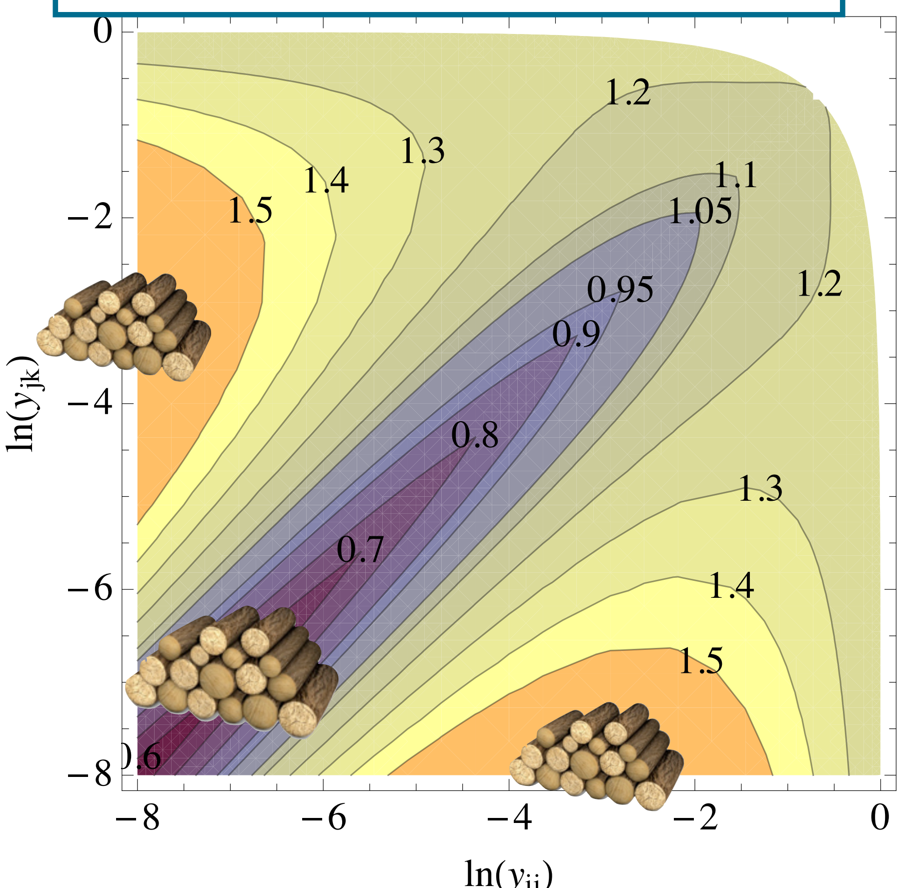
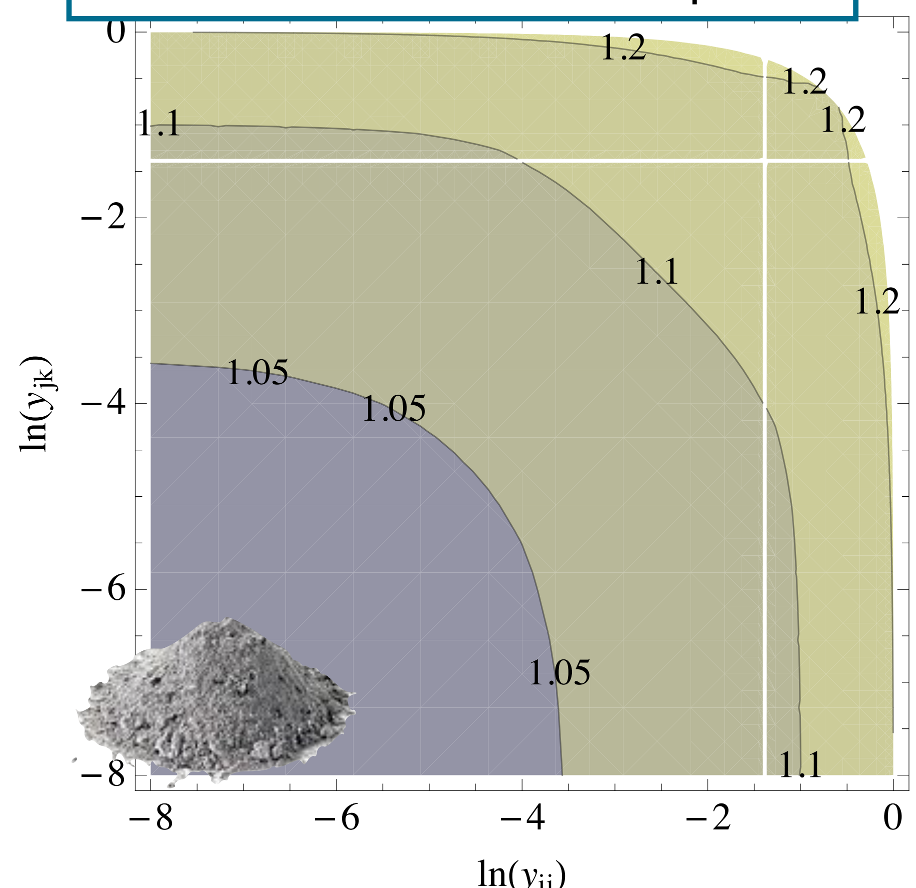
NLO and Helicity Amplitudes in VINCIA

Peter Skands (CERN TH)

Evolution in Antenna Mass



Evolution in Ariadne p_T



work with E. Laenen, L. Hartgring, M. Ritzmann, A. Larkoski, W. Giele, D. Kosower, J. Lopez-Villarejo

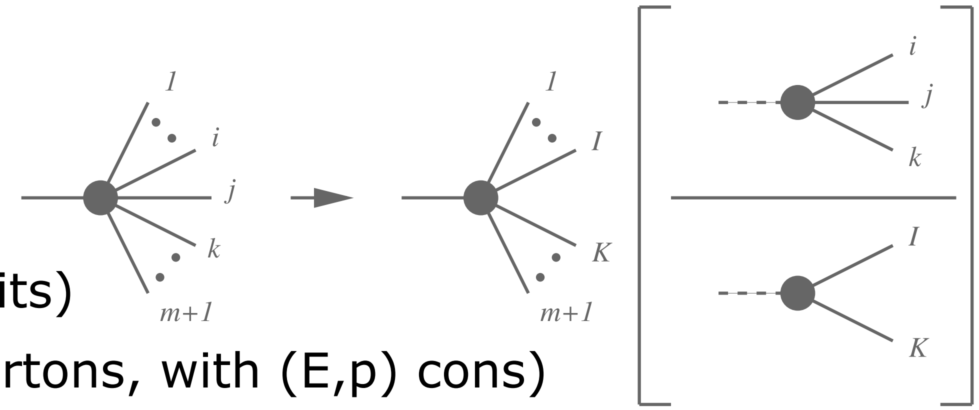
Workshop on Parton Showers and Resummation
IPPP, Durham, July 2013

VINCIA

Written as a Plug-in to PYTHIA 8
Current Version: VINCIA 1.1.00
C++ (~20,000 lines)

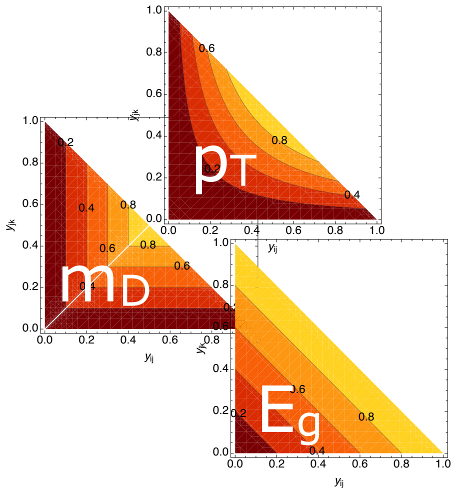
Virtual Numerical Collider with Interleaved Antennae

Giele, Kosower, Skands, PRD 78 (2008) 014026, PRD 84 (2011) 054003
Gehrmann-de Ridder, Ritzmann, Skands, PRD 85 (2012) 014013



Based on antenna factorization

- of Amplitudes (exact in both soft and collinear limits)
- of Phase Space (LIPS : 2 on-shell → 3 on-shell partons, with (E,p) cons)



Resolution Time

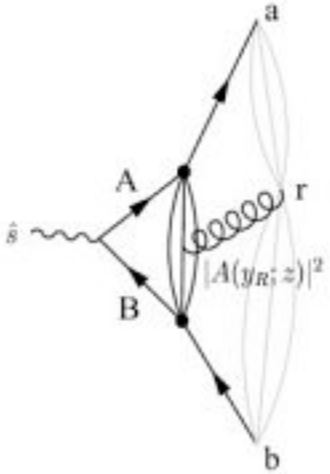
Infinite family of continuously deformable *Q_E*

Special cases: transverse momentum, dipole mass, energy

Radiation functions

Arbitrary non-singular coefficients, *ant_i*

+ Massive antenna functions for massive fermions *(c,b,t)*



Kinematics maps

Formalism derived for arbitrary 2→3 recoil maps, *$\kappa_{3\to2}$*

Default: massive generalization of Kosower’s antenna maps

vincia.hepforge.org

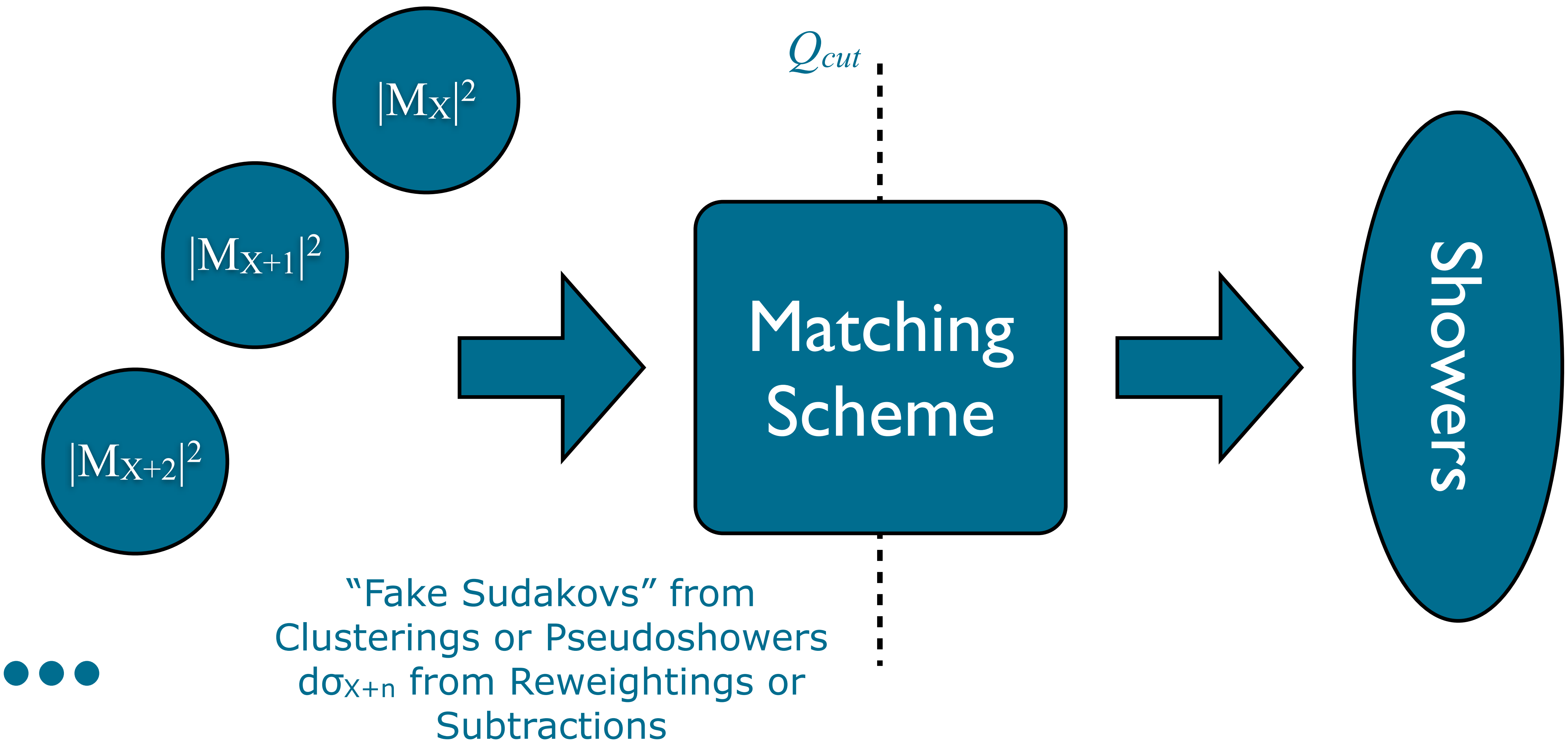
Matrix-Element Matching

Standard Paradigm:

Have ME for X, X+1,…, X+n;

Want to combine and add showers → "The Soft Stuff"

Double counting, IR divergences, multiscale logs



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Scale hierarchies: smaller single-scale phase-space region

Powers of alphaS pile up

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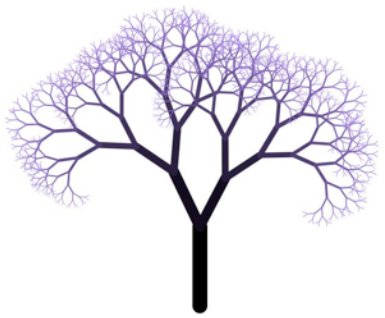
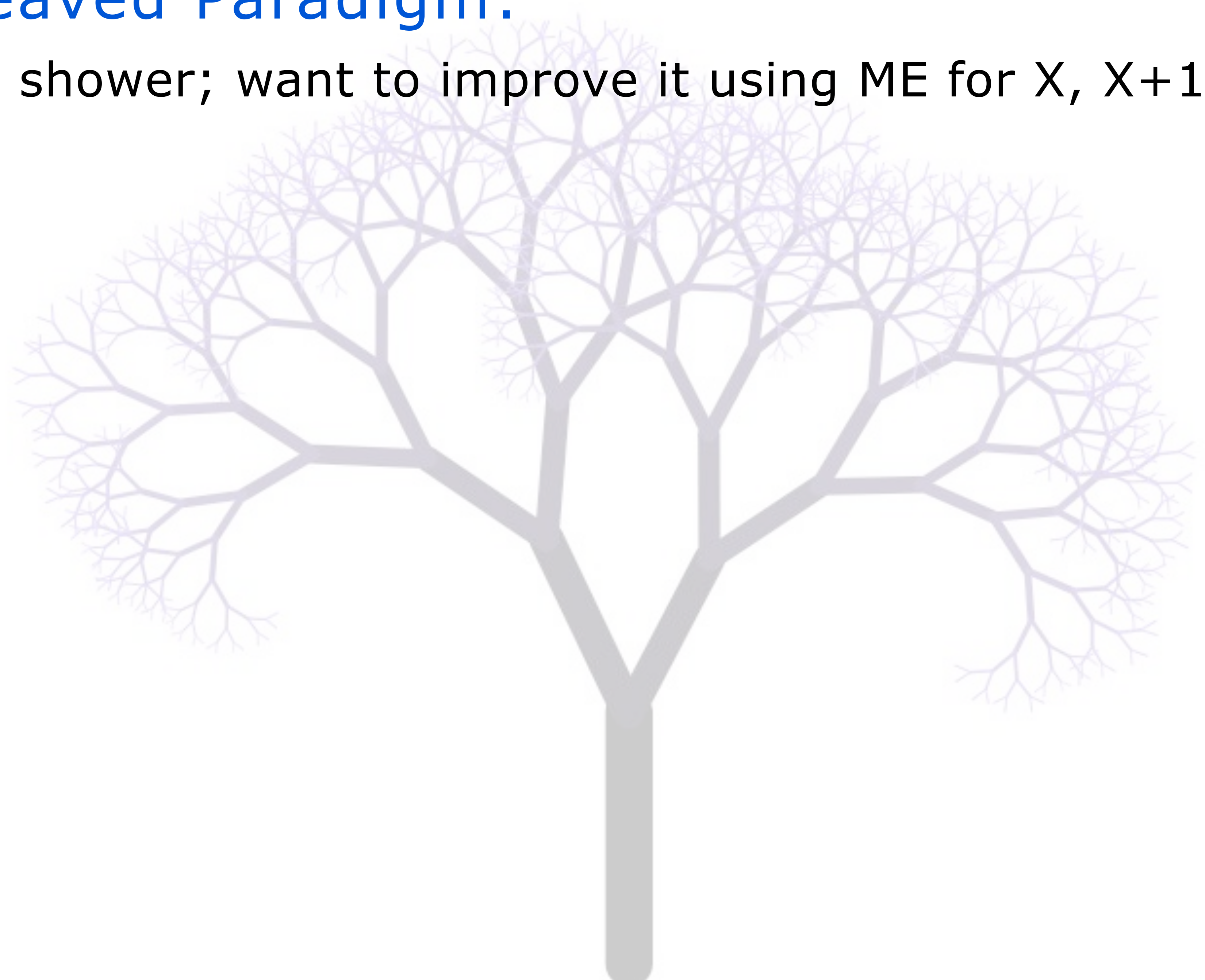
Better Starting Point: a QCD fractal?



Matrix-Element Corrections

Interleaved Paradigm:

Have shower; want to improve it using ME for X, X+1, …, X+n.



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Interpret all-orders shower structure as a trial distribution

Quasi-scale-invariant: intrinsically multi-scale (resums logs)

Unitary: automatically unweighted (& IR divergences → multiplicities)

More precise expressions imprinted via veto algorithm: ME corrections at LO, NLO, and more? → soft *and* hard

No additional phase-space generator or σ_{X+n} calculations → **fast**

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Existing Approaches:

First Order: PYTHIA and POWHEG

Beyond First Order: PYTHIA → too complicated. POWHEG → very active, still mostly in framework of standard paradigm. GENEVA?

