of (n-1) 4-vectors

Iterate until A or B (you choose which):

A: all distance measures larger than something

B: you reach a specified number of jets

Look at event at:

specific resolution

specific n_{jets}

Why k_T (or p_T or ΔR)?

Attempt to (approximately) capture universal jet-within-jet-witin-jet… behavior

Approximate full matrix element

$$\frac{|M^{(0)}_{X+1}(s_{i1},s_{1k},s)|^2}{|M^{(0)}_{X}(s)|^2} = 4\pi\alpha_s C_F \left(\frac{2s_{ik}}{s_{i1}s_{1k}} + \ldots\right)$$

"Eikonal"
(universal, always there)

by Leading-Log limit of QCD → universal dominant terms

$$\frac{\mathrm{d}s_{i1}\mathrm{d}s_{1k}}{s_{i1}s_{1k}} \to \frac{\mathrm{d}p_\perp^2}{p_\perp^2}\frac{\mathrm{d}z}{z(1-z)} \to \frac{\mathrm{d}E_1}{\min(E_i,E_1)}\frac{\mathrm{d}\theta_{i1}}{\theta_{i1}} \quad (E_1 \ll E_i,\ \theta_{i1} \ll 1) \qquad ,\ldots$$

Rewritings in soft/collinear limits

"smallest" k_T (or p_T or θ_{ij}, or …) → largest Eikonal

Types of Algorithms

2. "Cone" type

Take your 4-vectors. Select a procedure for which "test cones" to draw

- Different names for different procedures
  - Seeded : start from hardest 4-vectors (and possibly combinations thereof, e.g., CDF midpoint algorithm) = "seeds"
  - Unseeded : smoothly scan over entire event, trying everything
- Sum momenta inside test cone → new test cone direction
- Iterate until stable (test cone direction = momentum sum direction)

Warning: seeded algorithms are INFRARED UNSAFE

Safe vs Unsafe Jets

May look pretty similar in experimental environment …

But it's not nice to your theory friends …

Unsafe: badly divergent in pQCD → large IR corrections:

$$\text{IR Sensitive Corrections} \propto \alpha_s^n \log^m \left(\frac{Q^2_{\text{UV}}}{Q^2_{\text{IR}}} \right) \quad , \quad m \leq 2n$$

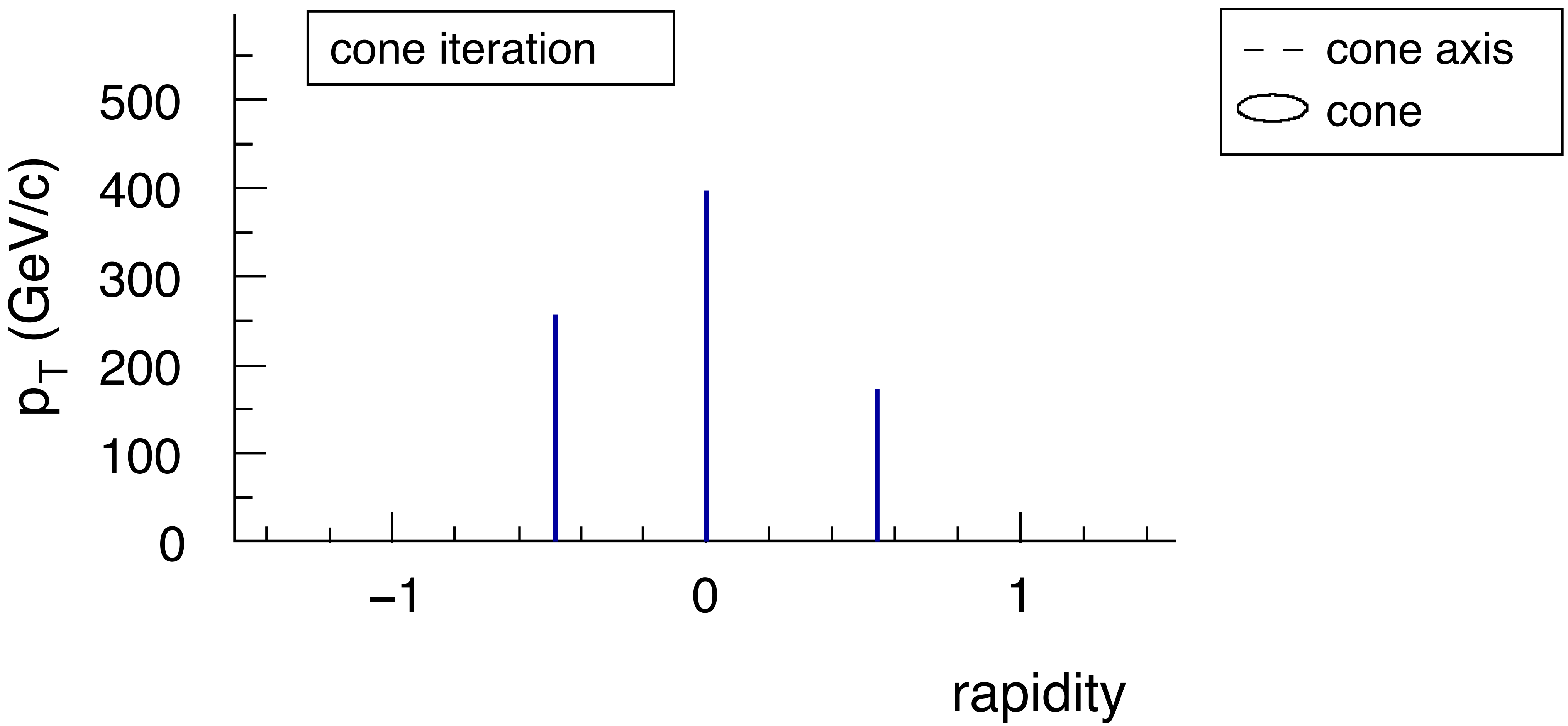
Even if we have a hadronization model with which to compute these corrections, the dependence on it → larger uncertainty

Safe → IR corrections power suppressed:

$$\text{IR Safe Corrections} \propto \frac{Q^2_{\text{IR}}}{Q^2_{\text{UV}}}$$

Can still be computed (MC) but can also be neglected (pure pQCD)

Let's look at a specific example …

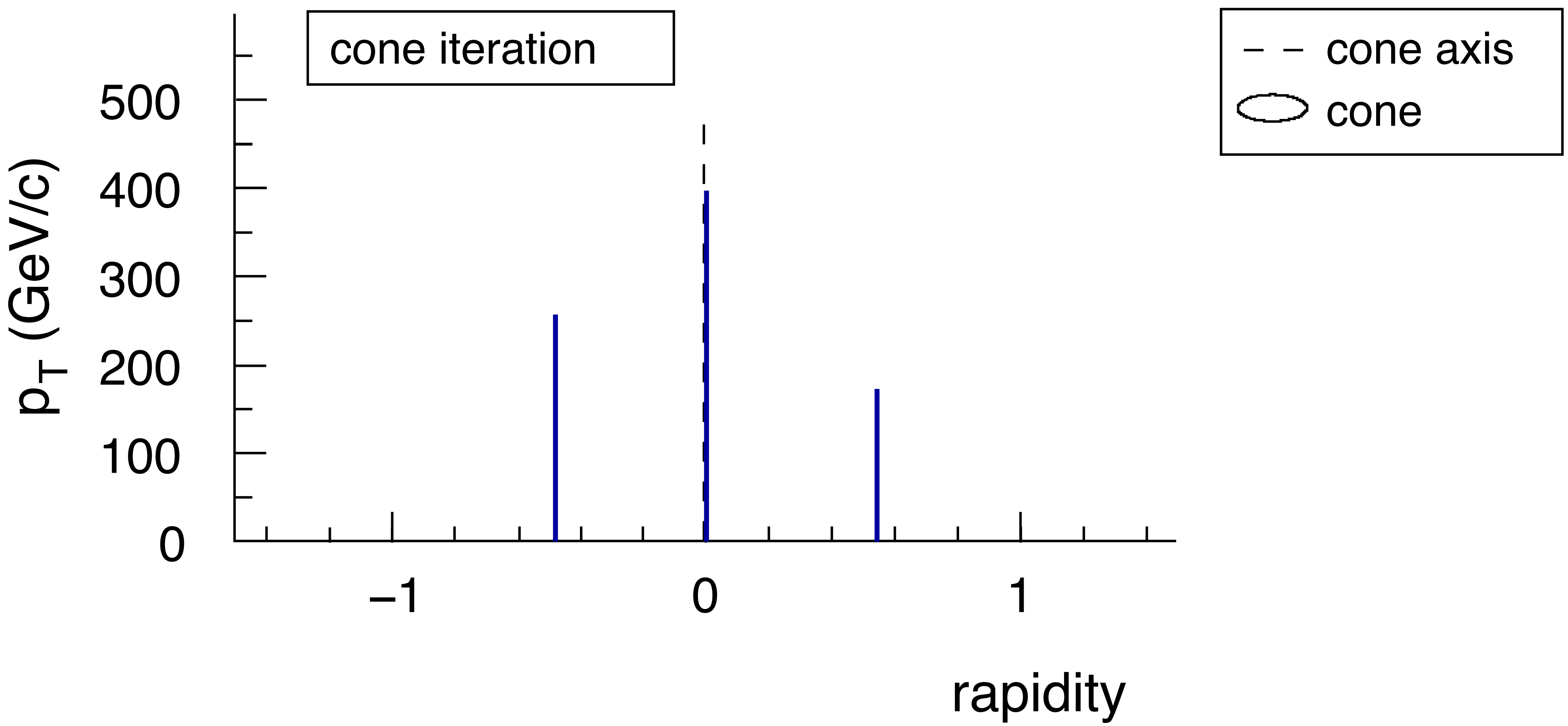


Collinear splitting can modify the hard jets: ICPR algorithms are **collinear unsafe** $\Longrightarrow$ perturbative calculations give ∞