Markov is Crucial

LO: Giele, Kosower, Skands, PRD 84 (2011) 054003
NLO: Hartgring, Laenen, Skands, arXiv:1303.4974

Problems:

Traditional parton showers are history-dependent (non-Markovian) → Number of generated terms (possible clustering histories) grows like $2^N N!$

+ Complicated kinematics

+ Dead zones

Parton- (or Catani-Seymour) Shower:
After 2 branchings: 8 terms
After 3 branchings: 48 terms
After 4 branchings: 384 terms

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Solutions: Markovian Evolution, Matched Antenna Showers, and Smooth Ordering

No need to ever cluster back more than one step

→ Number of generated terms grows like N

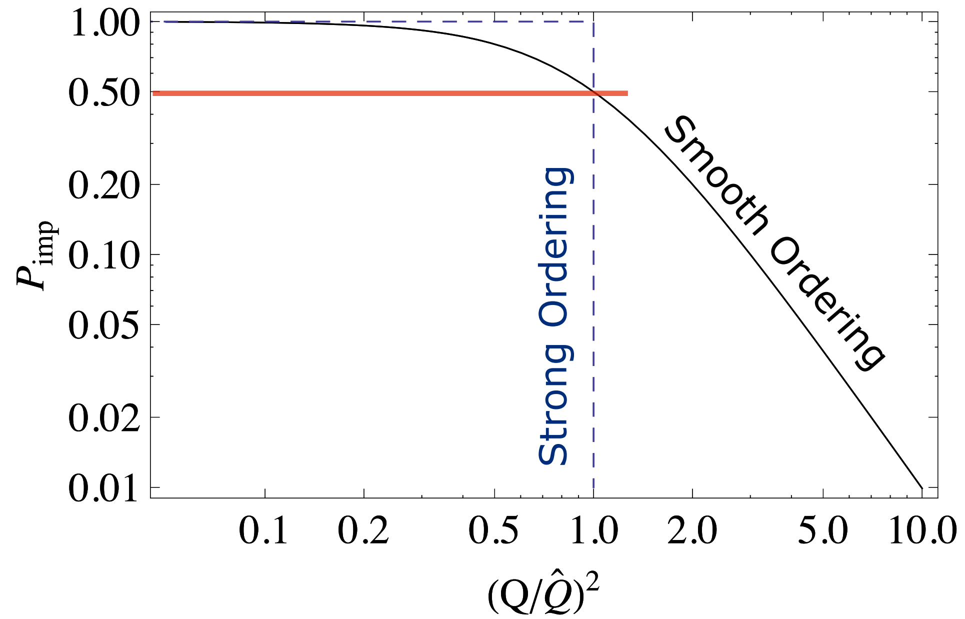
+ Simple expansions

+ Dead zones merely suppressed

Markovian Antenna Shower:
After 2 branchings: 2 terms
After 3 branchings: 3 terms
After 4 branchings: 4 terms

What is Smooth Ordering?

Giele, Kosower, Skands, PRD 84 (2011) 054003



$$P_{\rm strong} = \Theta\left(\hat{p}_\perp^2 - p_\perp^2\right)$$

$$P_{\rm smooth} = \frac{\hat{p}_\perp^2}{\hat{p}_\perp^2 + p_\perp^2} \otimes \frac{1}{p_\perp^2}$$

$A_{2\to3}$

Strongly Ordered Limit

$$\frac{1}{p_\perp^2}\left(1 - \mathcal{O}\left(\frac{p_\perp^2}{\hat{p}_\perp^2}\right)\right)$$

Strongly Unordered

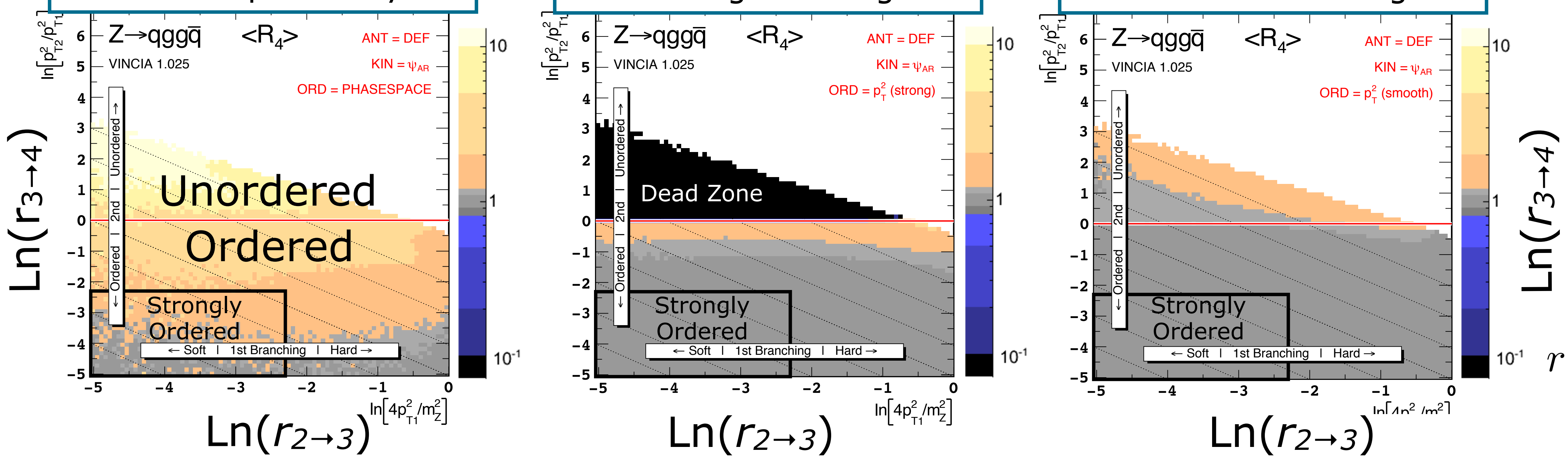
$$\frac{\hat{p}_\perp^2}{p_\perp^4}\left(1 - \mathcal{O}\left(\frac{\hat{p}_\perp^2}{p_\perp^2}\right)\right)$$

NB: Antenna Phase Spaces still nested
(antenna masses strongly ordered and decreasing)

Phase Space Only

Strong Ordering

Smooth Ordering

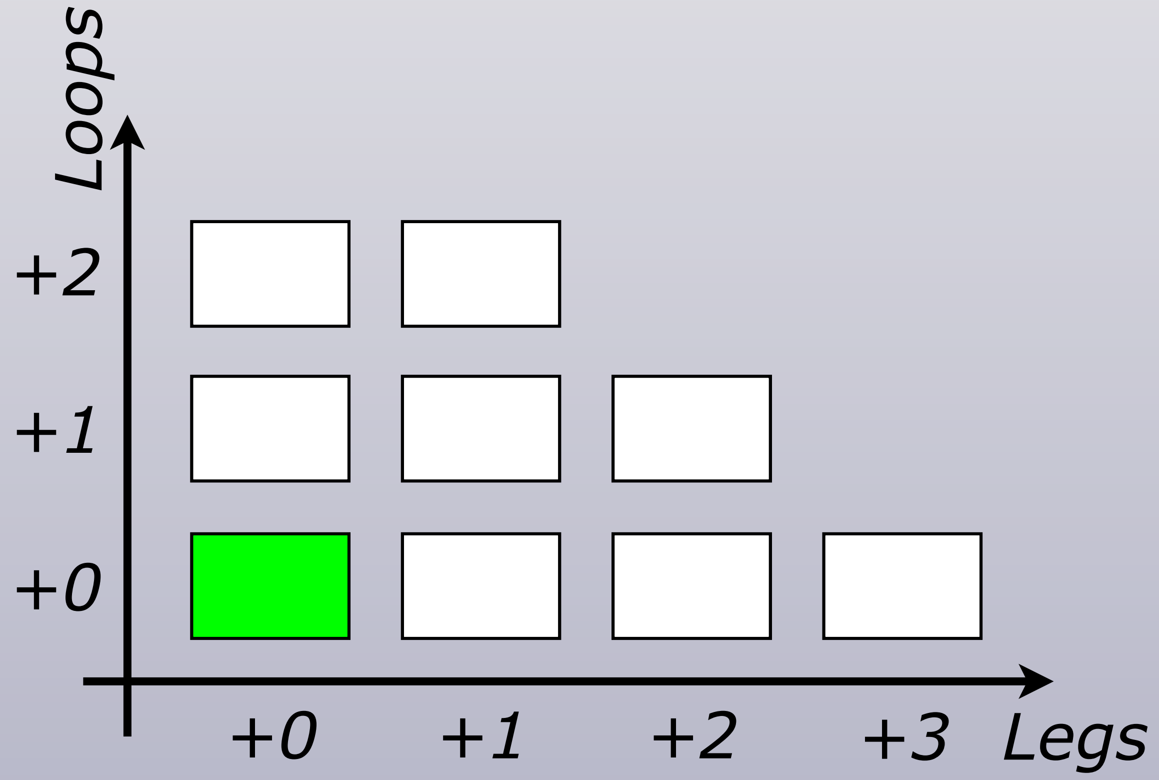


$$r = \frac{q^2}{\hat{q}^2}$$

New: Markovian pQCD

Start at Born level

$|M_F|^2$



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"Higher-Order Corrections To Timelike Jets"
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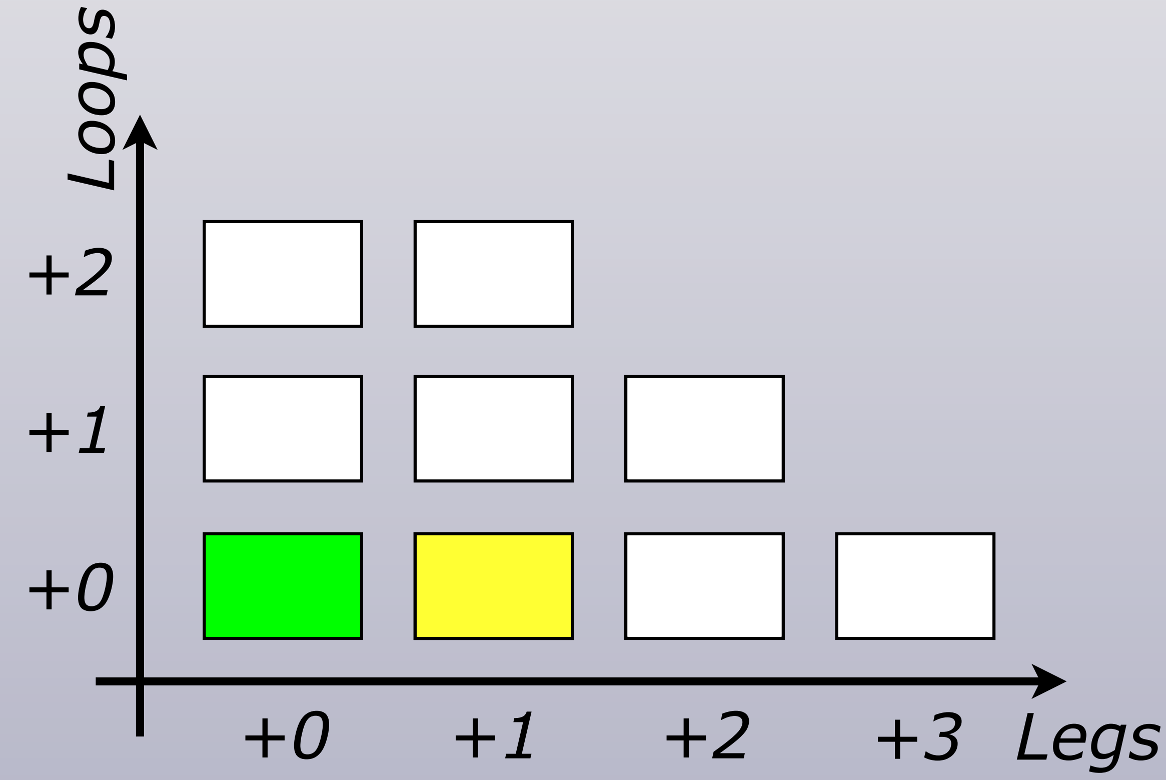
$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i \in \mathrm{ant}} a_i \, |M_F|^2$$



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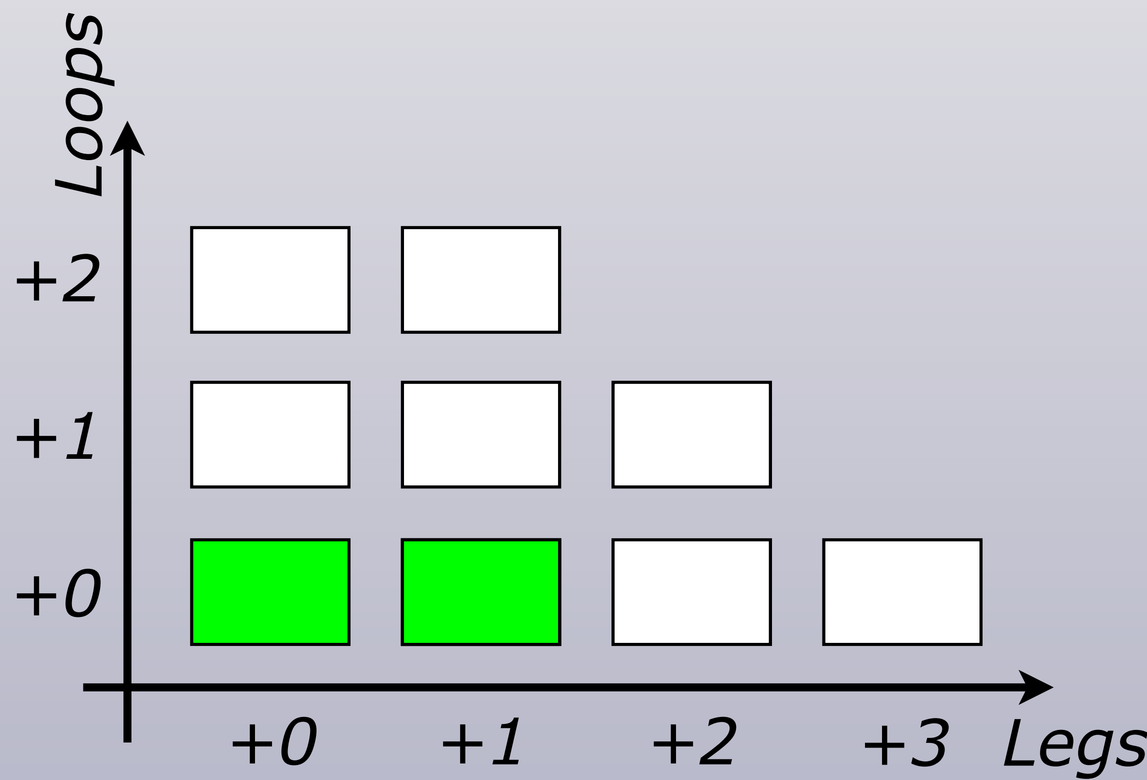

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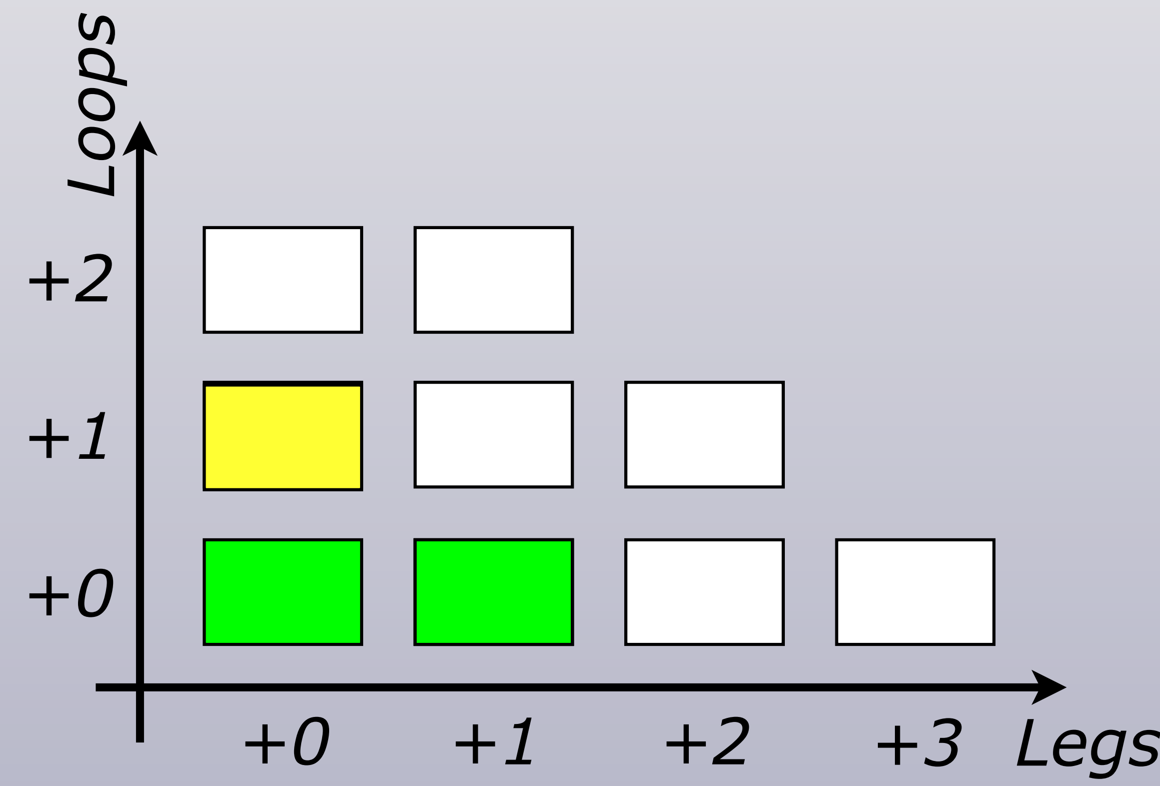

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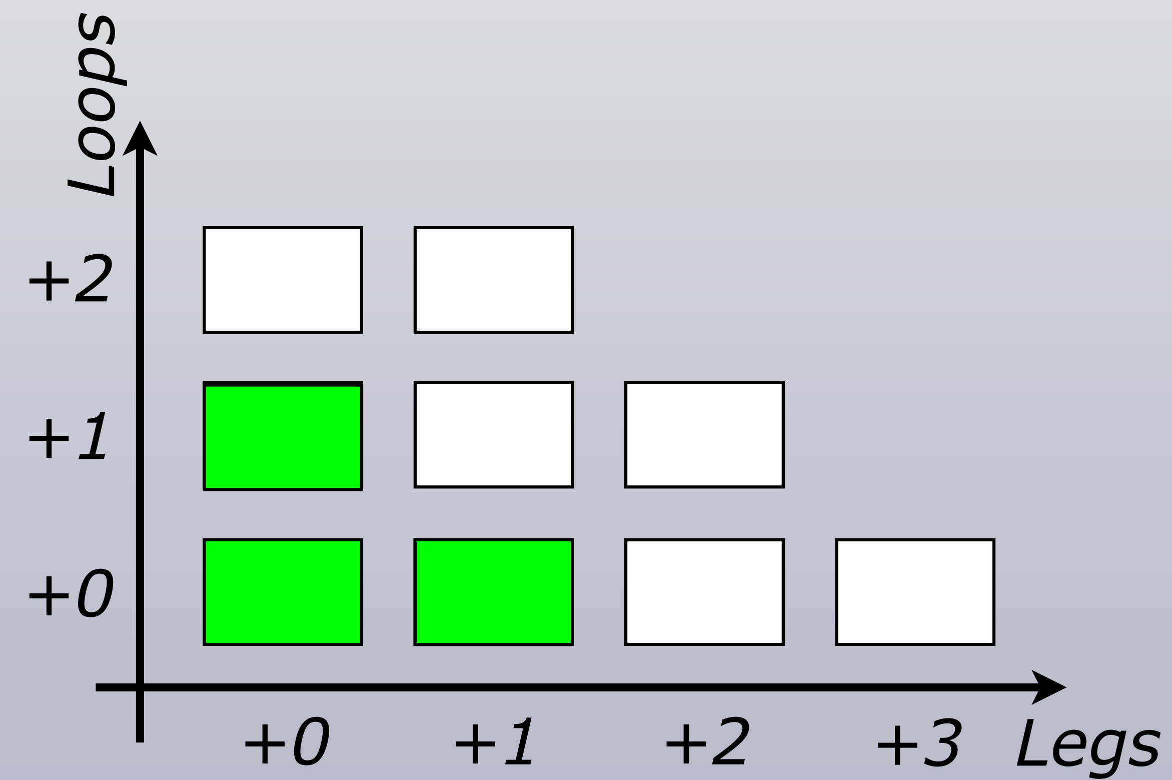