Slides from G. Salam

ICPR iteration issue

Iterative Cone Progressive Removal



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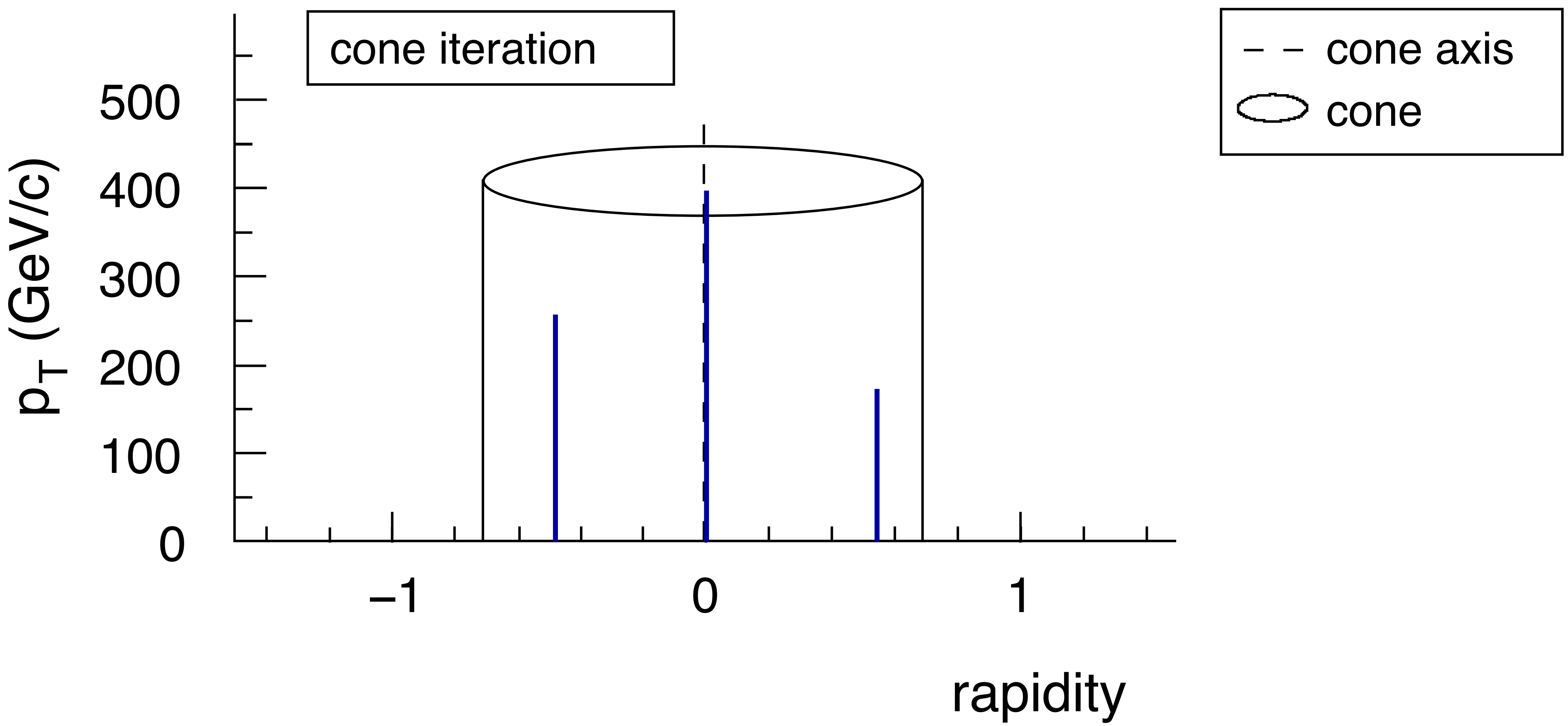

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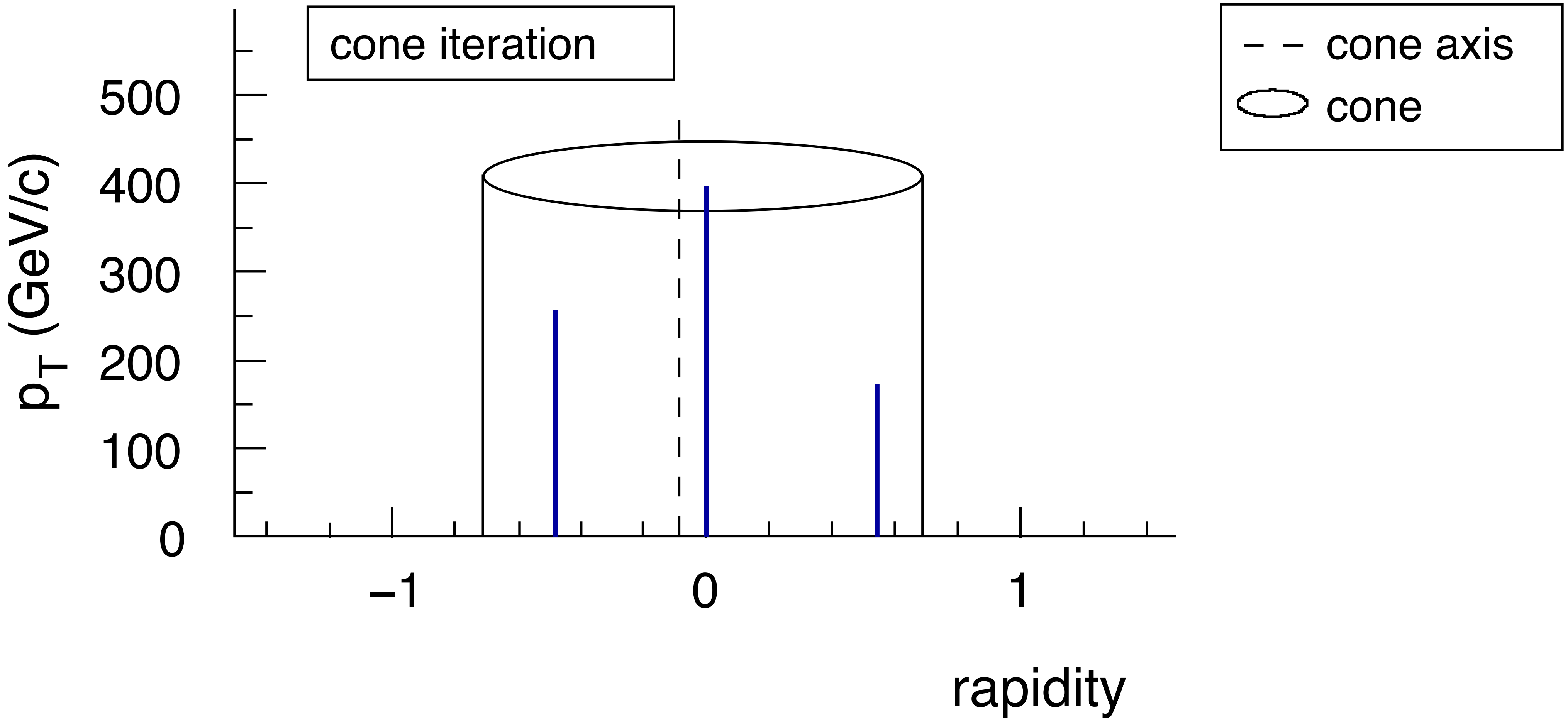