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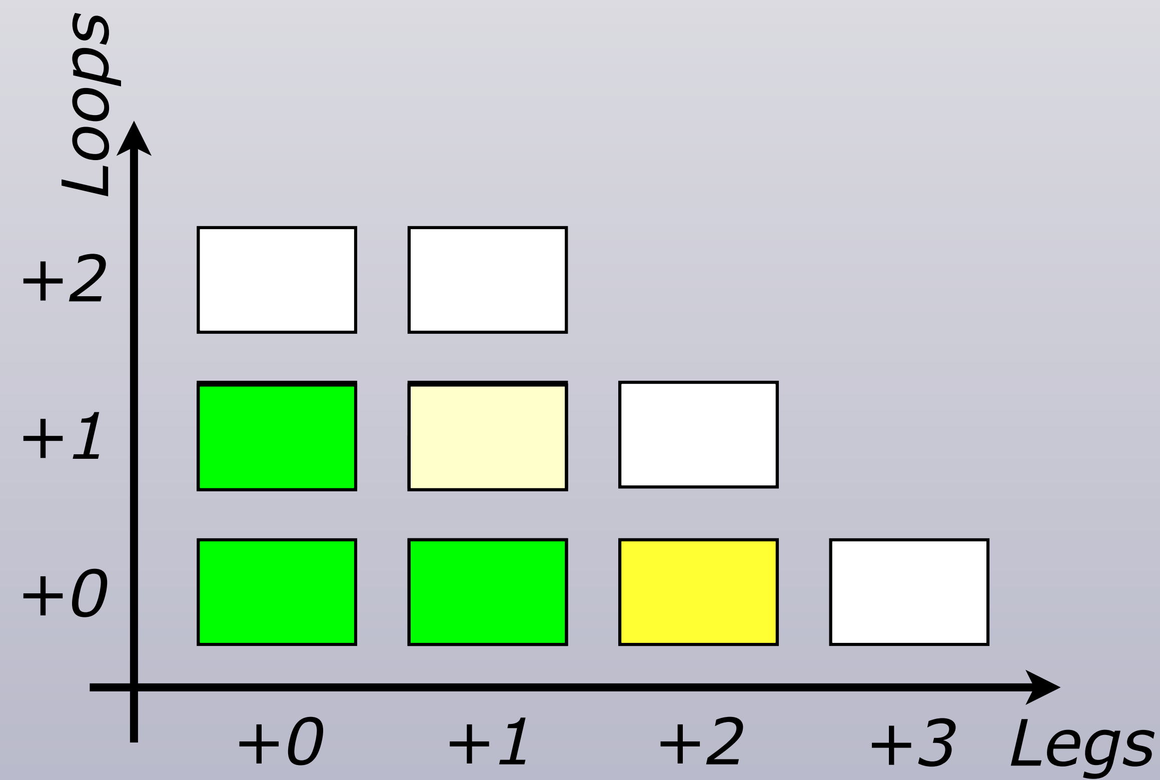

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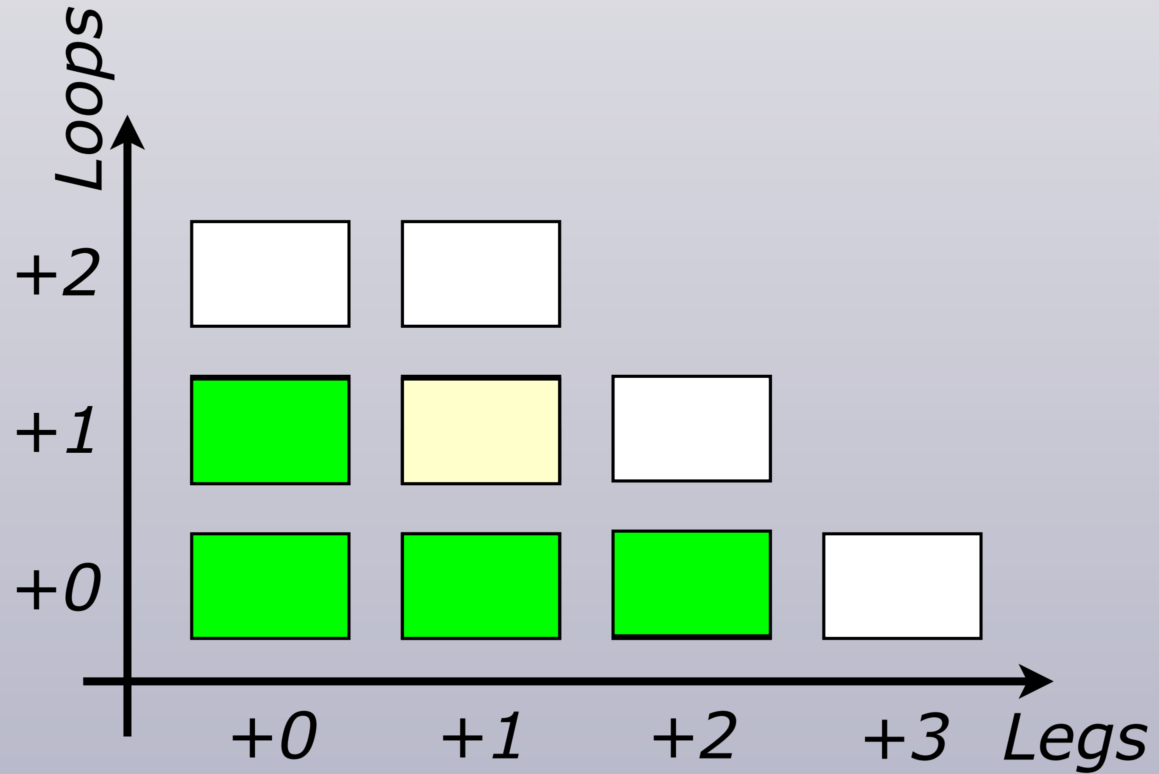

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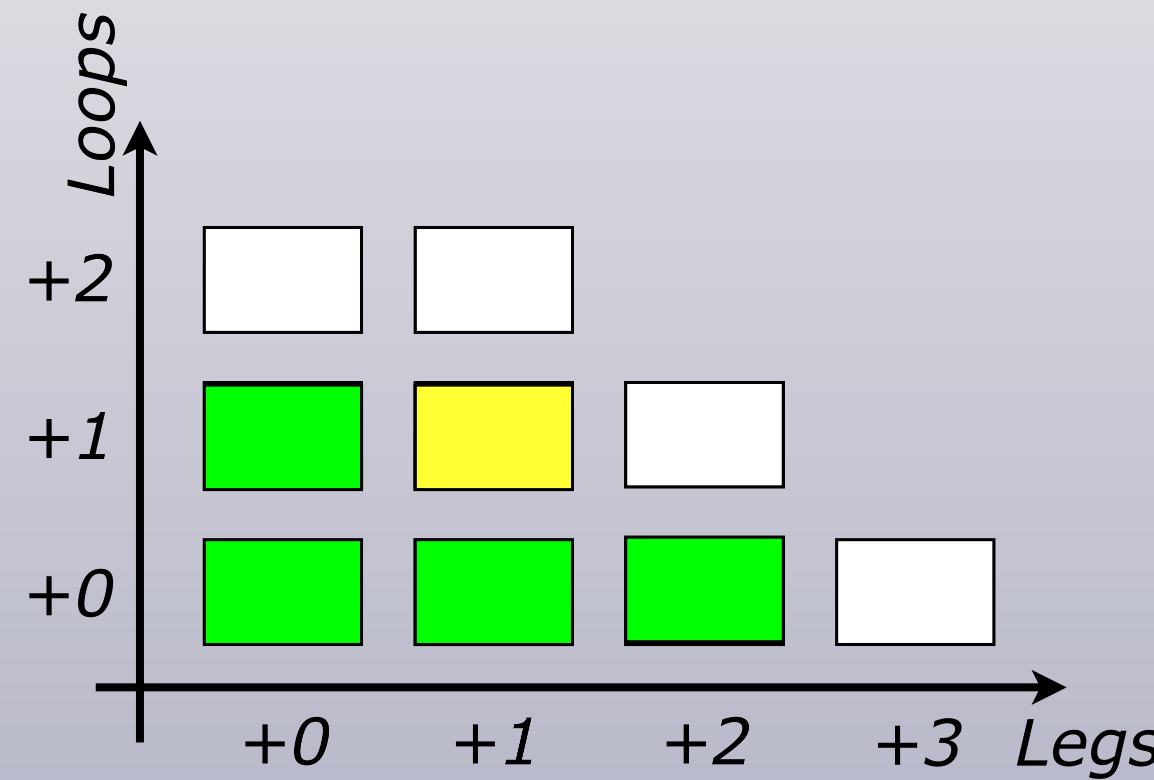

The VINCIA Code + PYTHIA 8

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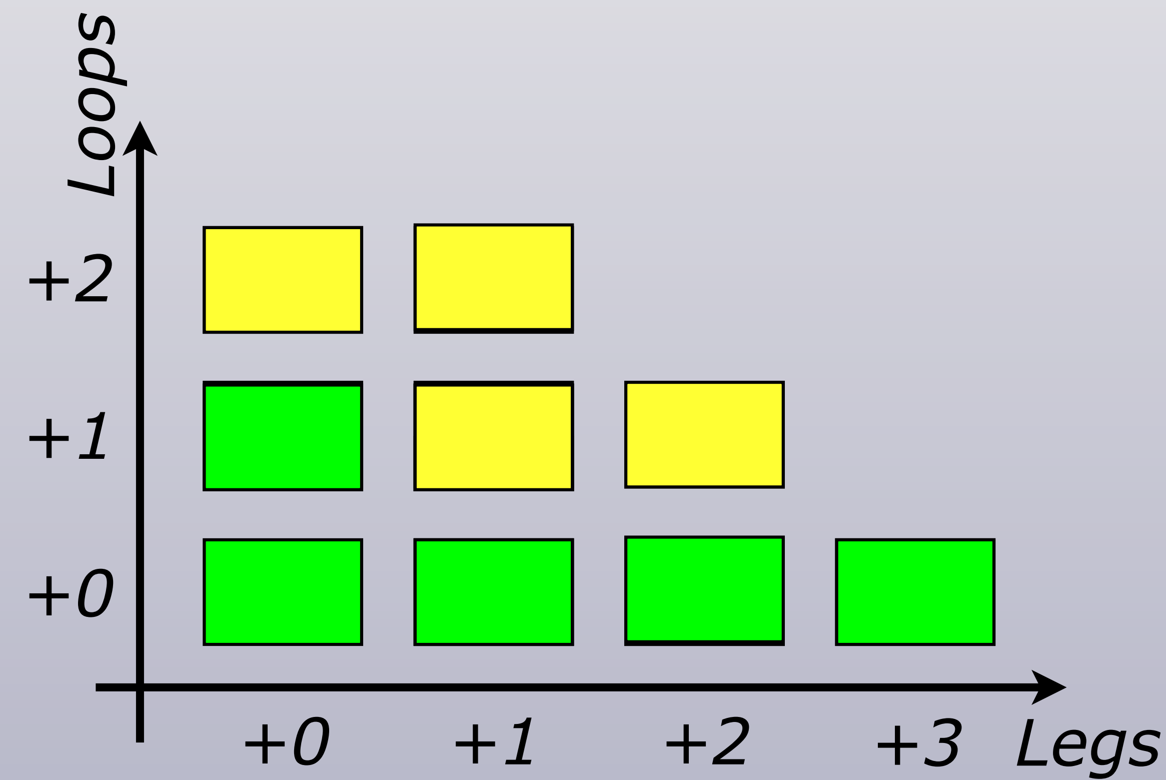

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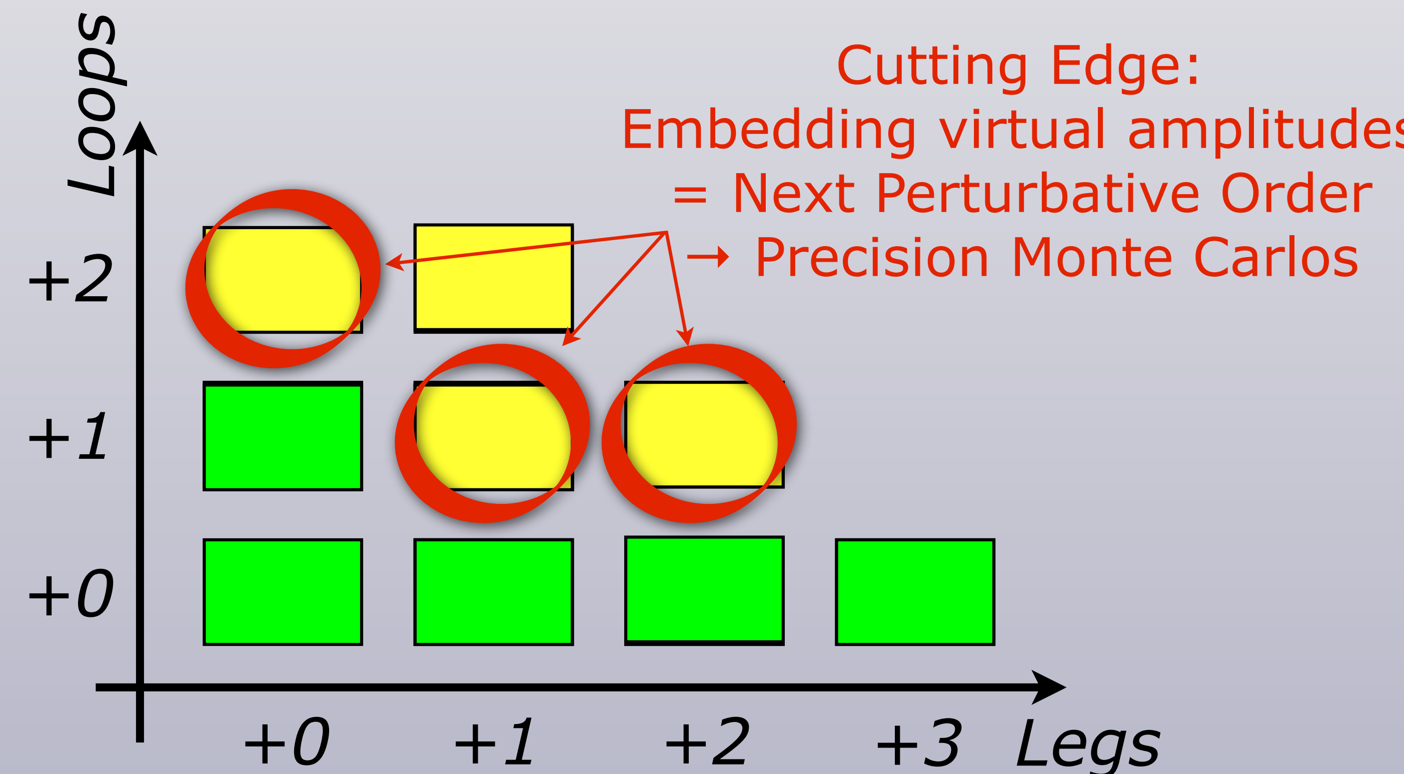

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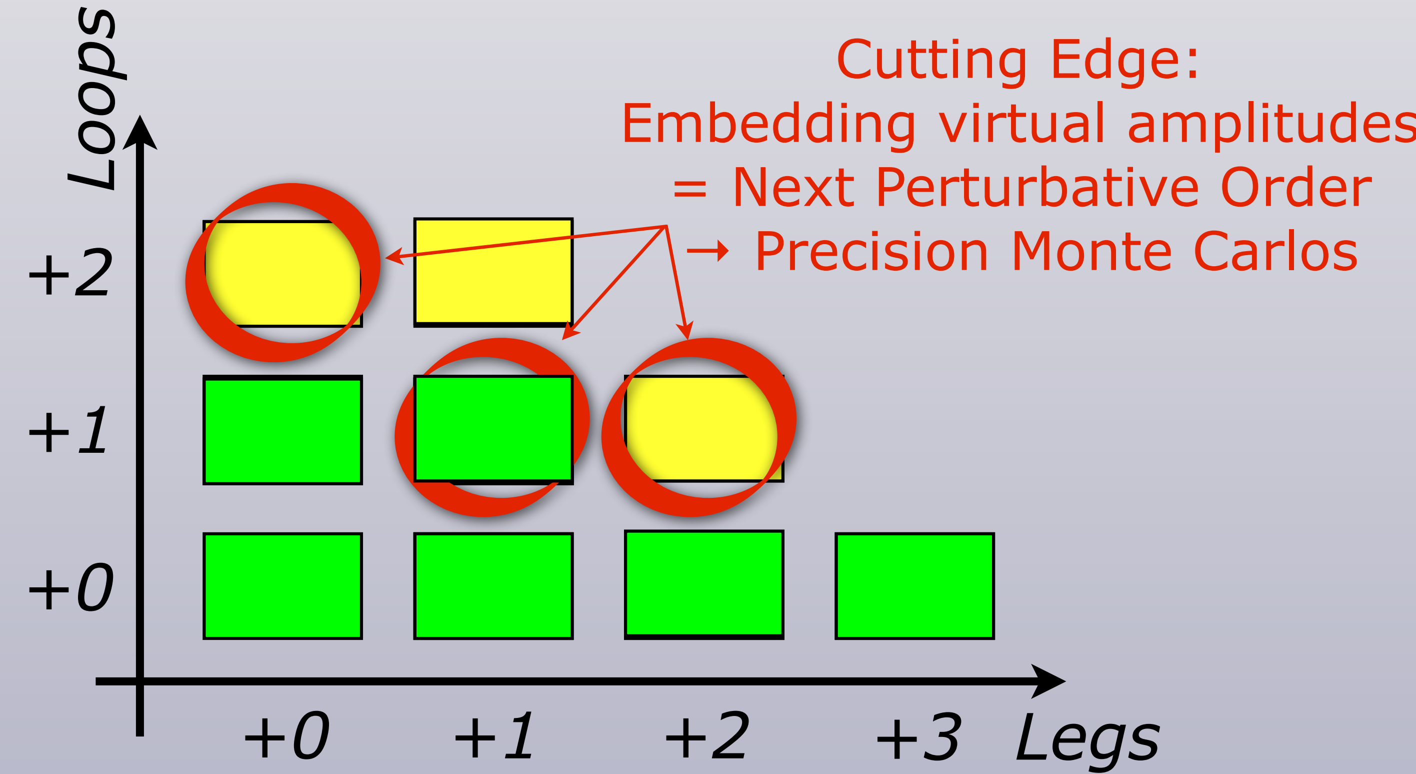

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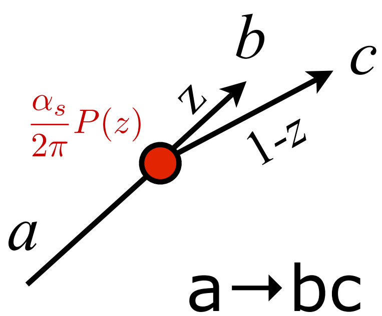
Helicities

Larkoski, Peskin, PRD 81 (2010) 054010
Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

Traditional parton showers use the standard Altarelli-Parisi kernels, P(z) = helicity sums/averages over:

$P(z)$	$++$	$-+$	$+-$	$--$
$g_+ \to gg:$	$1/z(1-z)$	$(1-z)^3/z$	$z^3/(1-z)$	0
$g_+ \to q\bar{q}:$	-	$(1-z)^2$	z^2	-
$q_+ \to qg:$	$1/(1-z)$	-	$z^2/(1-z)$	-
$q_+ \to gq:$	$1/z$	$(1-z)^2/z$	-	-



a→bc

Generalize these objects to dipole-antennae

E.g.,

$$q\bar{q} \to qg\bar{q}$$

$++ \to +++$ MHV

$++ \to +-+$ NMHV

$+- \to ++-$ P-wave

$+- \to +--$ P-wave

→ Can trace helicities through shower

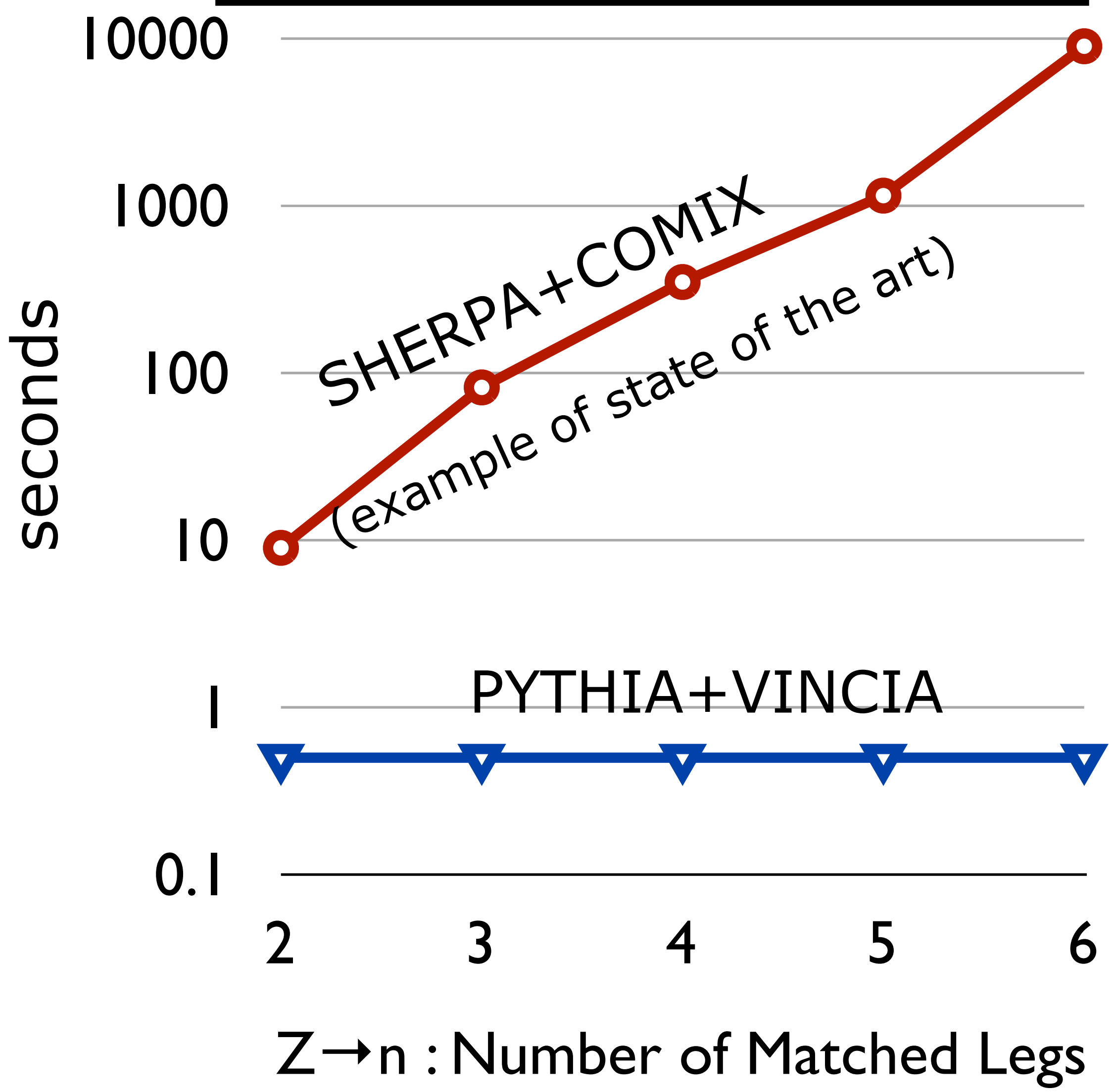
→ Eliminates contribution from unphysical helicity configurations

→ Can match to individual helicity amplitudes rather than helicity sum
→ Fast! (gets rid of another factor 2^N)

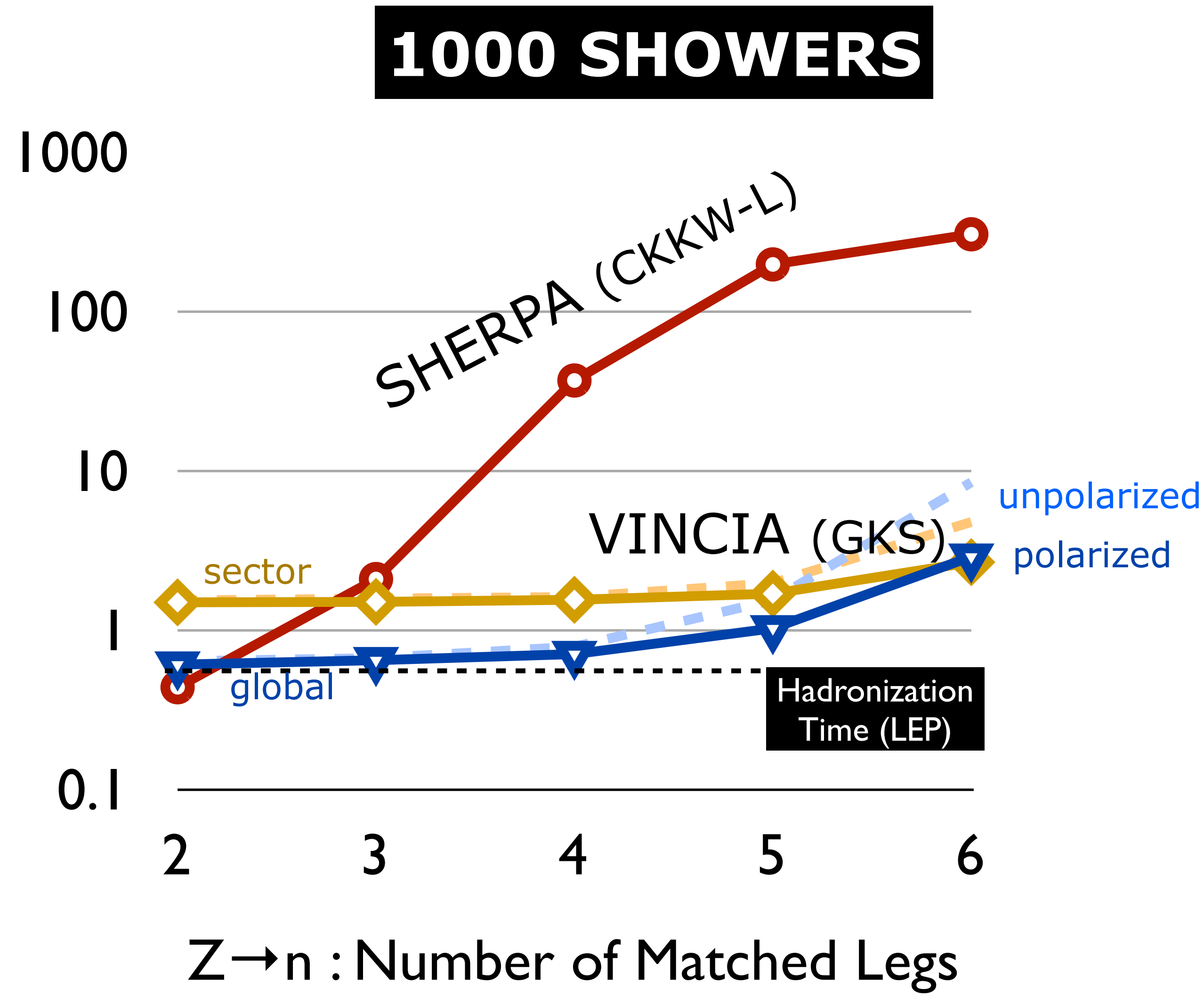
Speed

Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

1. Initialization time
(to pre-compute cross sections and warm up phase-space grids)



2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)



Z→udscb ; Hadronization OFF ; ISR OFF ; udsc MASSLESS ; b MASSIVE ; E_{CM} = 91.2 GeV ; Q_{match} = 5 GeV
SHERPA 1.4.0 (+COMIX) ; PYTHIA 8.1.65 ; VINCIA 1.0.29 + MADGRAPH 4.4.26 ;
gcc/gfortran v 4.7.1 -O2 ; single 3.06 GHz core (4GB RAM)

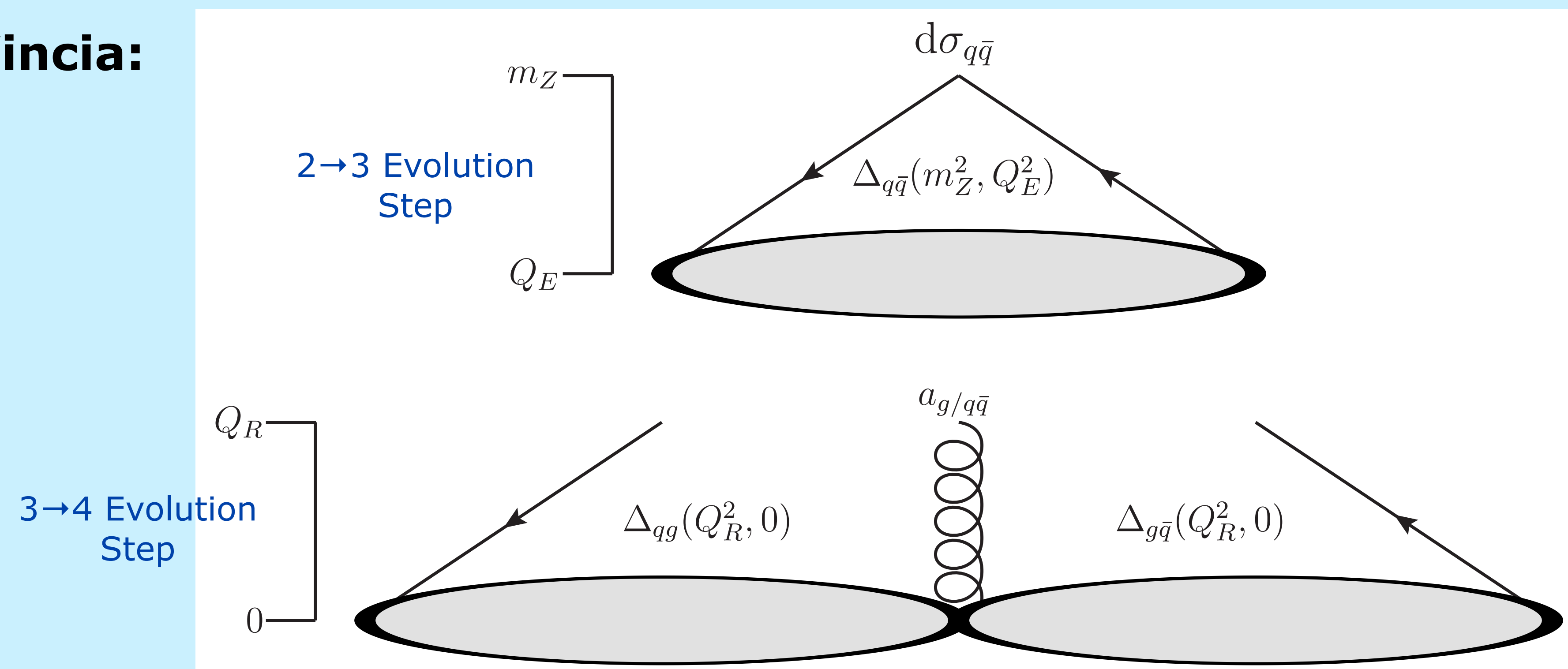
Hartgring, Laenen, Skands, arXiv:1303.4974

Getting Serious: 2nd order (1st order ~ POWHEG)

Fixed Order: Exclusive 3-jet rate (3 and only 3 jets), at Q = Qhad

$$\text{Exact} \rightarrow \underbrace{|M_1^0|^2}_{\text{Born}} + \underbrace{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}_{\text{Virtual}} + \underbrace{\int_0^{Q^2_{\mathrm{had}}} \frac{\mathrm{d}\Phi_2}{\mathrm{d}\Phi_1} |M_2^0|^2}_{\text{Unresolved Real}}$$

Vincia:



$$\text{Approximate} \rightarrow \underbrace{(1 + V_0)}_{V_0 = \alpha_s/\pi} \underbrace{|M_1^0|^2}_{\mu_R} \underbrace{\Delta_2(m_Z^2, Q_1^2)}_{2\rightarrow 3\ \text{Evolution}} \underbrace{\Delta_3(Q_{R1}^2, Q_{\mathrm{had}}^2)}_{3\rightarrow 4\ \text{Evolution}} ,$$

Loop Corrections

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

$$
\begin{aligned}
V_{1Z}(q,g,\bar{q}) \;=\; & \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right) \\
& + \frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right] \\
& + \frac{\alpha_s C_A}{2\pi}\left[8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\,A_{g/q\bar{q}}^{\mathrm{std}} + 8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{g/q\bar{q}}\right.
\end{aligned}
$$

$\mathbf{Q_1}$ = 3-parton Resolution Scale

$$
\begin{aligned}
& \left. - \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,(1-O_{Ej})\,A_{g/qg}^{\mathrm{std}} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{g/qg}\right]
\end{aligned}
$$

$\mathbf{O_{Ej}}$ = Gluon-Emission Ordering Function

$$
\begin{aligned}
& + \frac{\alpha_s n_F}{2\pi}\left[-\sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\,P_{Aj}\,A_{\bar{q}/qg}^{\mathrm{std}} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{\bar{q}/qg}\right.
\end{aligned}
$$

$\mathbf{O_{Sj}}$ = Gluon-Splitting Ordering Function

$$
\left. -\frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\right], \qquad (72)
$$

The "Ariadne" Log

Loop Corrections

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

V_0

$$
\begin{aligned}
V_{1Z}(q,g,\bar{q}) \;=\; & \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right) \\
& + \frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right] \\
& + \frac{\alpha_s C_A}{2\pi}\Bigg[8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\;A_{g/q\bar{q}}^{\mathrm{std}} \;+\; 8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/q\bar{q}}
\end{aligned}
$$

$\mathbf{Q_1}$ = 3-parton Resolution Scale

$$
- \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;(1-O_{Ej})\;A_{g/qg}^{\mathrm{std}} \;+\; \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/qg}\Bigg]
$$

$\mathbf{O_{Ej}}$ = Gluon-Emission Ordering Function

$$
+ \frac{\alpha_s n_F}{2\pi}\Bigg[-\sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\;P_{Aj}\;A_{\bar{q}/qg}^{\mathrm{std}} \;+\; \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{\bar{q}/qg}
$$

$\mathbf{O_{Sj}}$ = Gluon-Splitting Ordering Function

$$
-\frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\Bigg]\,, \qquad (72)
$$

The "Ariadne" Log

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

V0 μR

$$
\begin{aligned}
V_{1Z}(q,g,\bar{q}) \quad = \quad & \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right) \\
& + \frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right] \\
& + \frac{\alpha_s C_A}{2\pi}\left[8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\;A_{g/q\bar{q}}^{\mathrm{std}} \;+\; 8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/q\bar{q}}\right. \\
& \qquad \left. - \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;(1-O_{Ej})\;A_{g/qg}^{\mathrm{std}} \;+\; \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/qg}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-\sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\;P_{Aj}\;A_{\bar{q}/qg}^{\mathrm{std}} \;+\; \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{\bar{q}/qg}\right. \\
& \qquad \left. -\frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\right], \qquad (72)
\end{aligned}
$$

Q_1 = 3-parton Resolution Scale

O_{Ej} = Gluon-Emission Ordering Function

O_{Sj} = Gluon-Splitting Ordering Function

The "Ariadne" Log

Loop Corrections

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

V0 μR

$$
\begin{aligned}
V_{1Z}(q,g,\bar{q}) \;=\; & \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right) \\
& + \frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right] \\
& + \frac{\alpha_s C_A}{2\pi}\bigg[8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\; A_{g/q\bar{q}}^{\mathrm{std}} \;+\; 8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/q\bar{q}} \\
& \qquad - \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\;(1-O_{Ej})\;A_{g/qg}^{\mathrm{std}} \;+\; \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/qg}\bigg] \\
& + \frac{\alpha_s n_F}{2\pi}\bigg[-\sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\;P_{Aj}\;A_{\bar{q}/qg}^{\mathrm{std}} \;+\; \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{\bar{q}/qg} \\
& \qquad - \frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\bigg]\,, \qquad (72)
\end{aligned}
$$

Gluon Emission IR Singularity (std antenna integral)

Gluon Splitting IR Singularity (std antenna integral)

Q_1 = 3-parton Resolution Scale

O_{Ej} = Gluon-Emission Ordering Function

O_{Sj} = Gluon-Splitting Ordering Function

The "Ariadne" Log

Loop Corrections

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

$$
\begin{aligned}
V_{1Z}(q,g,\bar{q}) \;=\;& \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right)\\
&+ \frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right]\\
&+ \frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right]\\
&+ \frac{\alpha_s C_A}{2\pi}\Bigg[8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\;A_{g/q\bar{q}}^{\mathrm{std}} + 8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/q\bar{q}}\\
&\quad - \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;(1-O_{Ej})\;A_{g/qg}^{\mathrm{std}} + \sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/qg}\Bigg]\\
&+ \frac{\alpha_s n_F}{2\pi}\Bigg[-\sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\;P_{Aj}\;A_{\bar{q}/qg}^{\mathrm{std}} + \sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{\bar{q}/qg}\\
&\quad - \frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\Bigg], \qquad (72)
\end{aligned}
$$

V_0

μ_R

Gluon Emission IR Singularity (std antenna integral)

Gluon Splitting IR Singularity (std antenna integral)

Standard (universal) 2→3 Sudakov Logs

Q_1 = 3-parton Resolution Scale

O_{Ej} = Gluon-Emission Ordering Function

O_{Sj} = Gluon-Splitting Ordering Function

The “Ariadne” Log

Loop Corrections

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

V0 μR

$$V_{1Z}(q,g,\bar{q}) = \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right)$$

Gluon Emission IR Singularity (std antenna integral)

$$+\frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right]$$

Gluon Splitting IR Singularity (std antenna integral)

$$+\frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right]$$

Standard (universal) 2→3 Sudakov Logs

$$+\frac{\alpha_s C_A}{2\pi}\left[8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\;A_{g/q\bar{q}}^{\mathrm{std}} + 8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/q\bar{q}}\right.$$

Q1 = 3-parton Resolution Scale

Standard (universal) 3→4 Sudakov Logs: CA

$$\left. -\sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;(1-O_{Ej})\;A_{g/qg}^{\mathrm{std}} + \sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{g/qg}\right]$$