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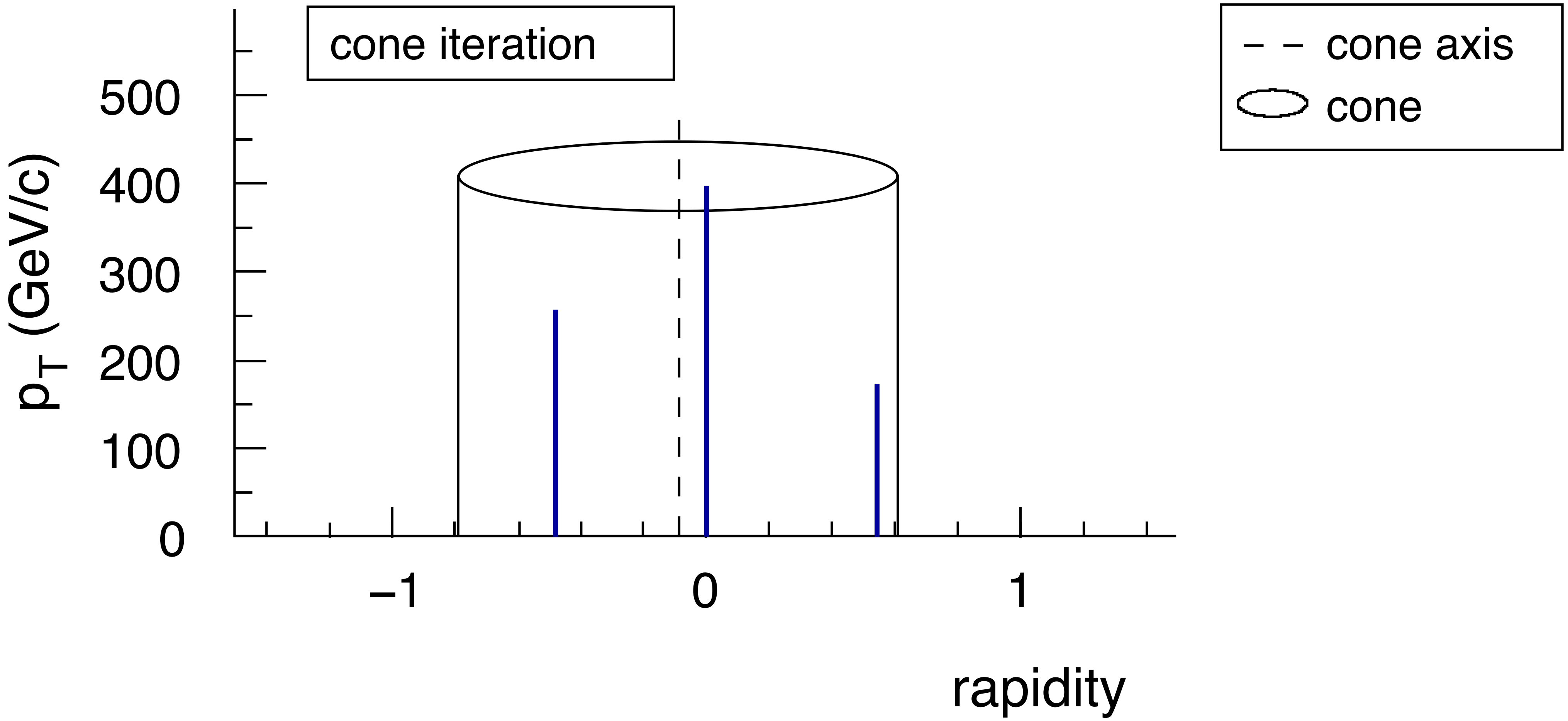

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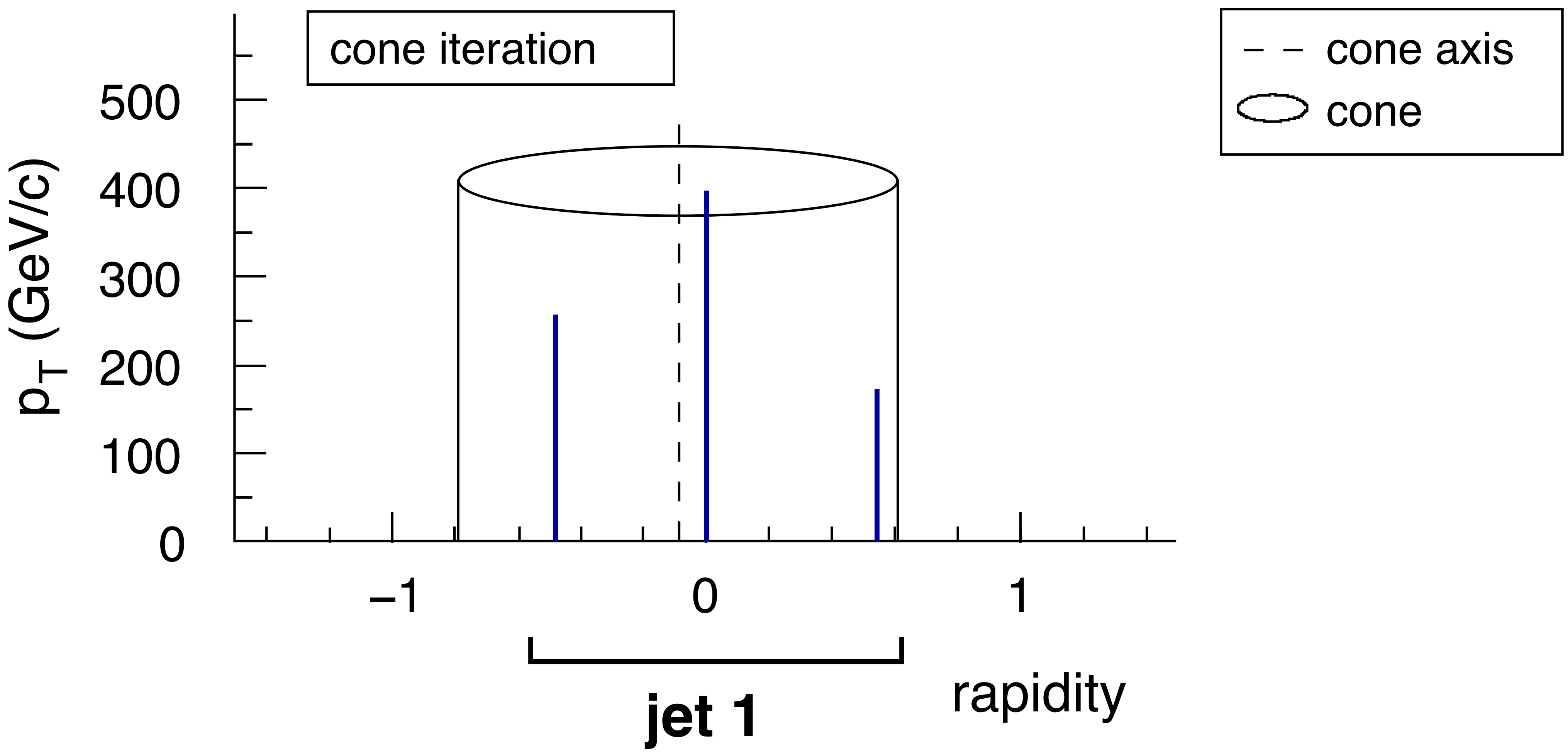