OEj = Gluon-Emission Ordering Function

$$+\frac{\alpha_s n_F}{2\pi}\left[-\sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\;P_{Aj}\;A_{\bar{q}/qg}^{\mathrm{std}} + \sum_{j=1}^{2}8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\;\delta A_{\bar{q}/qg}\right.$$

OSj = Gluon-Splitting Ordering Function

$$\left. -\frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\right], \qquad (72)$$

The "Ariadne" Log

Loop Corrections

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

$$
\begin{aligned}
V_{1Z}(q,g,\bar{q}) \;=\;& \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \underbrace{\frac{\alpha_s}{\pi}}_{V_0} - \frac{\alpha_s}{2\pi}\underbrace{\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu^2_{\mathrm{ME}}}{\mu^2_{\mathrm{PS}}}\right)}_{\mu_R} \\
&+ \frac{\alpha_s C_A}{2\pi}\left[-2I^{(1)}_{qg}(\epsilon,\mu^2/s_{qg}) - 2I^{(1)}_{qg}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right] \\
&+ \frac{\alpha_s n_F}{2\pi}\left[-2I^{(1)}_{qg,F}(\epsilon,\mu^2/s_{qg}) - 2I^{(1)}_{g\bar{q},F}(\epsilon,\mu^2/s_{qg}) - 1\right] \\
&+ \frac{\alpha_s C_A}{2\pi}\left[8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\,A^{\mathrm{std}}_{g/q\bar{q}} + 8\pi^2\int_{Q_1^2}^{m_Z^2}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{g/q\bar{q}}\right. \\
&\left. - \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,(1-O_{Ej})\,A^{\mathrm{std}}_{g/qg} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{g/qg}\right] \\
&+ \frac{\alpha_s n_F}{2\pi}\left[-\sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\,P_{Aj}\,A^{\mathrm{std}}_{\bar{q}/qg} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j}\mathrm{d}\Phi_{\mathrm{ant}}\,\delta A_{\bar{q}/qg}\right. \\
&\left. -\frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\right], \qquad (72)
\end{aligned}
$$

Gluon Emission IR Singularity (std antenna integral)

Gluon Splitting IR Singularity (std antenna integral)

Standard (universal) 2→3 Sudakov Logs

Q_1 = 3-parton Resolution Scale

Standard (universal) 3→4 Sudakov Logs: C_A

O_{Ej} = Gluon-Emission Ordering Function

Standard (universal) 3→4 Sudakov Logs: n_F

O_{Sj} = Gluon-Splitting Ordering Function

↓ appendix of our paper + functions in the code

The "Ariadne" Log

Loop Corrections

Hartgring, Laenen, Skands, arXiv:1303.4974

NLO Correction: Subtract and correct by difference

V_0 μ_R

$$
\begin{aligned}
V_{1Z}(q,g,\bar{q}) \;=\; & \left[\frac{2\,\mathrm{Re}[M_1^0 M_1^{1*}]}{|M_1^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right) \\
& + \frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right] \\
& + \frac{\alpha_s C_A}{2\pi}\left[8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\, A_{g/q\bar{q}}^{\mathrm{std}} + 8\pi^2\int_{Q_1^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\, \delta A_{g/q\bar{q}}\right. \\
& \left. - \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\,(1-O_{Ej})\, A_{g/qg}^{\mathrm{std}} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\, \delta A_{g/qg}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-\sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}(1-O_{Sj})\, P_{Aj}\, A_{\bar{q}/qg}^{\mathrm{std}} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\, \delta A_{\bar{q}/qg}\right. \\
& \left. - \frac{1}{6}\,\frac{s_{qg}-s_{g\bar{q}}}{s_{qg}+s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\right], \qquad (72)
\end{aligned}
$$

Gluon Emission IR Singularity (std antenna integral)

Gluon Splitting IR Singularity (std antenna integral)

Standard (universal) 2→3 Sudakov Logs

Q_1 = 3-parton Resolution Scale

Standard (universal) 3→4 Sudakov Logs: C_A

O_{Ej} = Gluon-Emission Ordering Function

Standard (universal) 3→4 Sudakov Logs: n_F

O_{Sj} = Gluon-Splitting Ordering Function

δA: Integrals over ME/PS corrections Done numerically

appendix of our paper
+ functions in the code

The "Ariadne" Log

1) IR Limits

Hartgring, Laenen, Skands, arXiv:1303.4974

Pole-subtracted one-loop matrix element

$$\text{SVirtual} = \left[\frac{2\,\mathrm{Re}[M_3^0 M_3^{1*}]}{|M_3^0|^2}\right]^{\mathrm{LC}} + \frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right]$$
$$+ \frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 1\right]$$

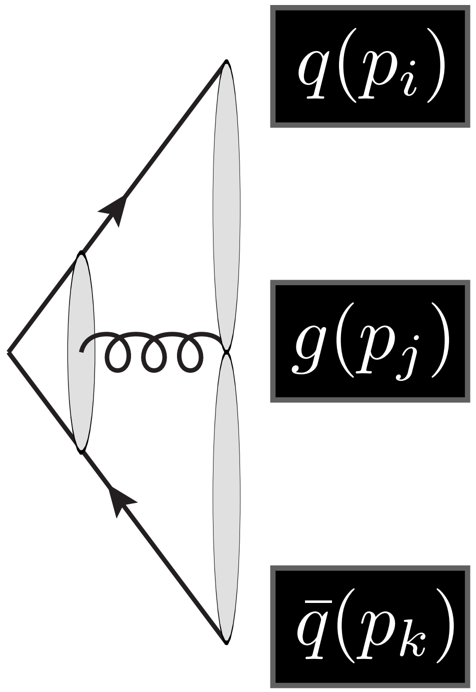
SVirtual	soft	$\left(-L^2 - \frac{10}{3}L - \frac{\pi^2}{6}\right) C_A + \frac{1}{3} n_F L$	$s_{qg} = s_{g\bar{q}} = y \to 0$
	hard collinear	$-\frac{5}{3} L C_A + \frac{1}{6} n_F L$	$s_{qg} = y \to 0,\, s_{g\bar{q}} \to s$

Second-Order Antenna Shower Expansion:

		strong	smooth	V_{3Z}
$p_\perp$	soft	$\left(L^2 - \frac{1}{3}L + \frac{\pi^2}{6}\right) C_A + \frac{1}{3} n_F L$	$\left(L^2 - \frac{1}{3}L - \frac{\pi^2}{6}\right) C_A + \frac{1}{3} n_F L$	$-\beta_0 L$
	hard collinear	$-\frac{1}{6} L C_A + \frac{1}{6} n_F L$	$\left(-\frac{1}{6}L - \frac{\pi^2}{6}\right) C_A + \frac{1}{6} n_F L$	$-\frac{1}{2}\beta_0 L$
m_D	soft	$\left(L^2 + \frac{3}{2}L - \frac{\pi^2}{6}\right) C_A$	$\left(L^2 + \frac{3}{2}L - \frac{\pi^2}{6}\right) C_A$	$-\frac{1}{2}\beta_0 L$
	hard collinear	$-\frac{1}{6} L C_A + \frac{1}{6} n_F L$	$\left(-\frac{1}{6}L - \frac{\pi^2}{3}\right) C_A + \frac{1}{6} n_F L$	$-\frac{1}{2}\beta_0 L$

Hartgring, Laenen, Skands, arXiv:1303.4974

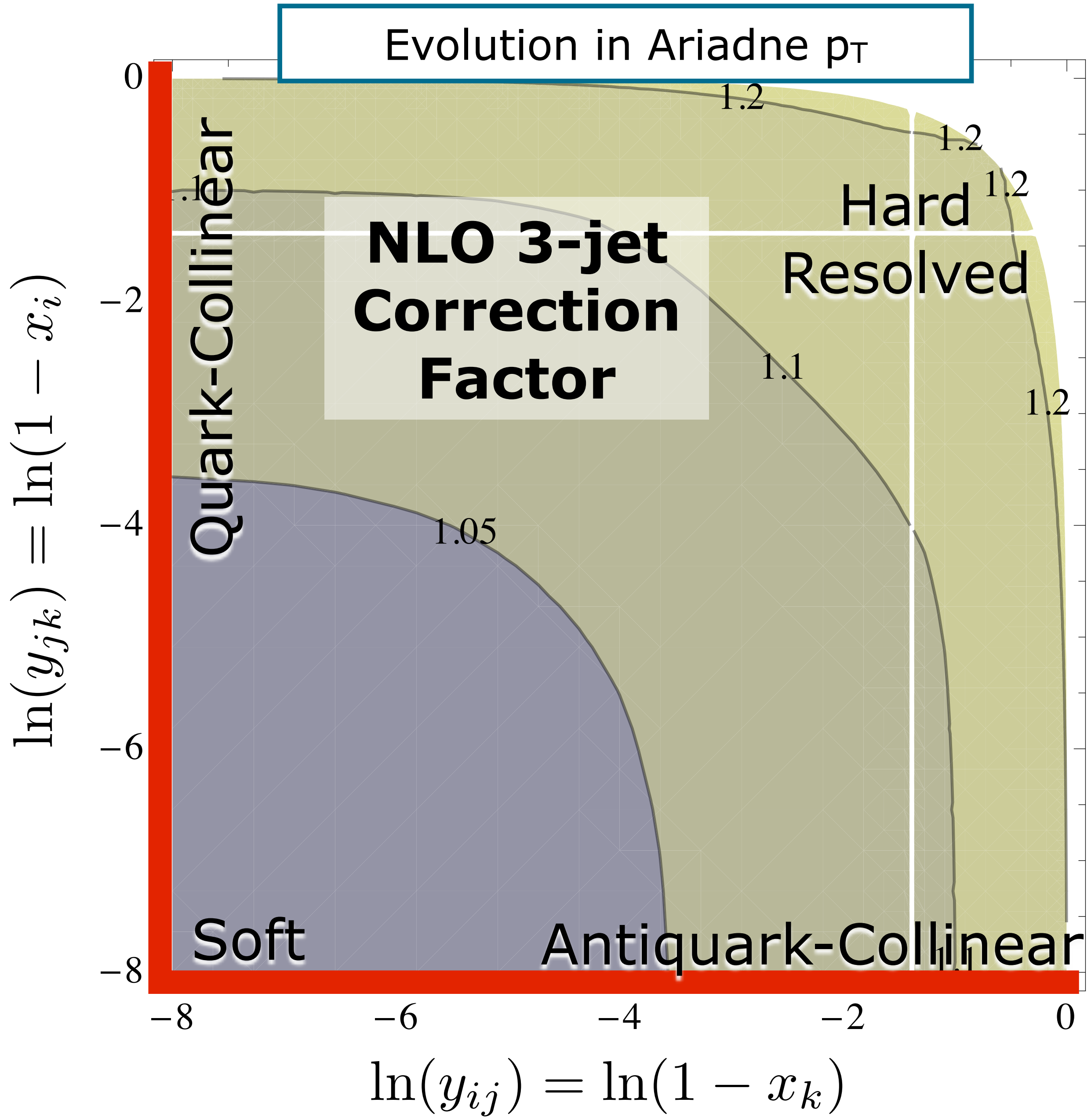
Z → Jets (NLO$_{2,3}$ + LO$_{2,3,4,5}$ + Shower)



Scaled Invariants

$$y_{ij} = \frac{2(p_i \cdot p_j)}{M_Z^2}$$

→ 0 when i || j
& when E_j → 0



Size of NLO Correction:
over 3-parton Phase Space

$\mu_R = p_T$

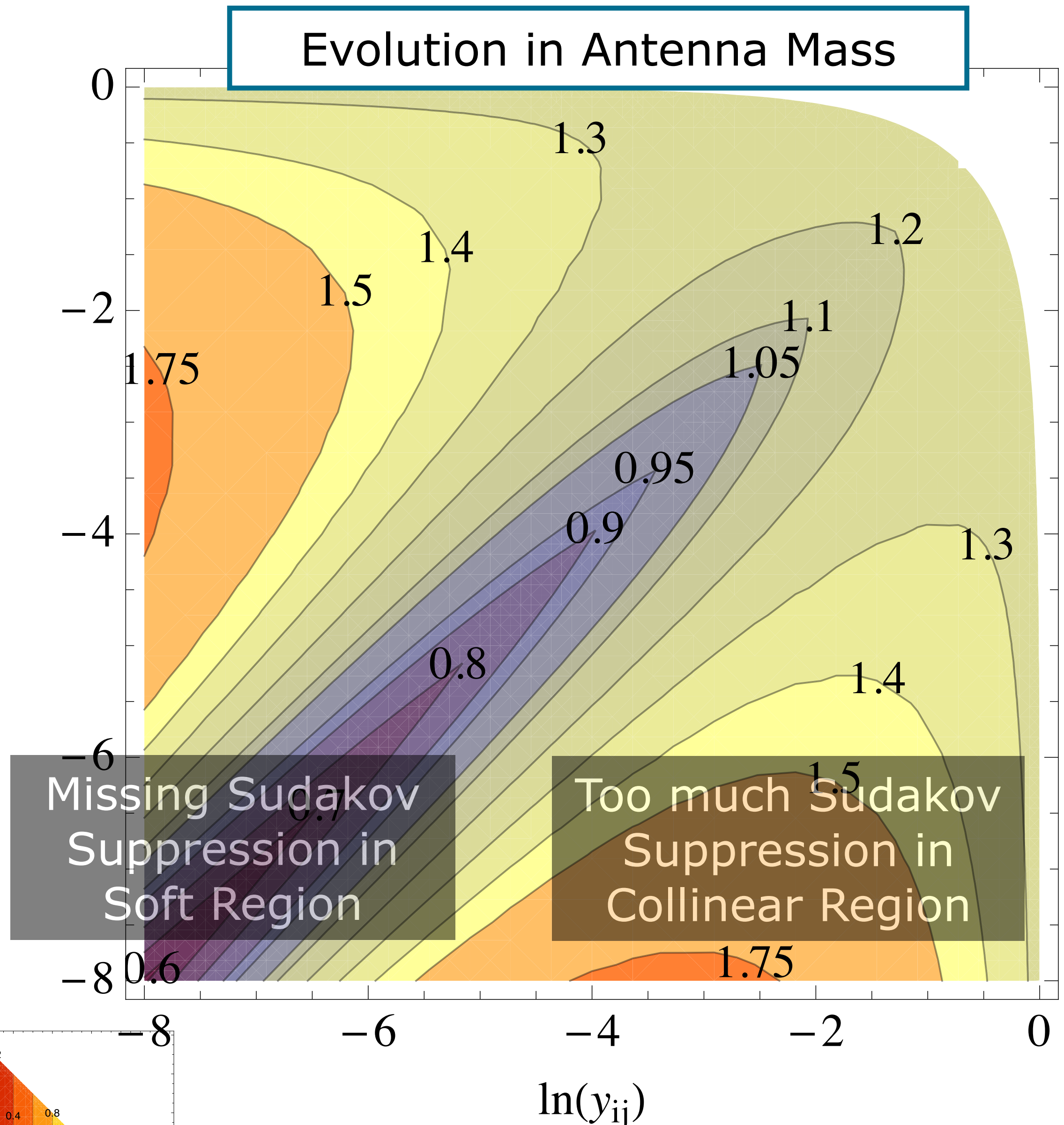
$\alpha_S(M_Z) = 0.12$

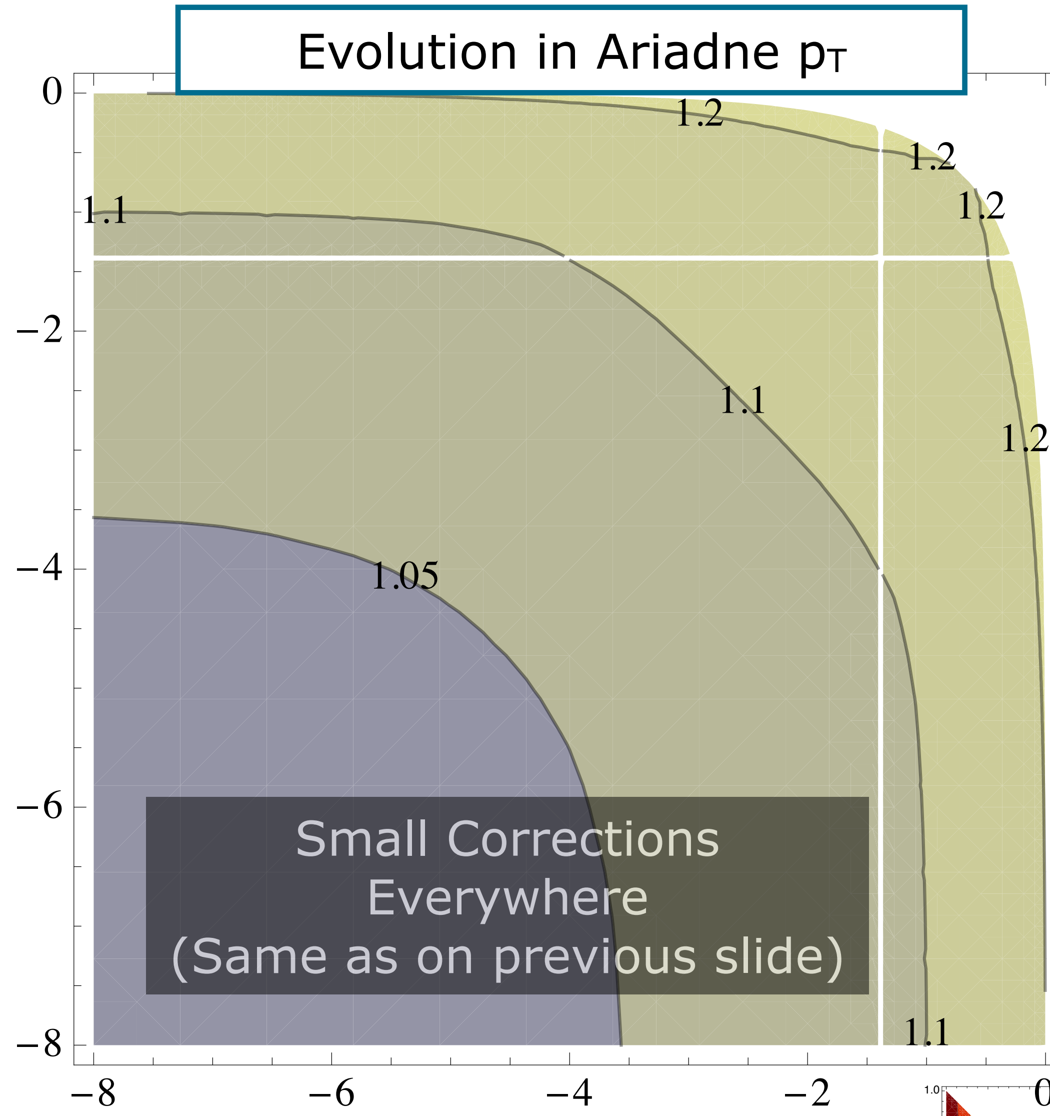
With CMW factor

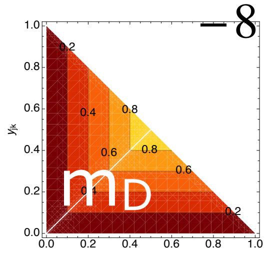
Evolution Variable

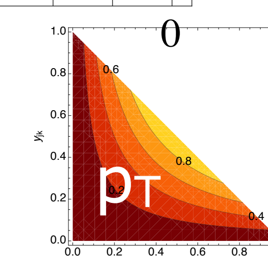
Hartgring, Laenen, Skands, arXiv:1303.4974