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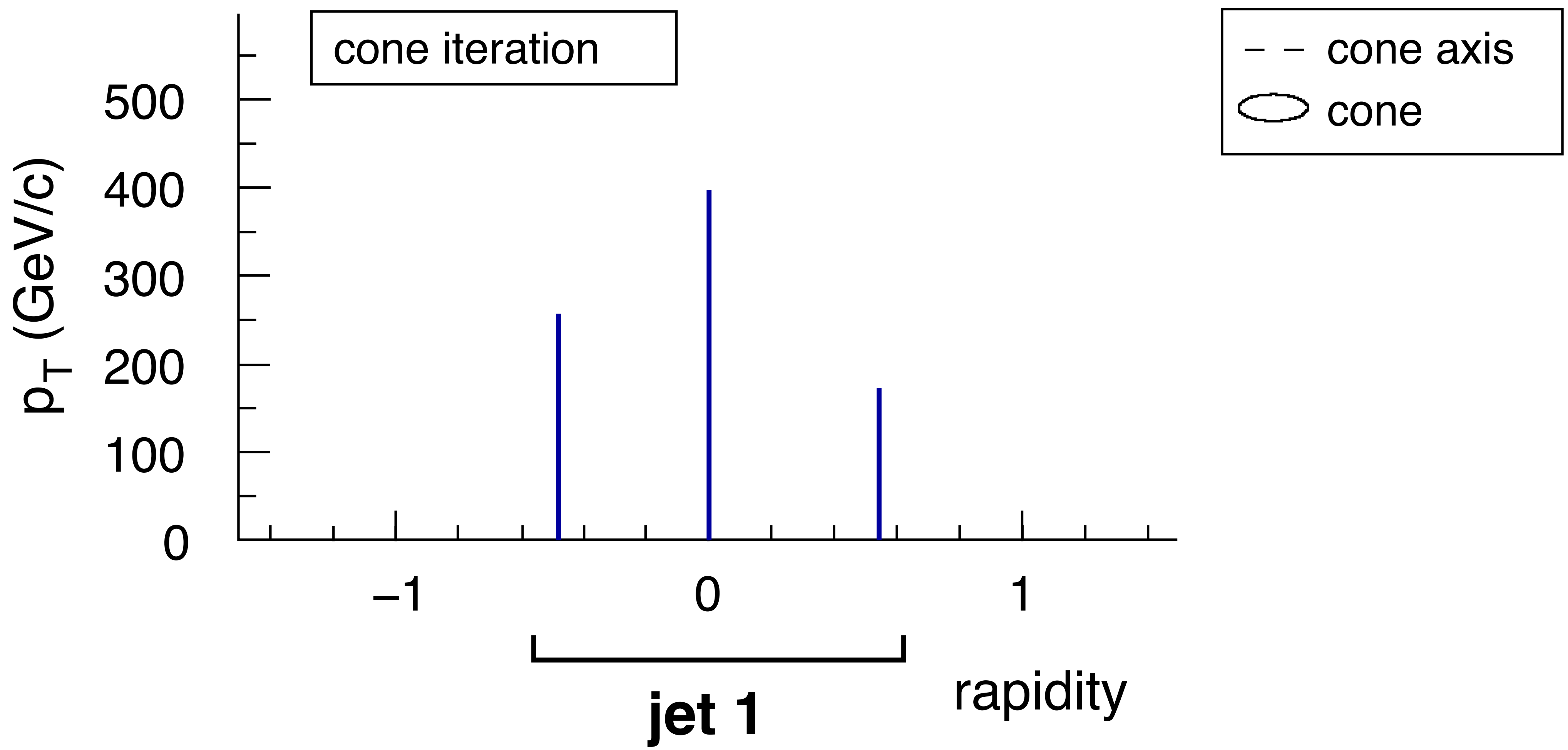

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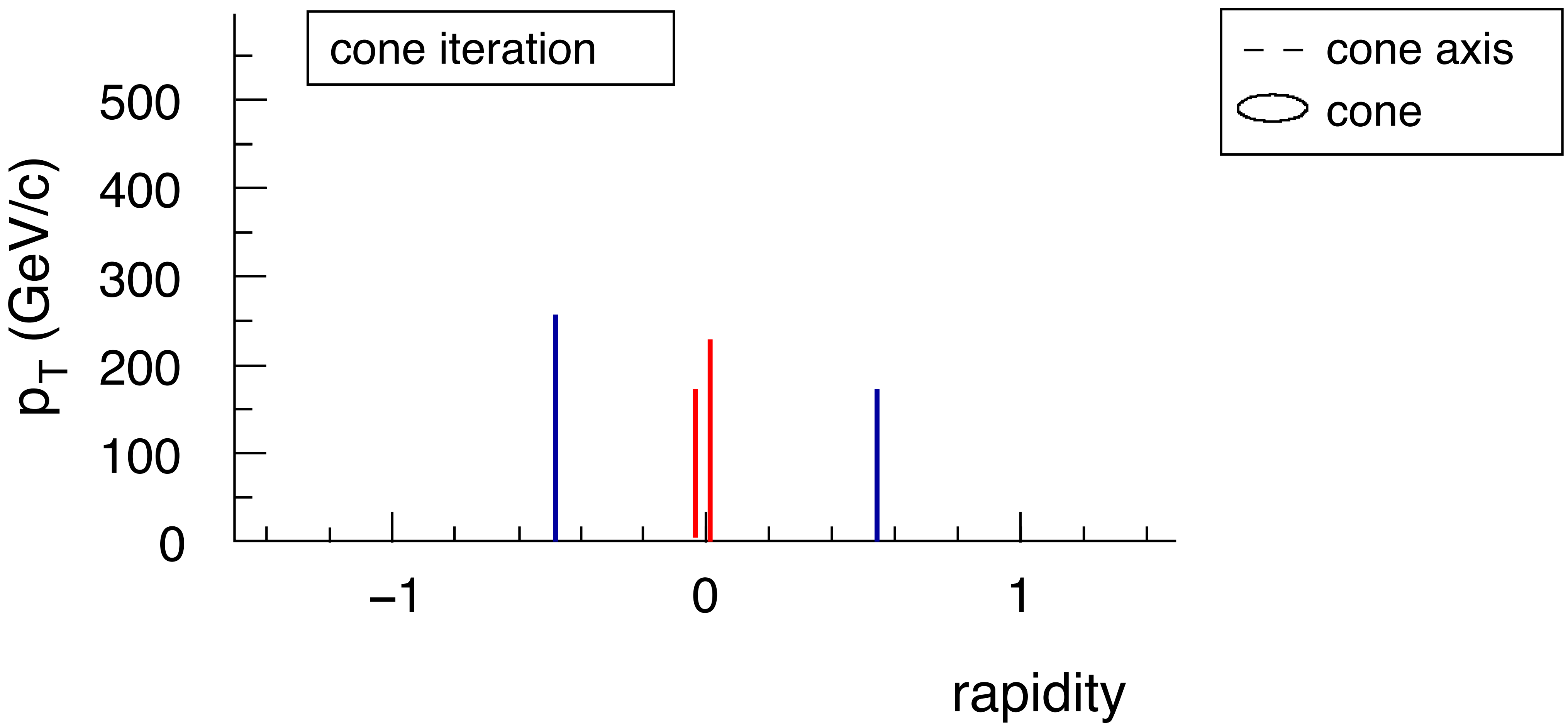

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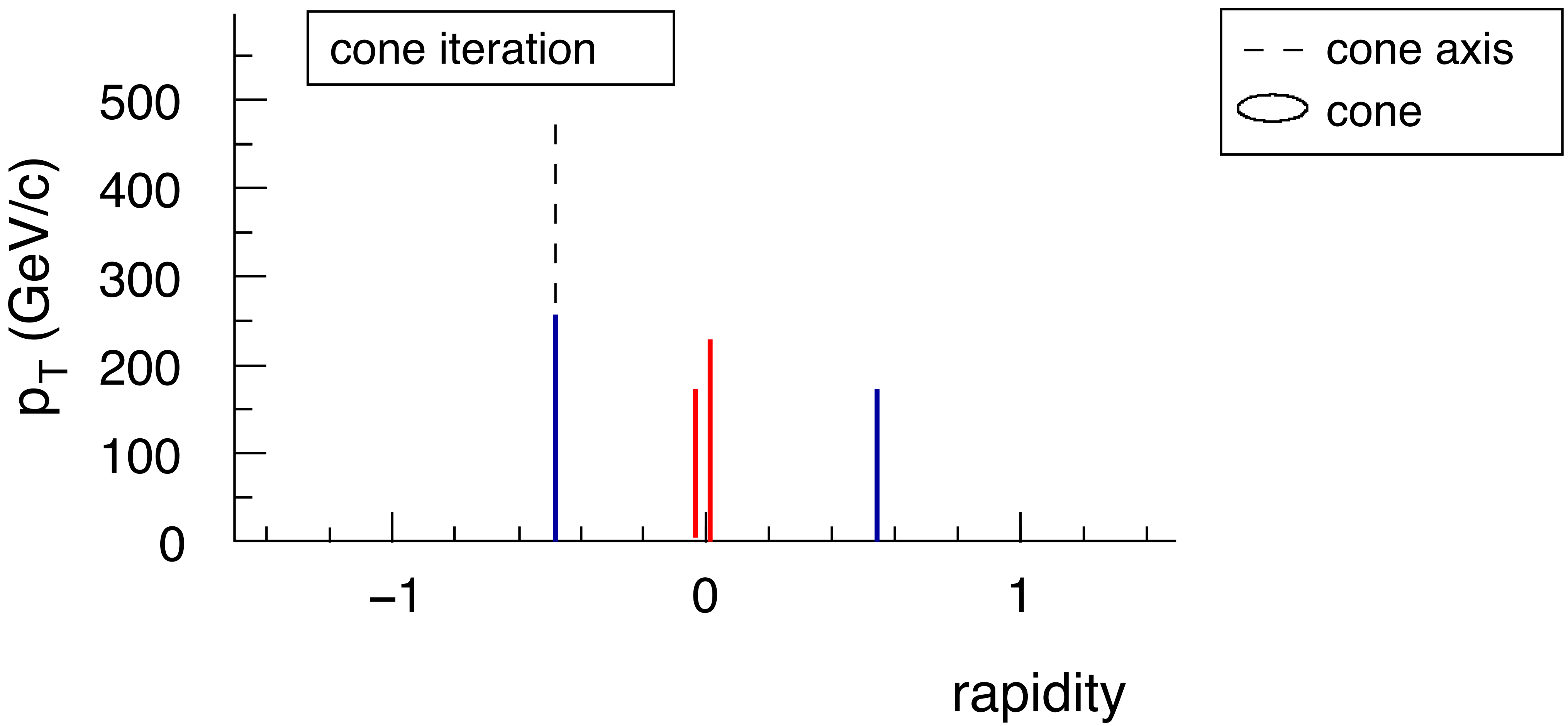

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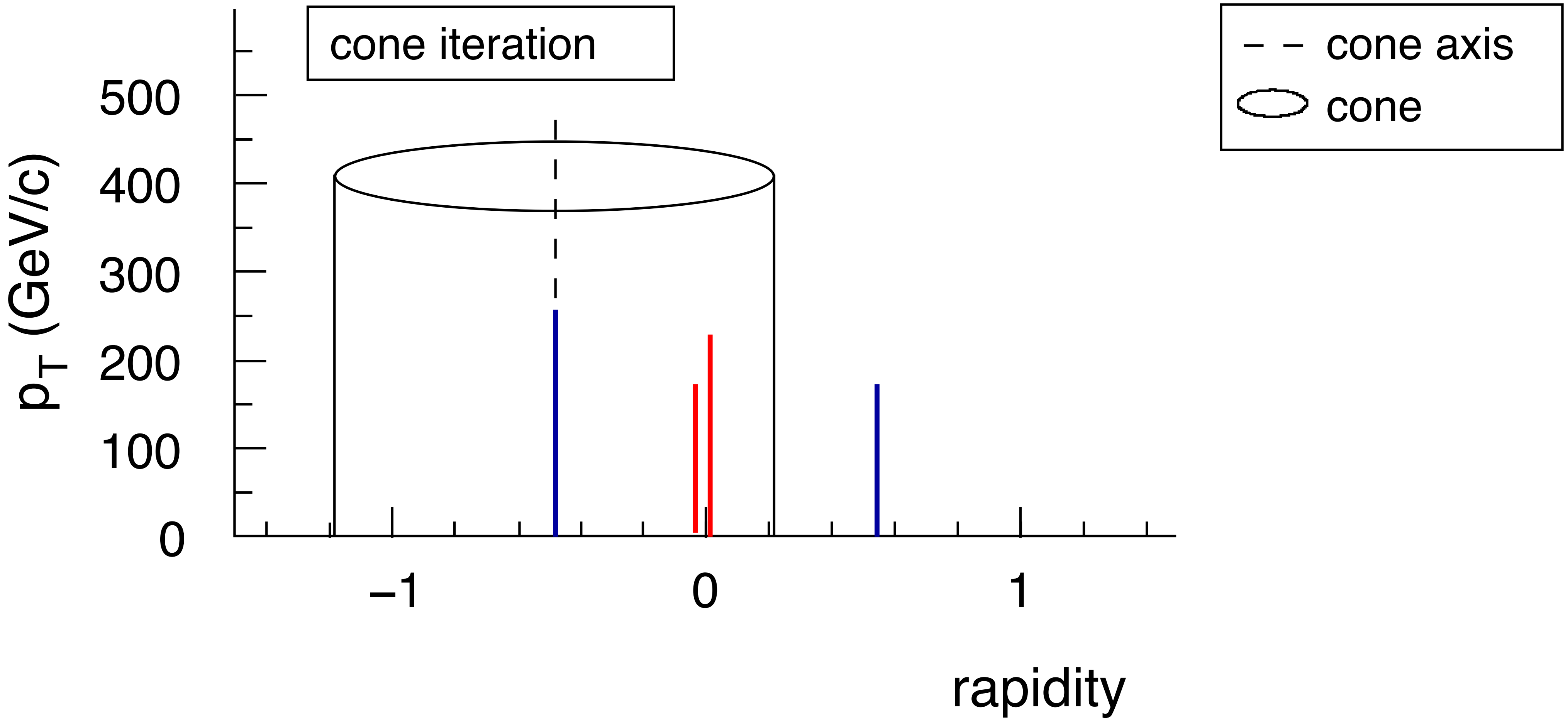

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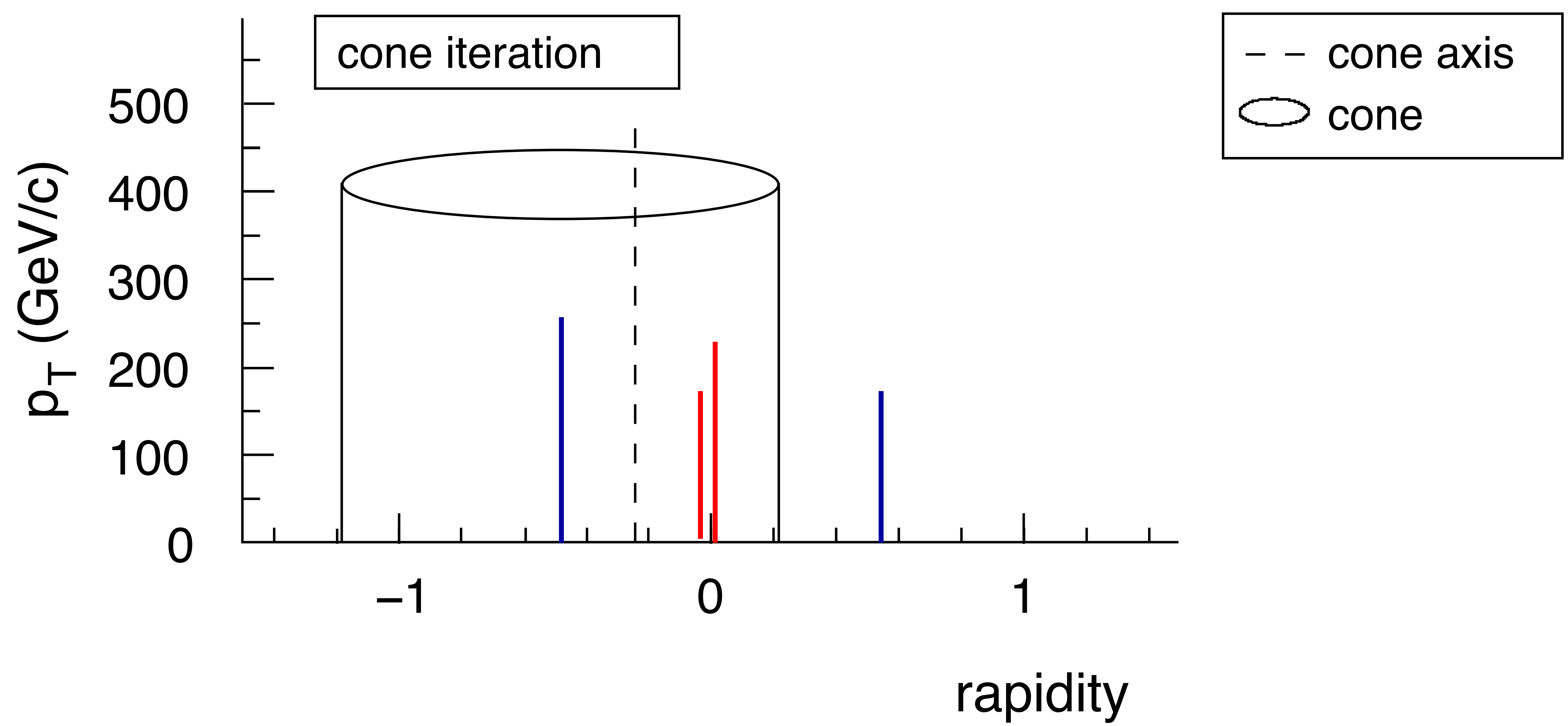