The choice of evolution variable (Q)





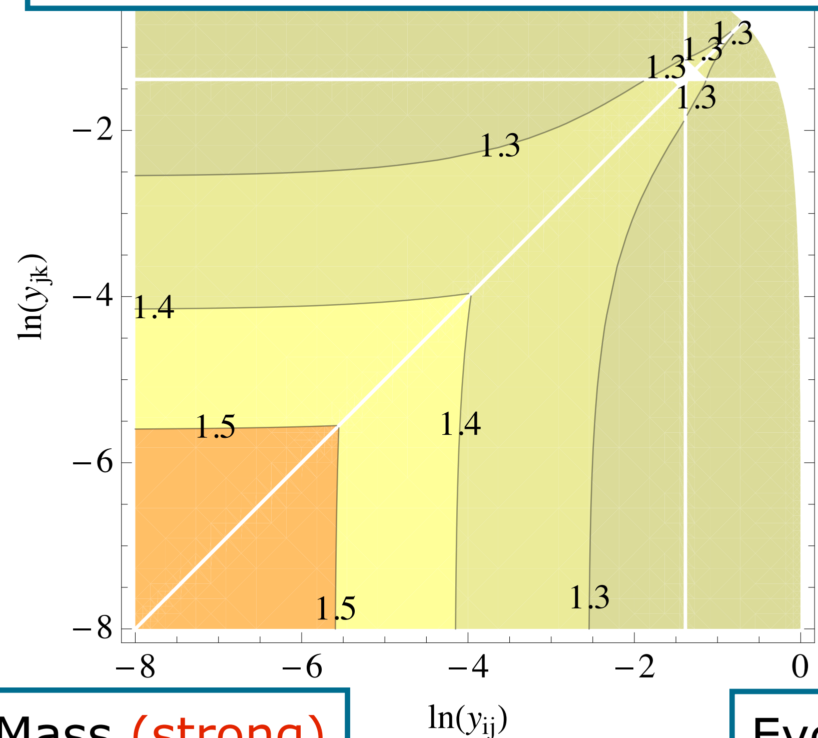




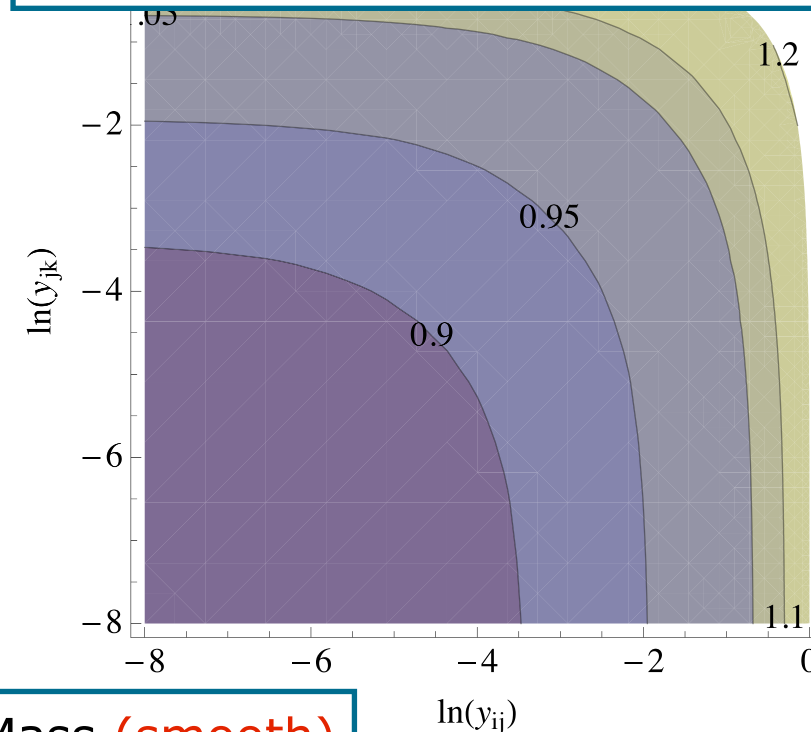
Parameters: $\alpha_S(M_Z) = 0.12$, $\mu_R = p_{Tg}$

Further Examples

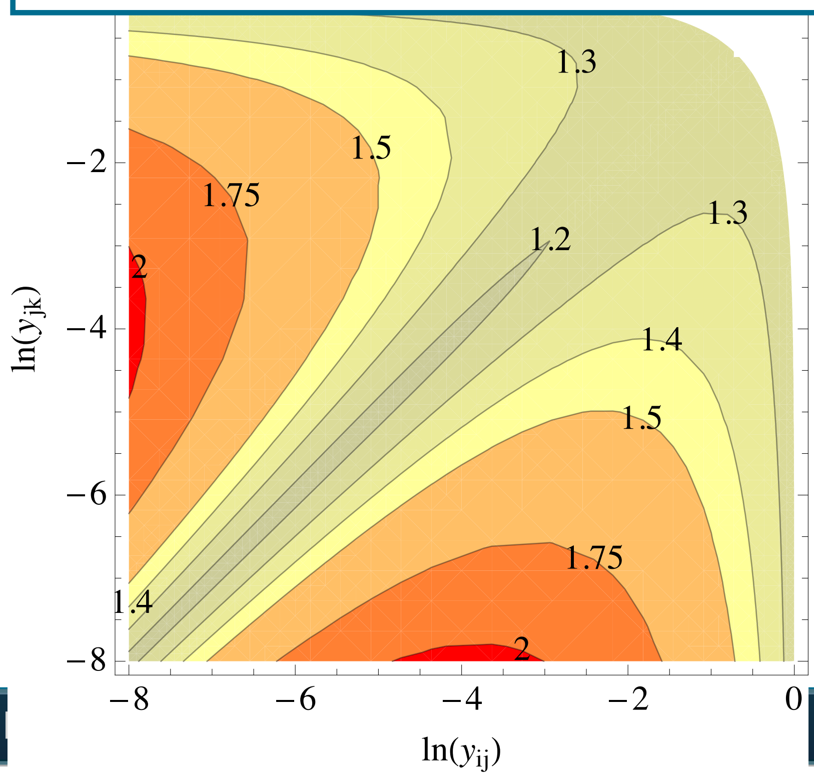
Evolution: pT (strong)
μ_R = Antenna Mass



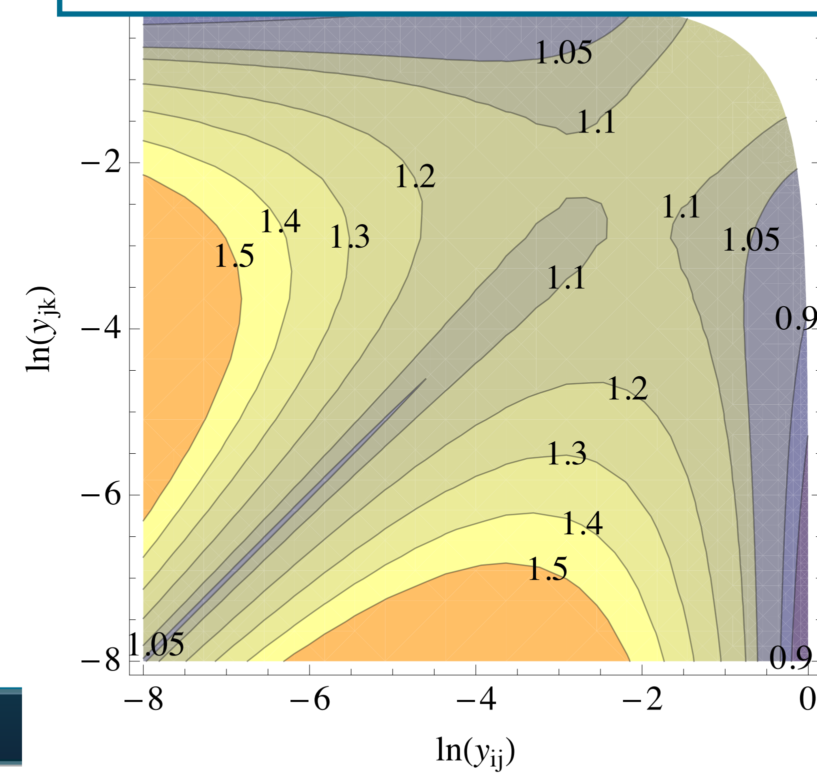
Evolution: pT (smooth)
μ_R = pT



Evolution: Antenna Mass (strong)
μ_R = Antenna Mass



Evolution: Antenna Mass (smooth)
μ_R = Antenna Mass



All plots use:
$\alpha_S = 0.12$

The proof of the pudding

Hartgring, Laenen, Skands, arXiv:1303.4974

New VINCIA NLO Tune

$\alpha_s(M_Z)^{CMW} = 0.122$

(with 2-loop running)

LO Tunes

(both VINCIA and PYTHIA)

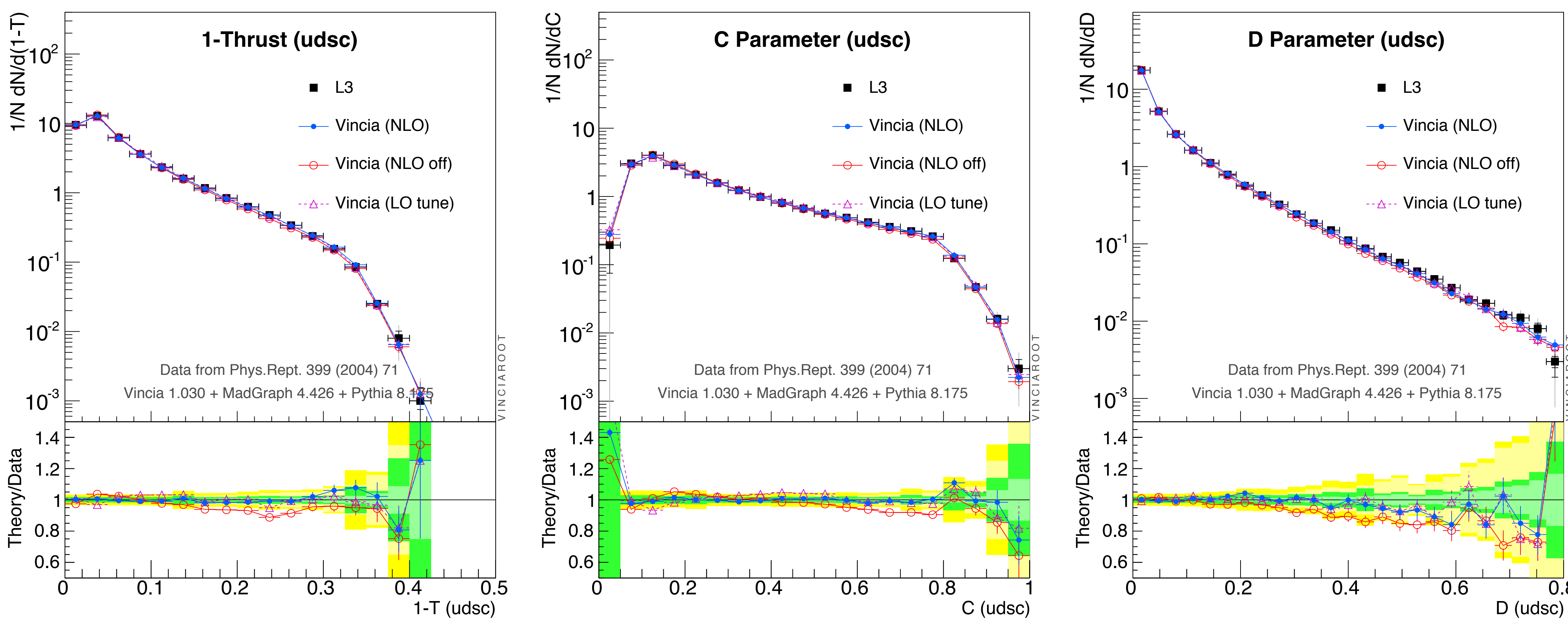
$\alpha_s(M_Z)^{MSbar} \sim 0.139$

(LO matrix elements give similar values, and also LO PDFs)

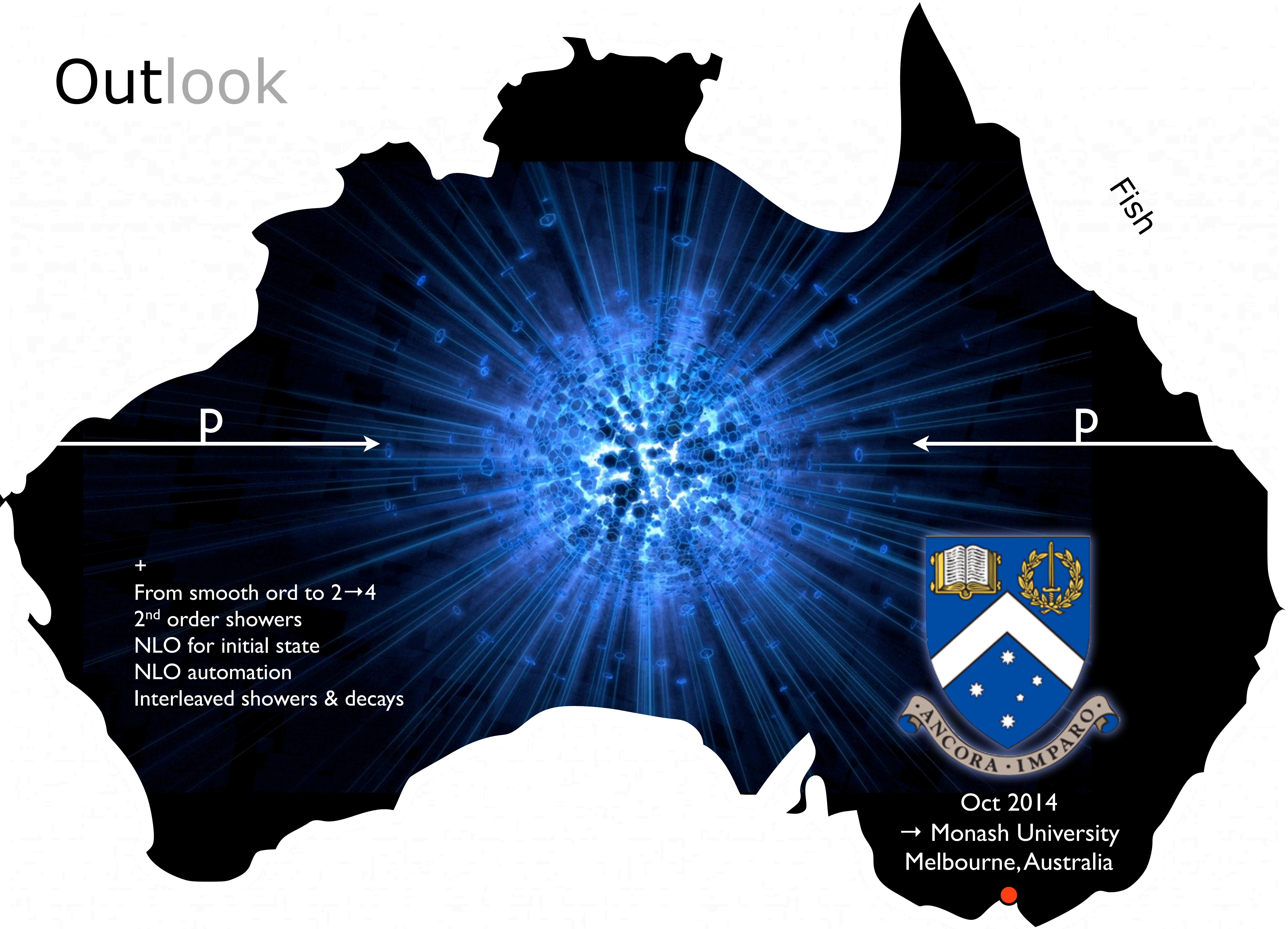
$\langle\chi^2\rangle$ **Shapes**	T	C	D	B_W	B_T
PYTHIA 8	0.4	0.4	0.6	0.3	0.2
VINCIA (LO)	0.2	0.4	0.4	0.3	0.3
VINCIA (NLO)	0.2	0.2	0.6	0.3	0.2

$\langle\chi^2\rangle$ **Frag**	N_{ch}	x	Mesons	Baryons
PYTHIA 8	0.8	0.4	0.9	1.2
VINCIA (LO)	0.0	0.5	0.3	0.6
VINCIA (NLO)	0.1	0.7	0.2	0.6

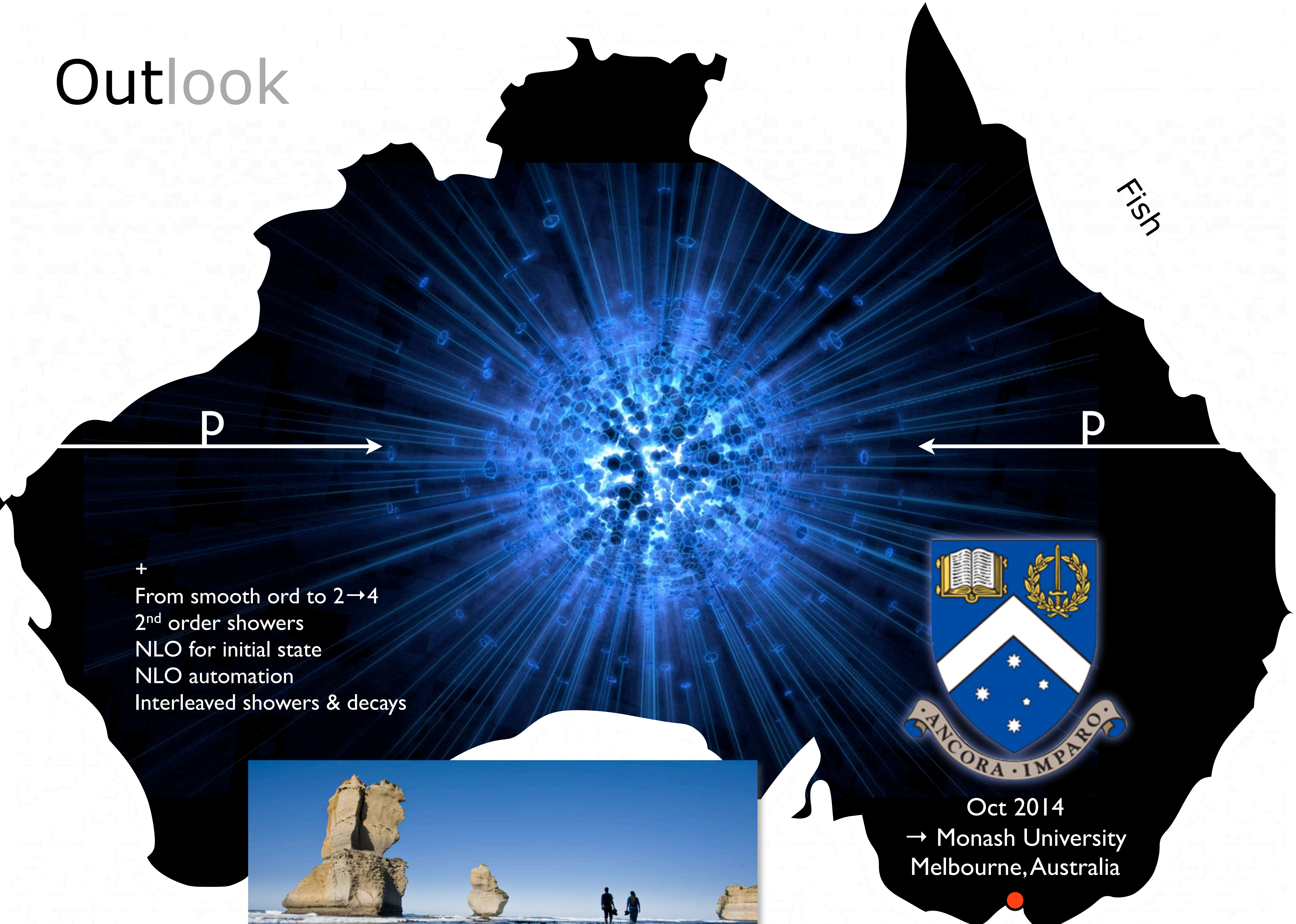
$\langle\chi^2\rangle$ **Jets**	r_{1j}^{exc}	$\ln(y_{12})$	r_{2j}^{exc}	$\ln(y_{23})$	r_{3j}^{exc}	$\ln(y_{34})$	r_{4j}^{exc}	$\ln(y_{45})$	r_{5j}^{exc}	$\ln(y_{56})$	r_{6j}^{inc}
PYTHIA 8	0.1	0.2	0.1	0.2	0.1	0.3	0.2	0.3	0.2	0.4	0.3
VINCIA (LO)	0.1	0.2	0.1	0.2	0.0	0.2	0.3	0.1	0.1	0.0	0.0
VINCIA (NLO)	0.2	0.4	0.1	0.3	0.1	0.3	0.2	0.2	0.1	0.2	0.1



Outlook


p
p
Fish
+
From smooth ord to 2→4
2nd order showers
NLO for initial state
NLO automation
Interleaved showers & decays
ANCORA · IMPARO
Oct 2014
→ Monash University
Melbourne, Australia

Outlook



+
From smooth ord to 2→4
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Outlook

p

p

+
From smooth ord to 2→4
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Oct 2014
→ Monash University
Melbourne, Australia

What we need

At NLO:

Functions, not events.

Colour-ordered helicity amplitudes (preferably already interfered with Born, but not essential; also $|\mathrm{Born}|^2$ can be useful eg for normalization & convention checks)

Return: μ_R , coeffs of $1/\varepsilon^2$ and $1/\varepsilon$ poles , finite piece (the latter preferably separated into a few pieces of different transcendentalities → eg can do analytic subtraction of $\ln^2$ piece, good for numerical stability)

→ Binoth Accord (though we'd still have to agree on specifications of colour order and helicity)

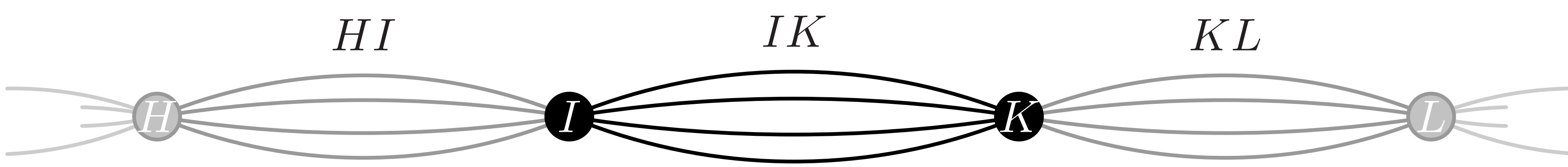
At NNLO (?):

2-loop interfered with Born and 1-loop × 1-loop

Return: μ_R , coeffs of $1/\varepsilon^n$, finite piece

Shower Types

Traditional vs Coherent vs Global vs Sector vs Dipole



	Coll(I)	Soft(IK)
Parton Shower (DGLAP)	a_I	$a_I + a_K$
Coherent Parton Shower (HERWIG *[12,40]*, PYTHIA6 *[11])*	$\Theta_I a_I$	$\Theta_I a_I + \Theta_K a_K$
Global Dipole-Antenna (ARIADNE *[17], GGG [36], WK [32],* VINCIA*)*	$a_{IK} + a_{HI}$	a_{IK}
Sector Dipole-Antenna (LP [41], VINCIA)	$\Theta_{IK} a_{IK} + \Theta_{HI} a_{HI}$	a_{IK}
Partitioned-Dipole Shower (SK [23], NS [42], DTW [24], PYTHIA8 *[38],* SHERPA*)*	$a_{I,K} + a_{I,H}$	$a_{I,K} + a_{K,I}$

Figure 2: Schematic overview of how the full collinear singularity of parton I and the soft singularity of the IK pair, respectively, originate in different shower types. (Θ_I and Θ_K represent angular vetos with respect to partons I and K, respectively, and Θ_{IK} represents a sector phase-space veto, see text.)

Sector Antennae

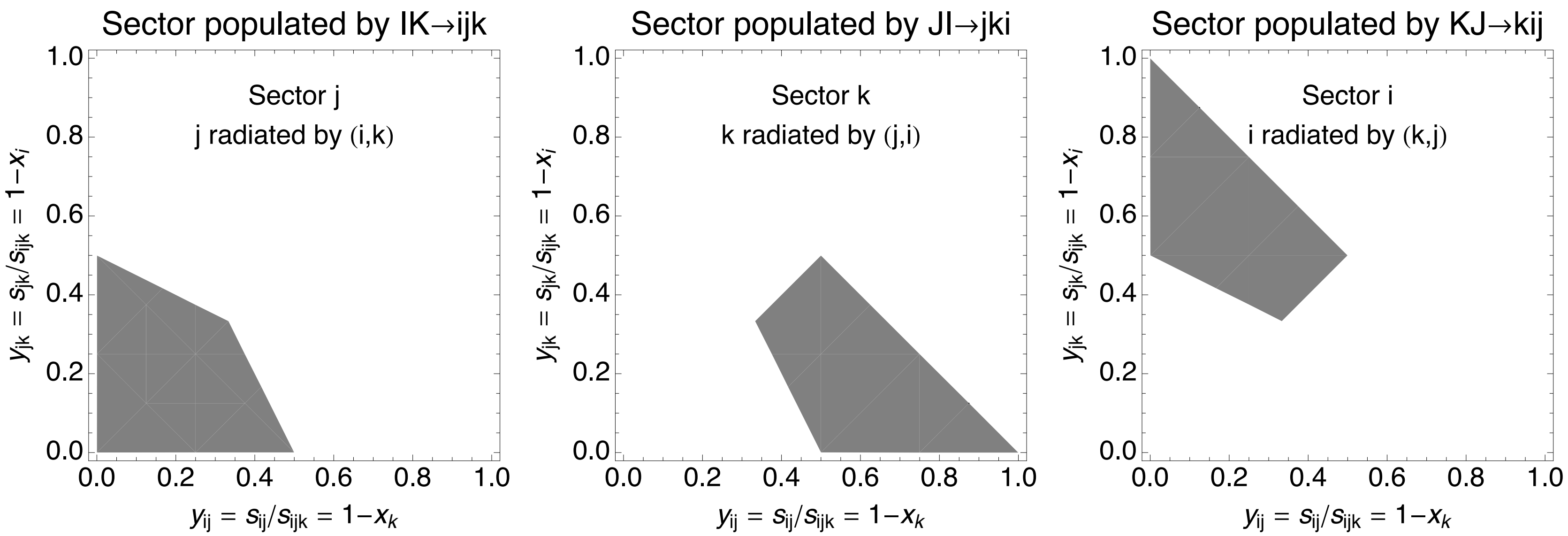
Global

$$\bar{a}^{\text{gl}}_{g/qg}(p_i,p_j,p_k) \xrightarrow{s_{jk}\to 0} \frac{1}{s_{jk}}\left(P_{gg\to G}(z) - \frac{2z}{1-z} - z(1-z)\right)$$

→ P(z) = Sum over two neigboring antennae

Sector

Only a single term in each phase space point



→ Full P(z) must be contained in every antenna

$$\begin{aligned}
\bar{a}^{\text{sct}}_{j/IK}(y_{ij},y_{jk}) = \bar{a}^{\text{gl}}_{j/IK}(y_{ij},y_{jk}) \quad &+ \quad \delta_{Ig}\delta_{H_K H_k}\left\{\delta_{H_I H_i}\delta_{H_I H_j}\left(\frac{1+y_{jk}+y_{jk}^2}{y_{ij}}\right)\right.\\
&+ \quad \left.\delta_{H_I H_j}\left(\frac{1}{y_{ij}(1-y_{jk})} - \frac{1+y_{jk}+y_{jk}^2}{y_{ij}}\right)\right\}\\
&+ \quad \delta_{Kg}\delta_{H_I H_i}\left\{\delta_{H_I H_j}\delta_{H_K H_k}\left(\frac{1+y_{ij}+y_{ij}^2}{y_{jk}}\right)\right.\\
&+ \quad \left.\delta_{H_K H_j}\left(\frac{1}{y_{jk}(1-y_{ij})} - \frac{1+y_{ij}+y_{ij}^2}{y_{jk}}\right)\right\}
\end{aligned}$$

Sector = Global + additional collinear terms
(from "neighboring" antenna)

The Denominator

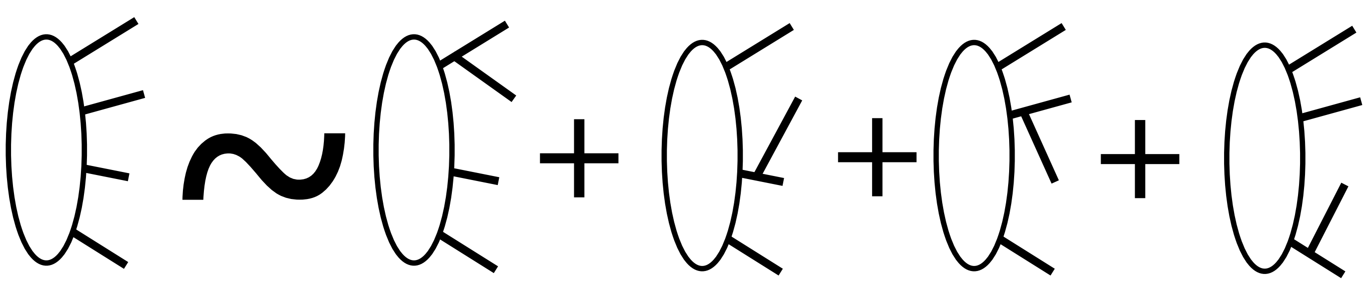
$$a_i \rightarrow \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2}$$

In a traditional parton shower, you would face the following problem:

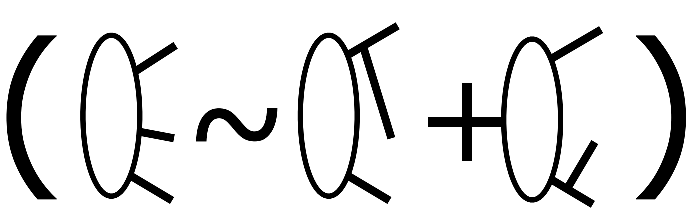
Existing parton showers are *not* really Markov Chains

Further evolution (restart scale) depends on *which* branching happened last
→ proliferation of terms

Number of histories contributing to n^{th} branching ∝ **$2^n n!$**



j = 2
→ 4 terms



j = 1
→ 2 terms

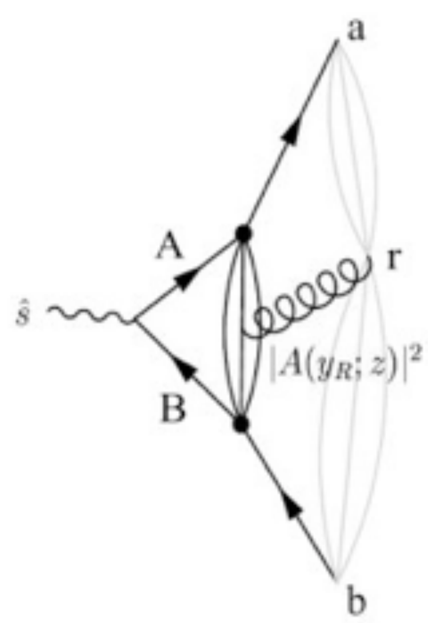
Parton- (or Catani-Seymour) Shower:
After 2 branchings: 8 terms
After 3 branchings: 48 terms
After 4 branchings: 384 terms

(+ parton showers have complicated and/or frame-dependent phase-space mappings, especially at the multi-parton level)

Matched Markovian Antenna Showers

Antenna showers: one term per parton *pair* **$2^n n! \rightarrow n!$**

Giele, Kosower, Skands, PRD 84 (2011) 054003



(+ generic Lorentz-invariant and on-shell phase-space factorization)

+ Change "shower restart" to Markov criterion:

Given an *n*-parton configuration, "ordering" scale is

$$Q_{ord} = min(Q_{E1}, Q_{E2}, \ldots, Q_{En})$$

Unique restart scale, independently of how it was produced

+ Matching: n! → n

Given an *n*-parton configuration, its phase space weight is:

$|M_n|^2$: Unique weight, independently of how it was produced

Matched Markovian Antenna Shower:	Parton- (or Catani-Seymour) Shower:
After 2 branchings: 2 terms	After 2 branchings: 8 terms
After 3 branchings: 3 terms	After 3 branchings: 48 terms
After 4 branchings: 4 terms	After 4 branchings: 384 terms

+ **Sector** antennae
→ 1 term at *any* order

Larkosi, Peskin,Phys.Rev. D81 (2010) 054010
Lopez-Villarejo, Skands, JHEP 1111 (2011) 150

Approximations

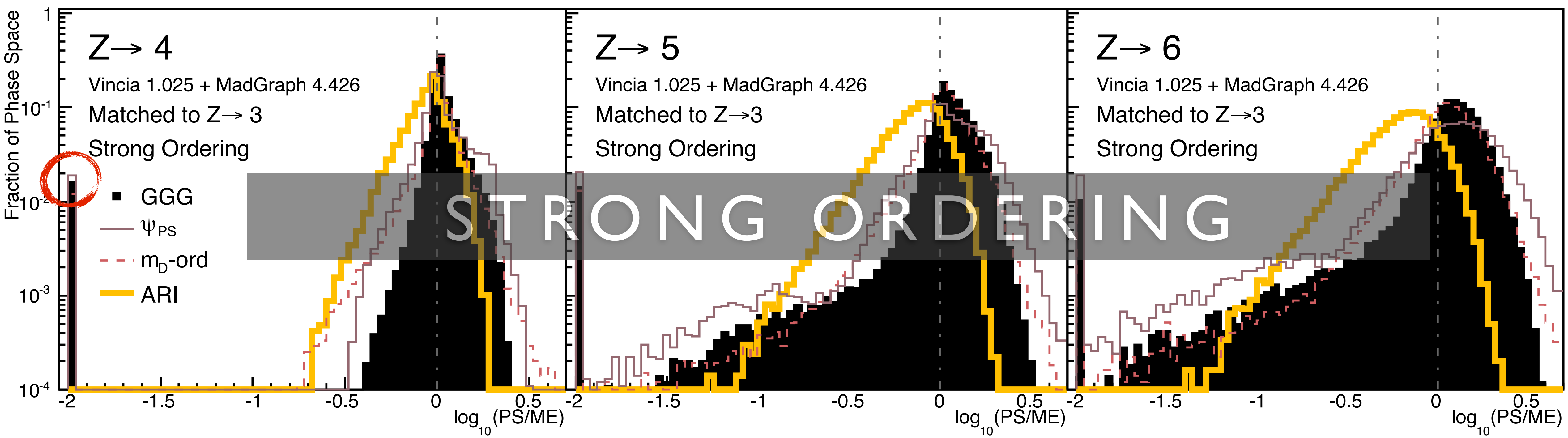
Q: How well do showers do?

Exp: Compare to data. Difficult to interpret; all-orders cocktail including hadronization, tuning, uncertainties, etc

Th: Compare products of splitting functions to full tree-level matrix elements

Plot distribution of **$\log_{10}$(PS/ME)**

(second order) (third order) (fourth order)



Dead Zone: 1-2% of phase space have no strongly ordered paths leading there* *fine from strict LL point of view: those points correspond to "unordered" non-log-enhanced configurations

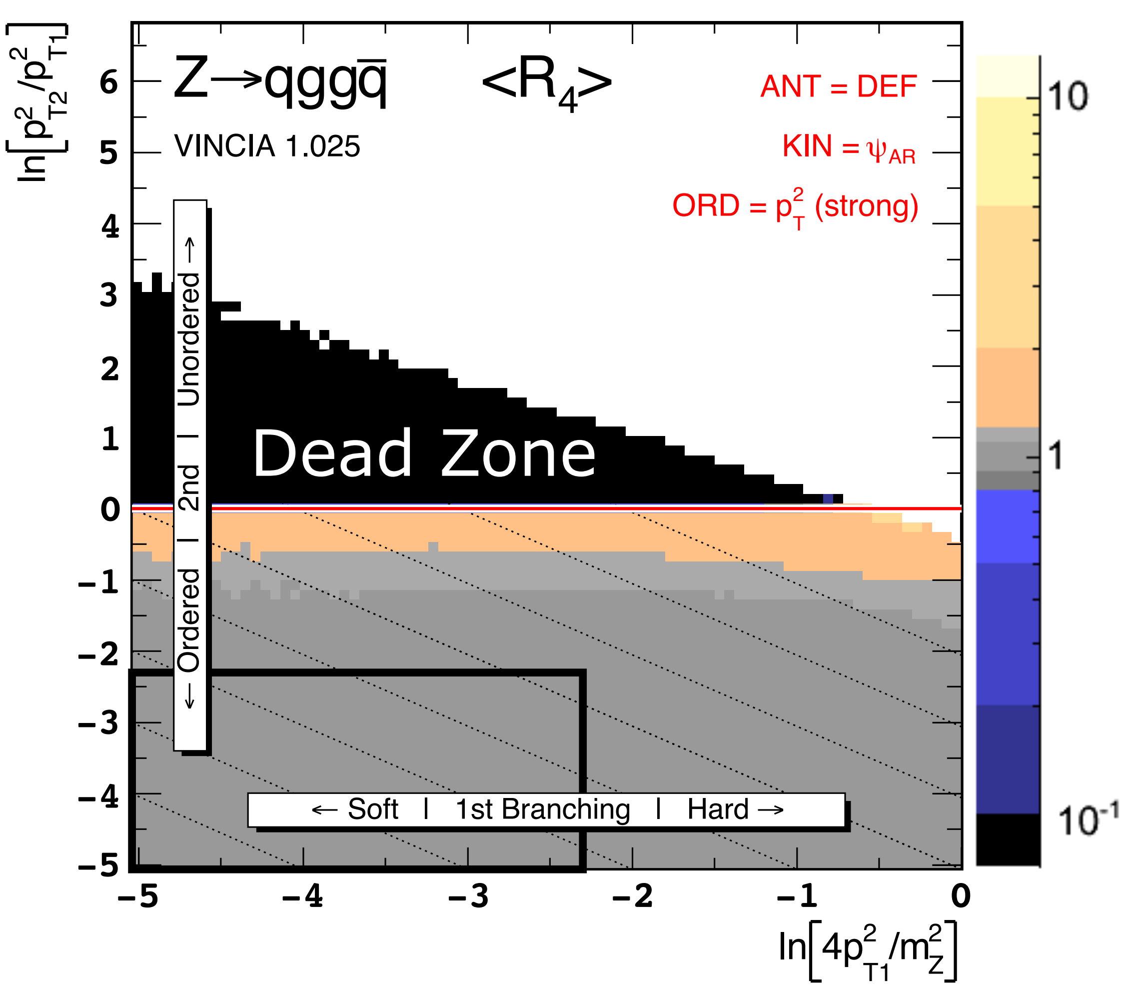
Generate Branchings *without* imposing strong ordering