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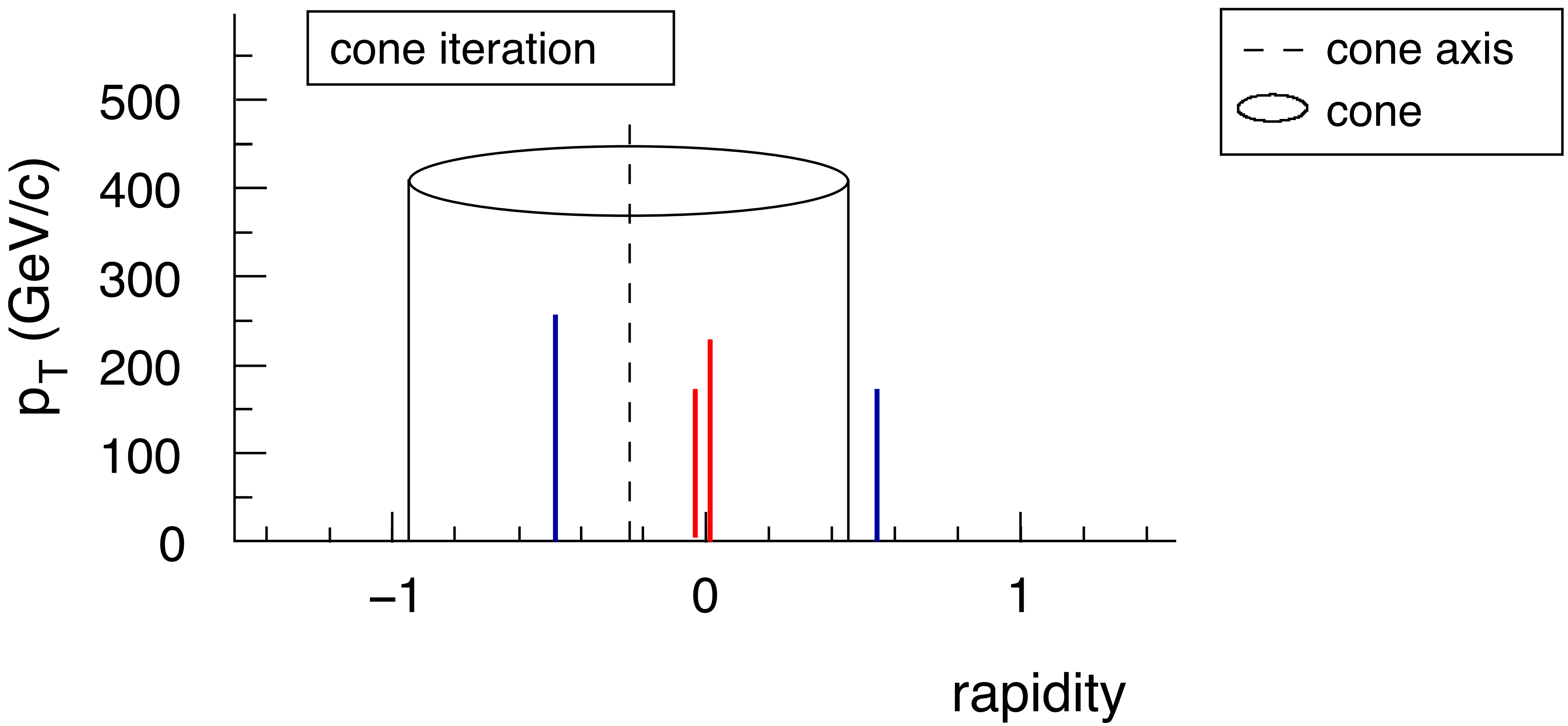

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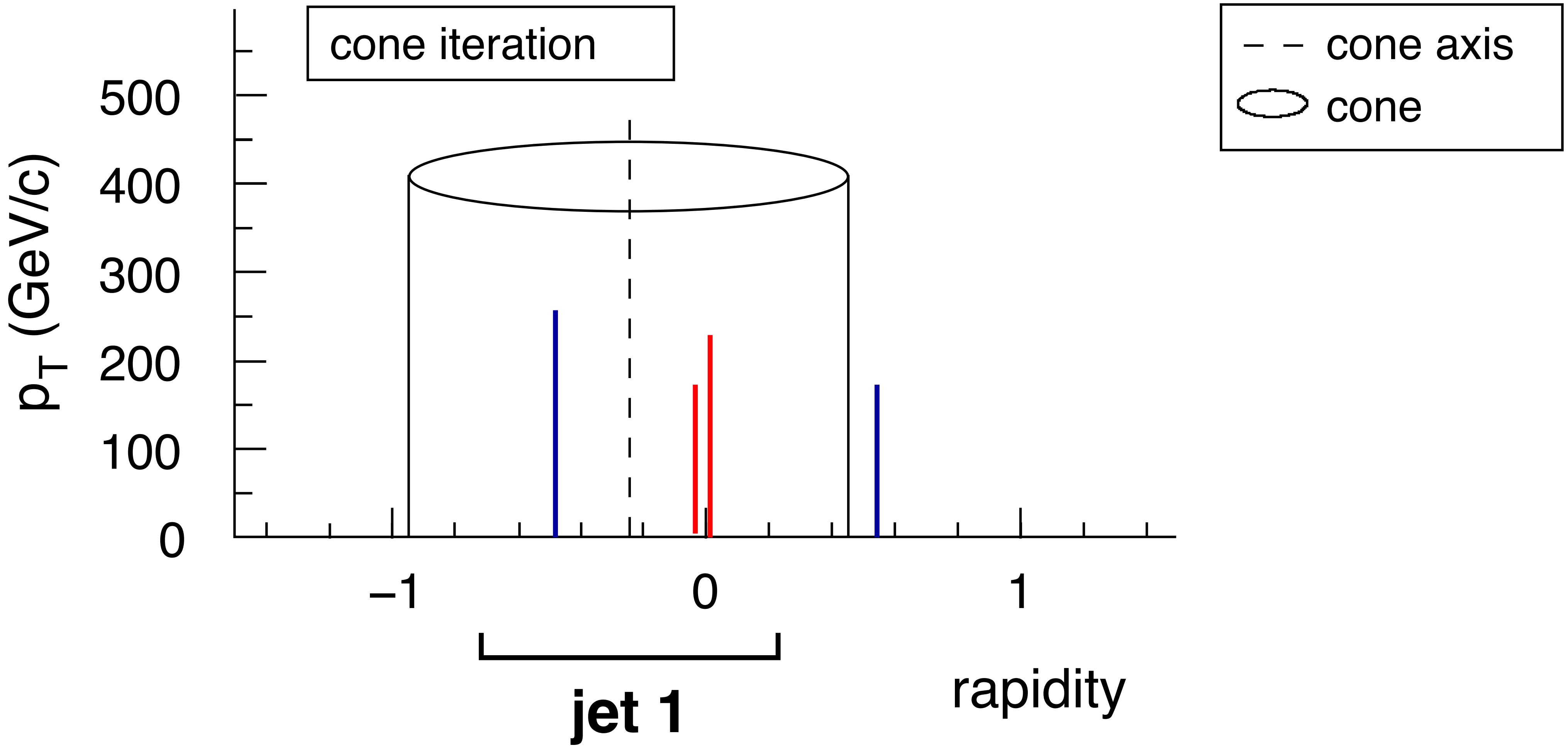

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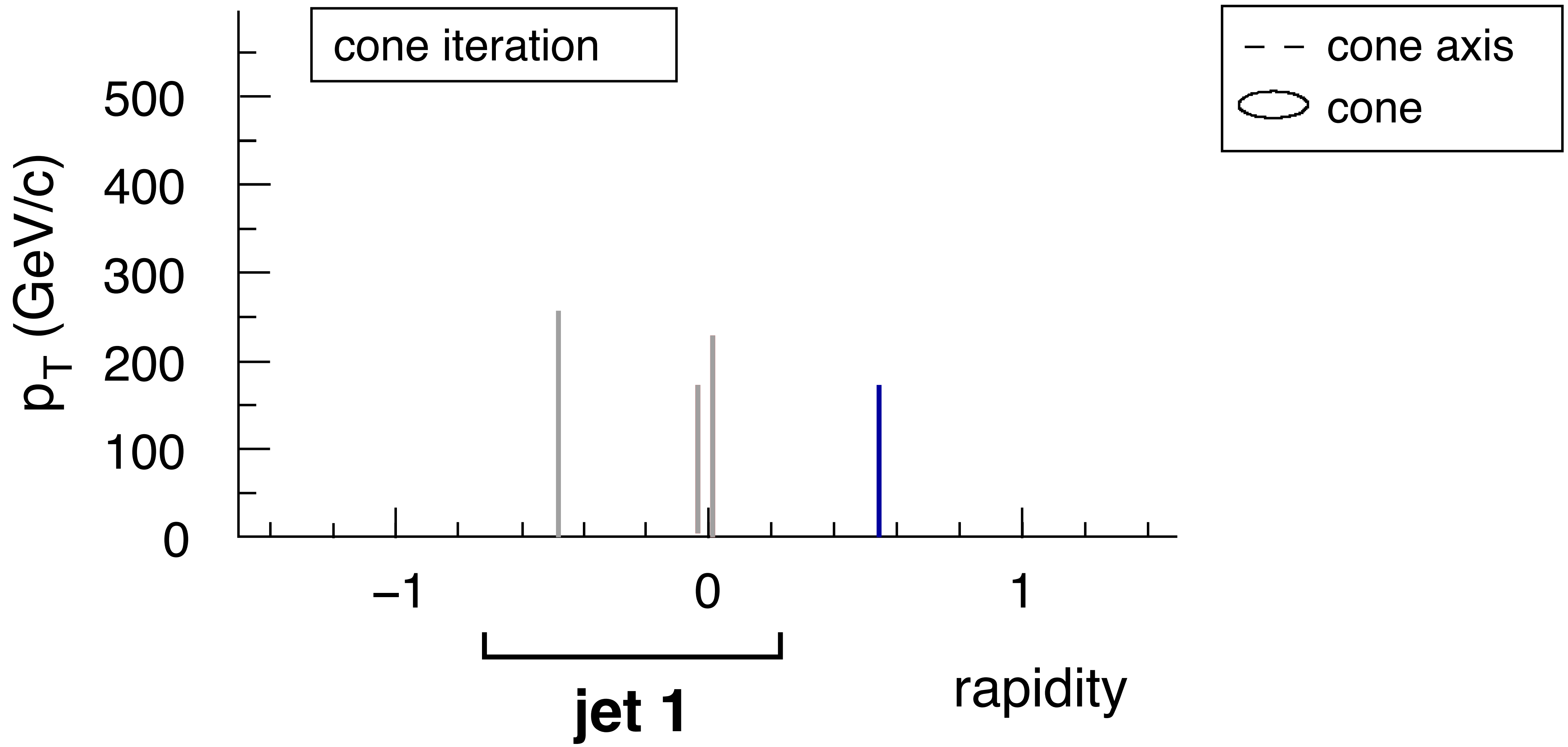

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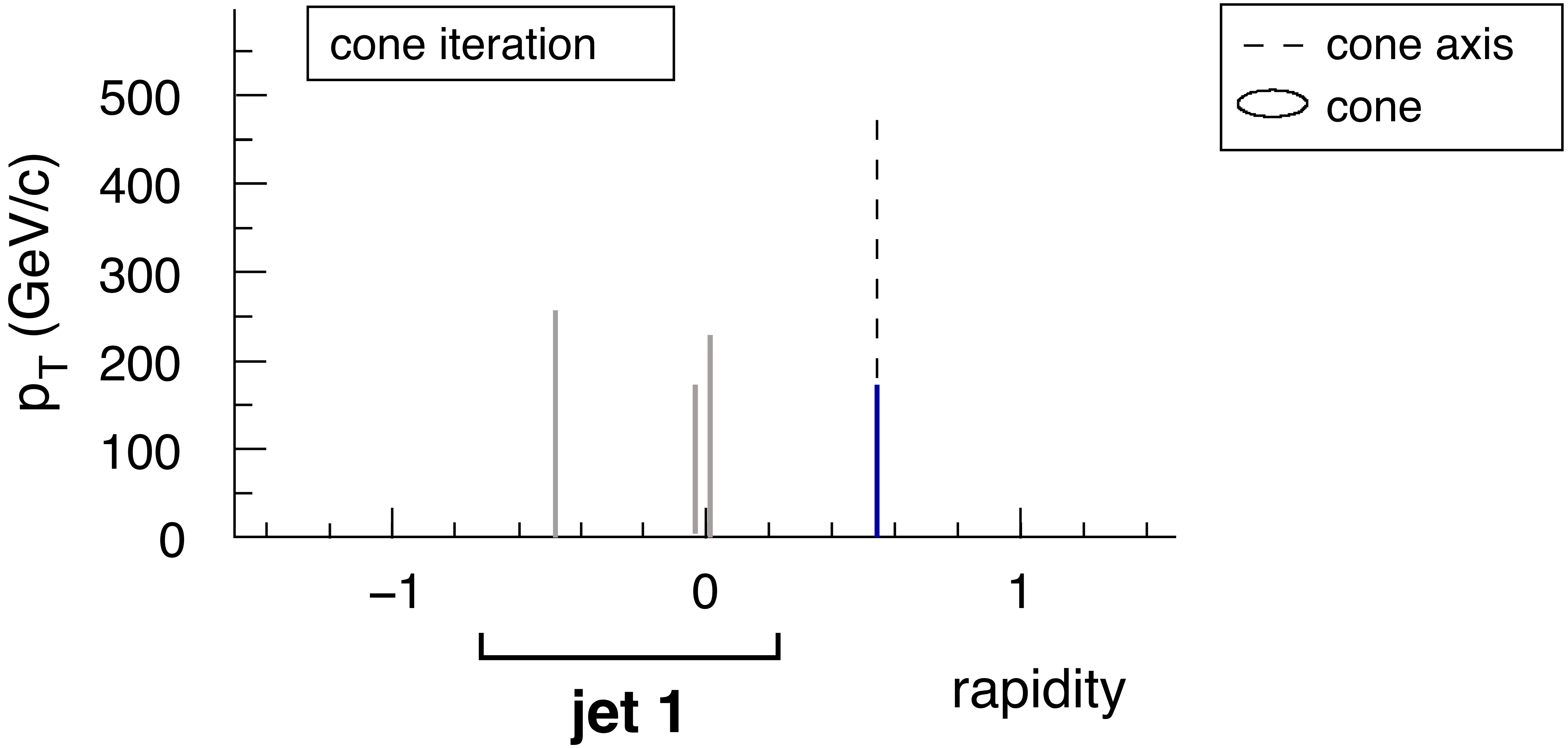

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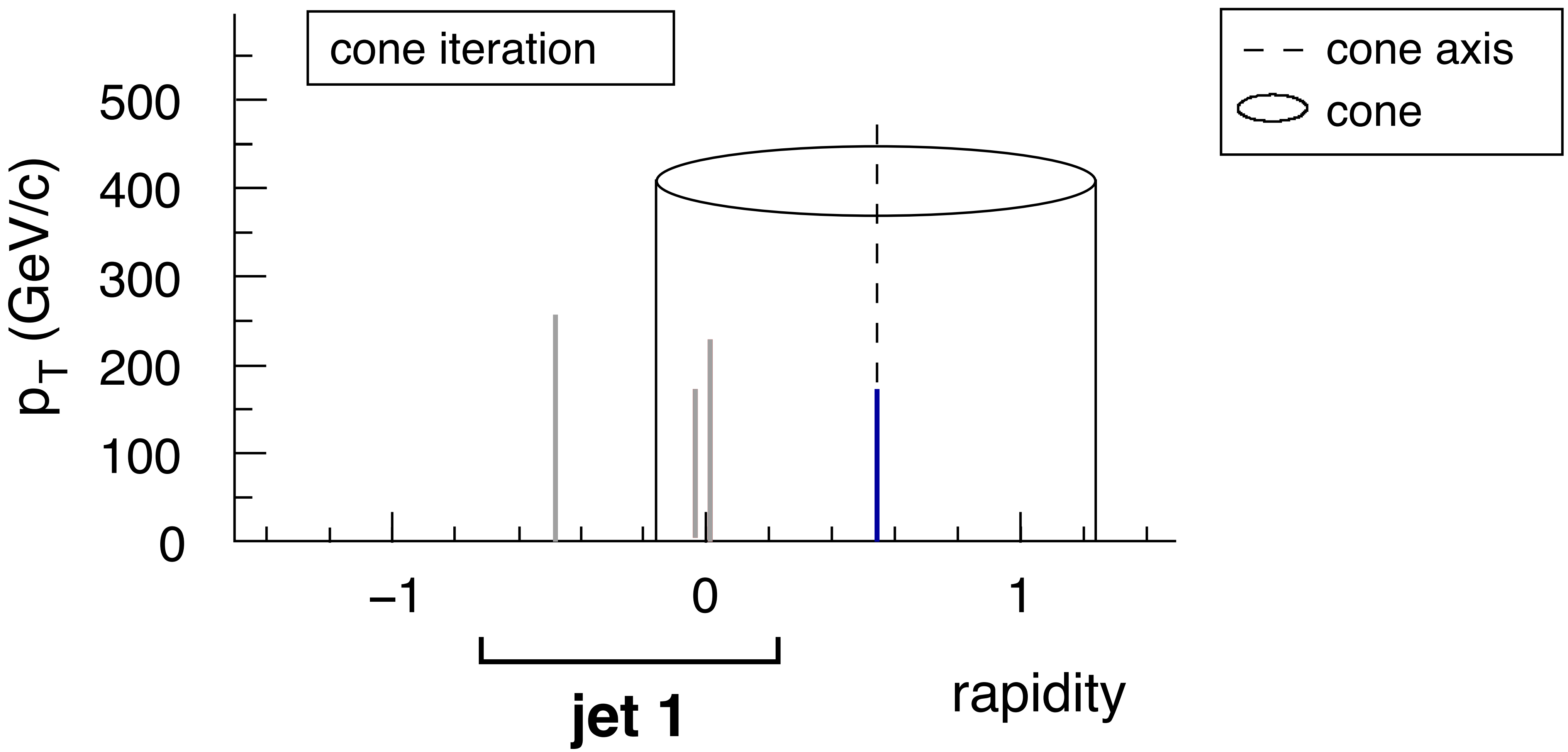

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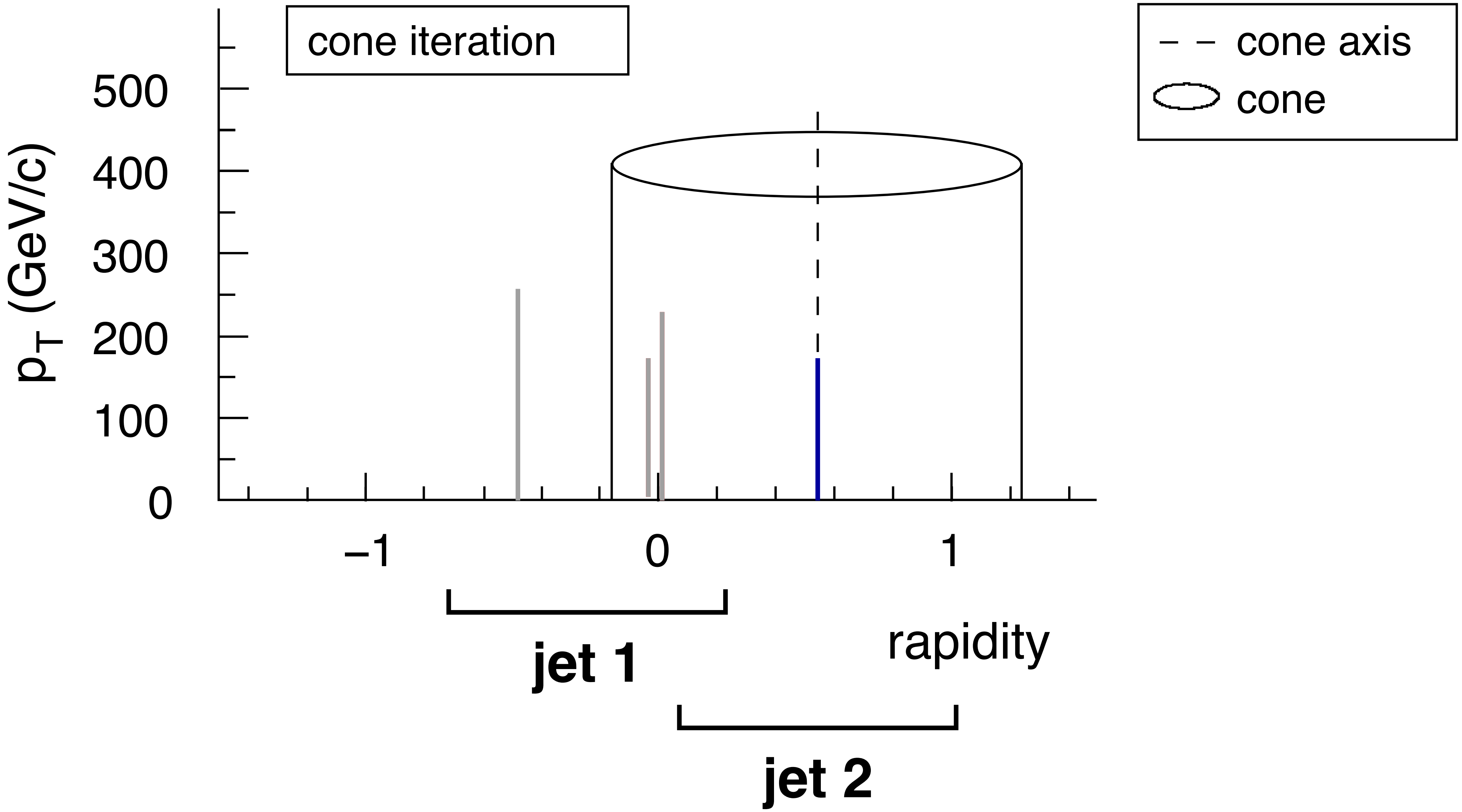

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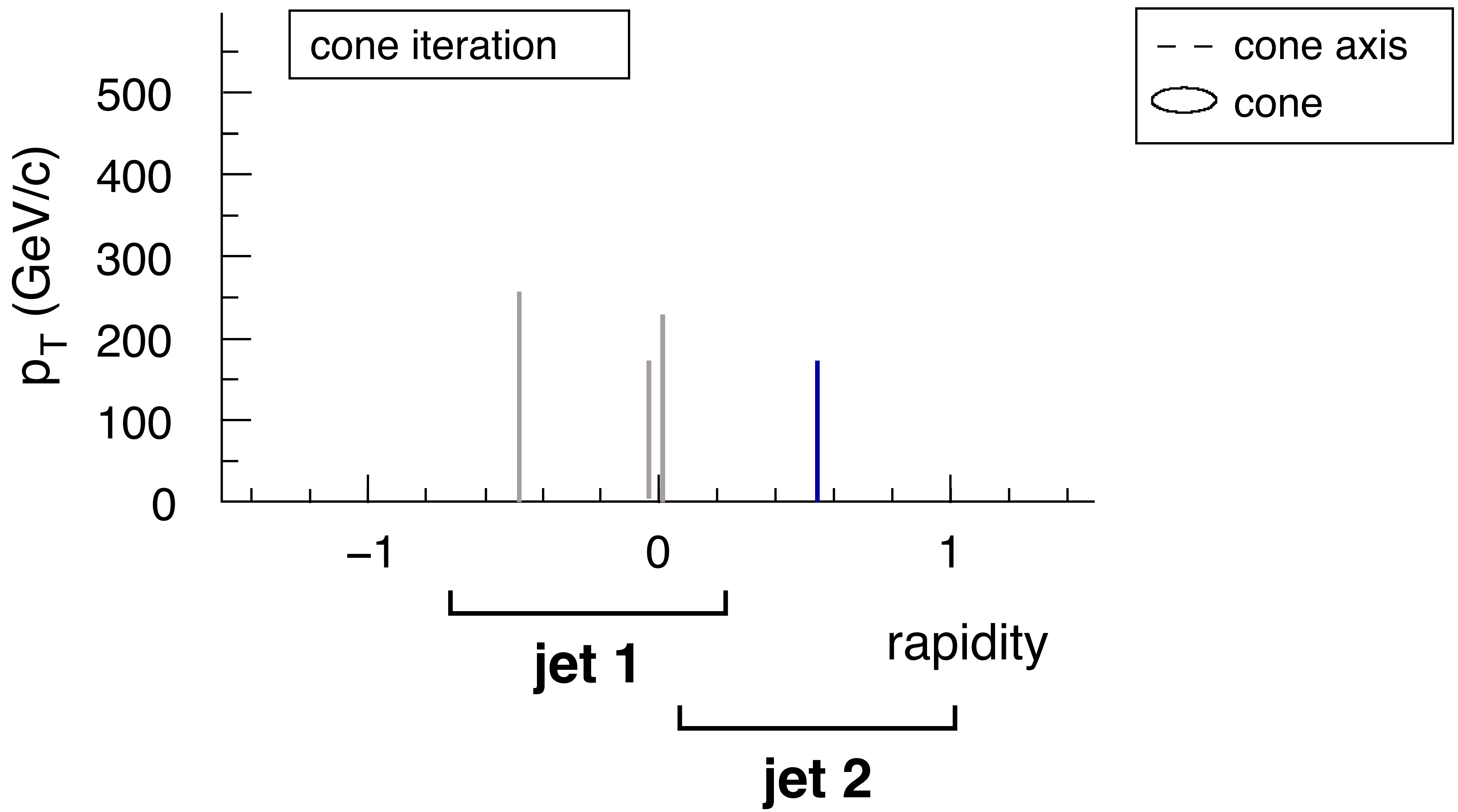

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Slides from G. Salam

Stereo Vision

Use IR Safe algorithms

To study short-distance physics

http://www.fastjet.fr/

These days, ≈ as fast as IR unsafe algos and widely implemented (e.g., FASTJET), including

"Cone-like": SiSCone, Anti-k_T, …

"Recombination-like": k_T,Cambridge/Aachen,Anti-k_T…

Image Credits: Richard Seaman

Then use IR Sensitive observables

E.g., number of tracks, identified particles, …

To explicitly check hadronization and models of IR physics

More about IR in lecture on soft QCD …