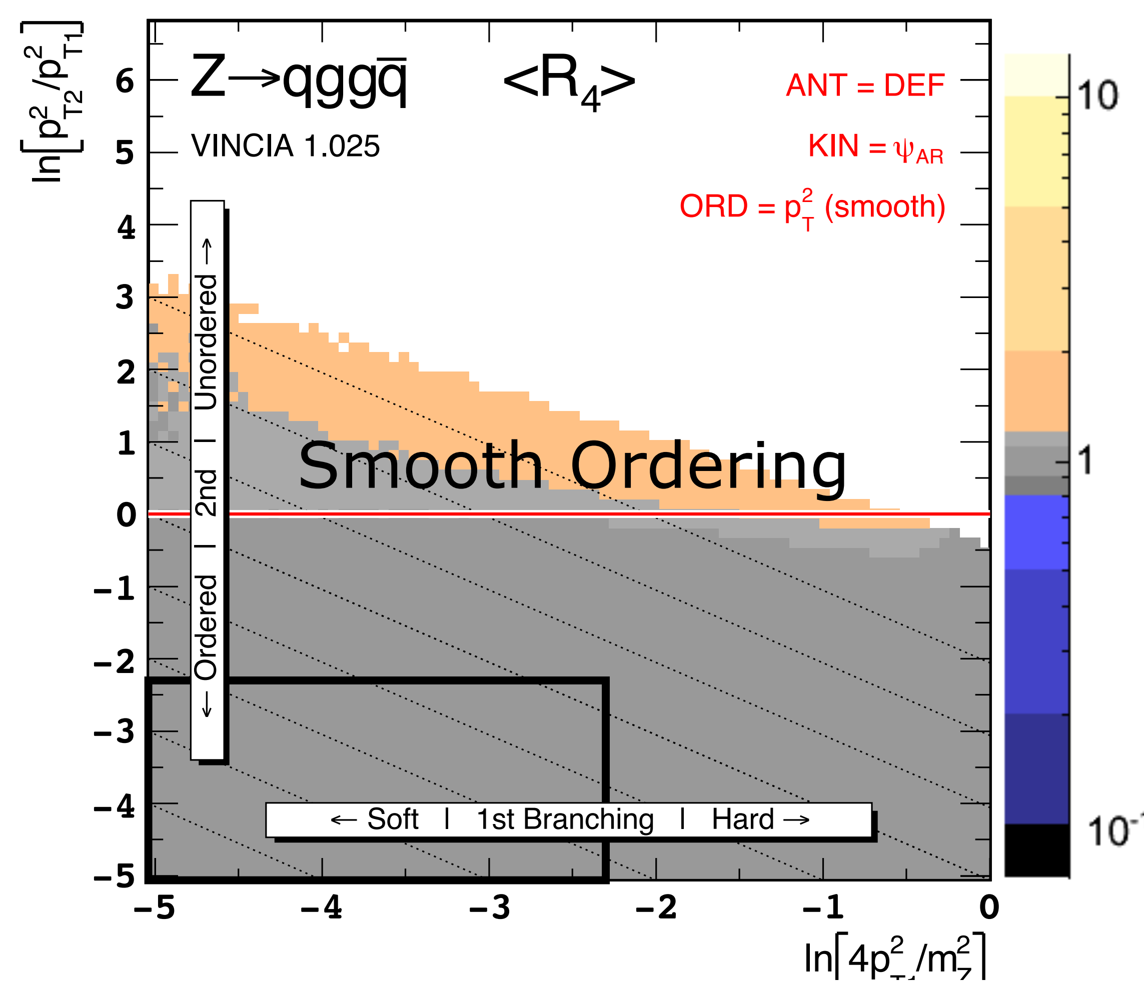
At each step, each dipole allowed to fill its entire phase space

Overcounting removed by matching
+ smooth ordering beyond matched multiplicities

$$\frac{\hat{p}_\perp^2}{\hat{p}_\perp^2 + p_\perp^2} P_{\mathrm{LL}}$$

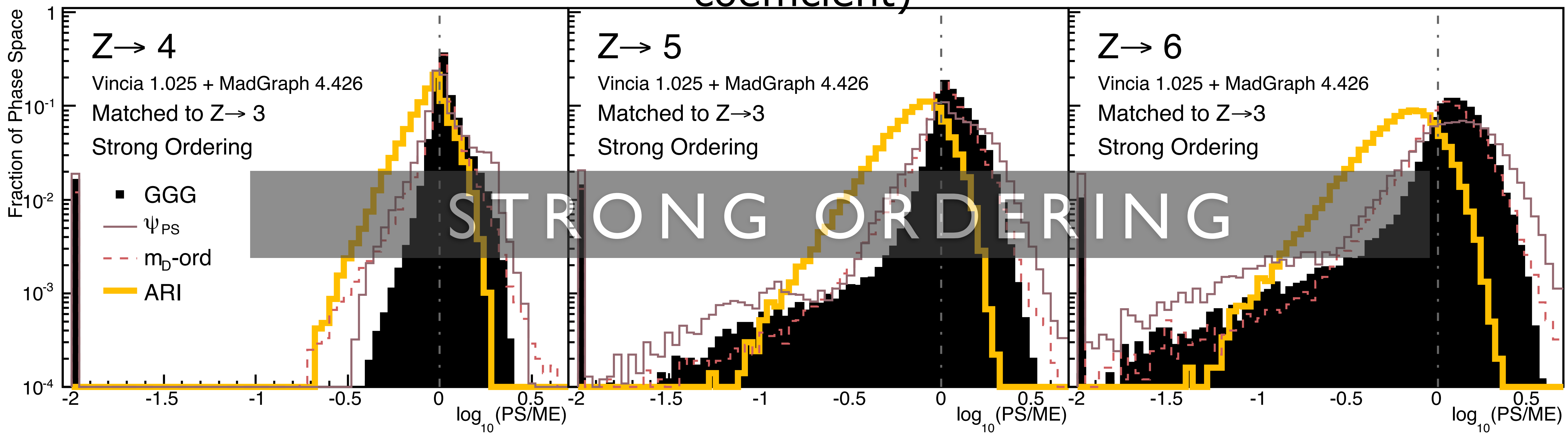
$\hat{p}_\perp^2$ last branching
$p_\perp^2$ current branching



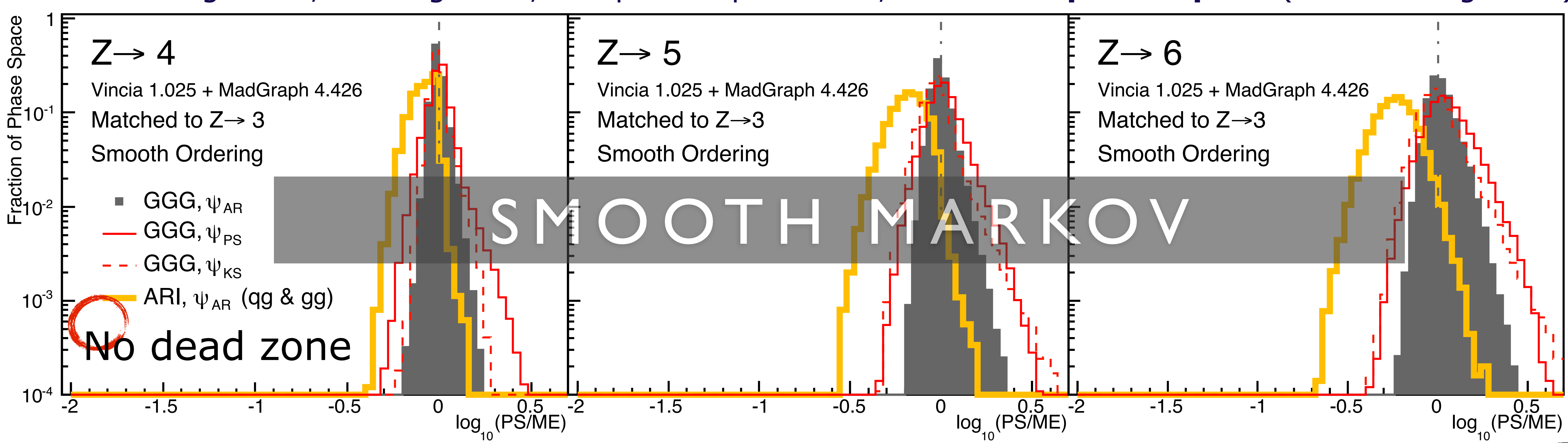


→ Better Approximations

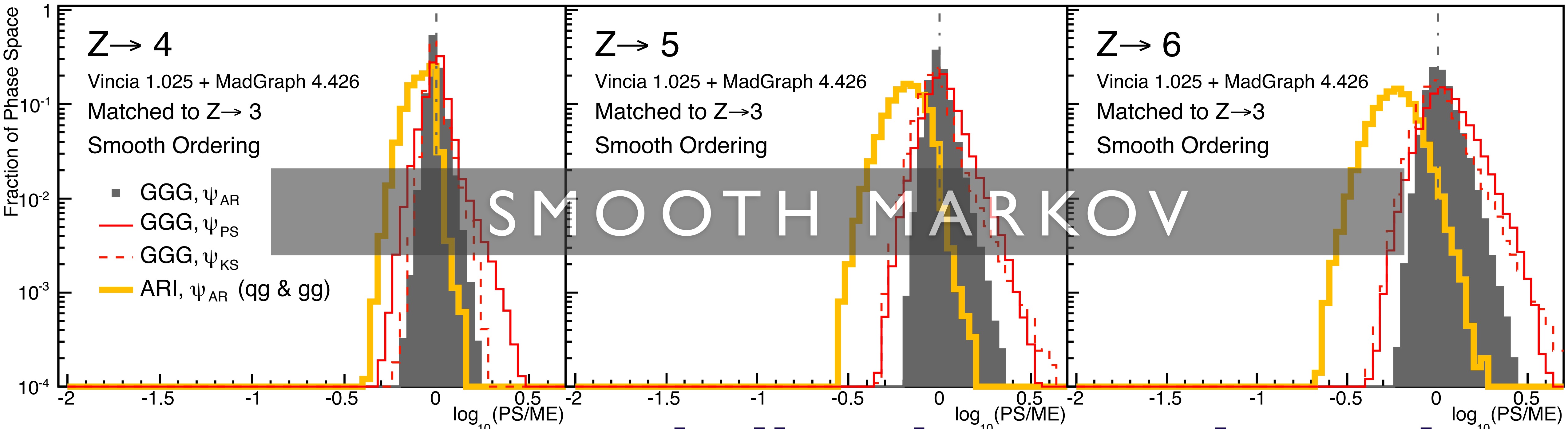
Distribution of **$\mathrm{Log}_{10}(\mathrm{PS}_{\mathrm{LO}}/\mathrm{ME}_{\mathrm{LO}})$** (inverse ~ matching coefficient)



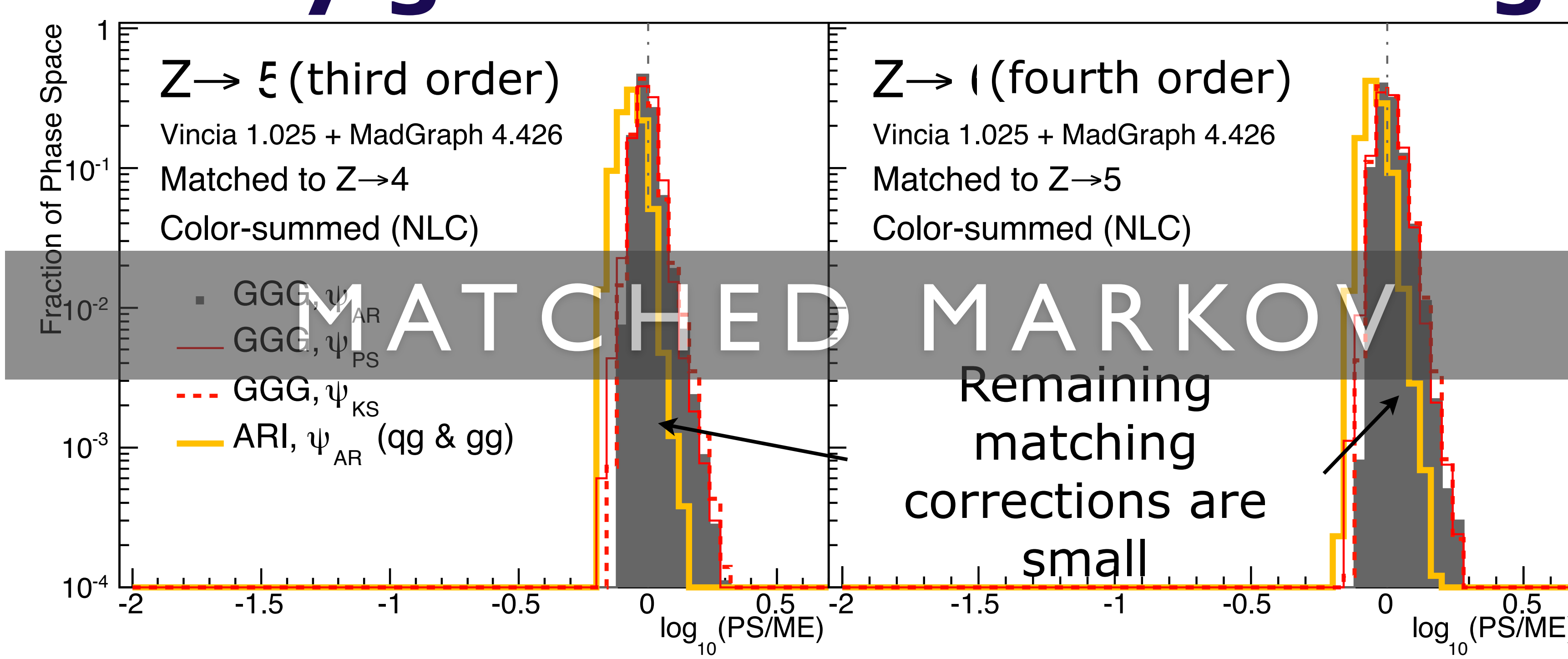
Leading Order, Leading Color, Flat phase-space scan, over **all of phase space** (no matching scale)



+ Matching (+ full colour)



→ A very good all-orders starting point

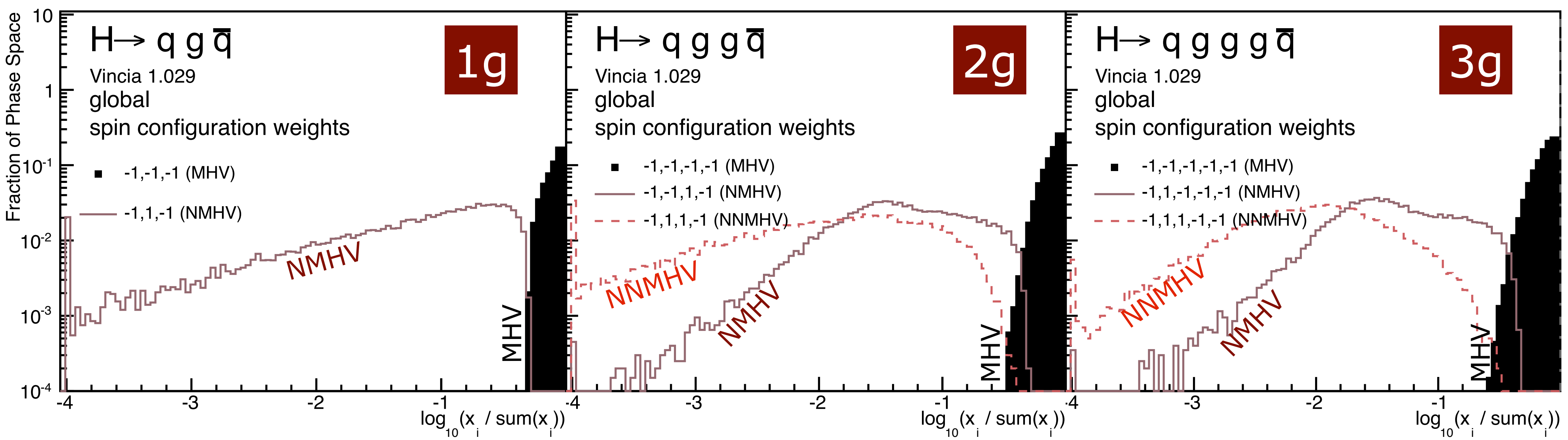


Helicity Contributions

Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

Flat phase-space scan. $H^0 \rightarrow qq + ng$. Size of helicity contributions.

Fixed Order



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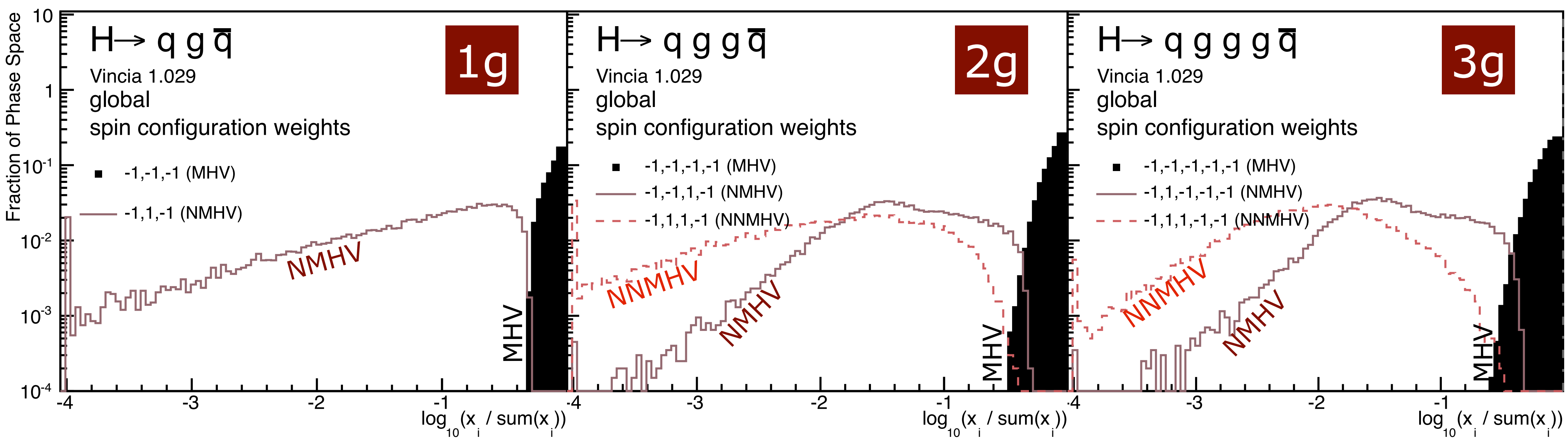
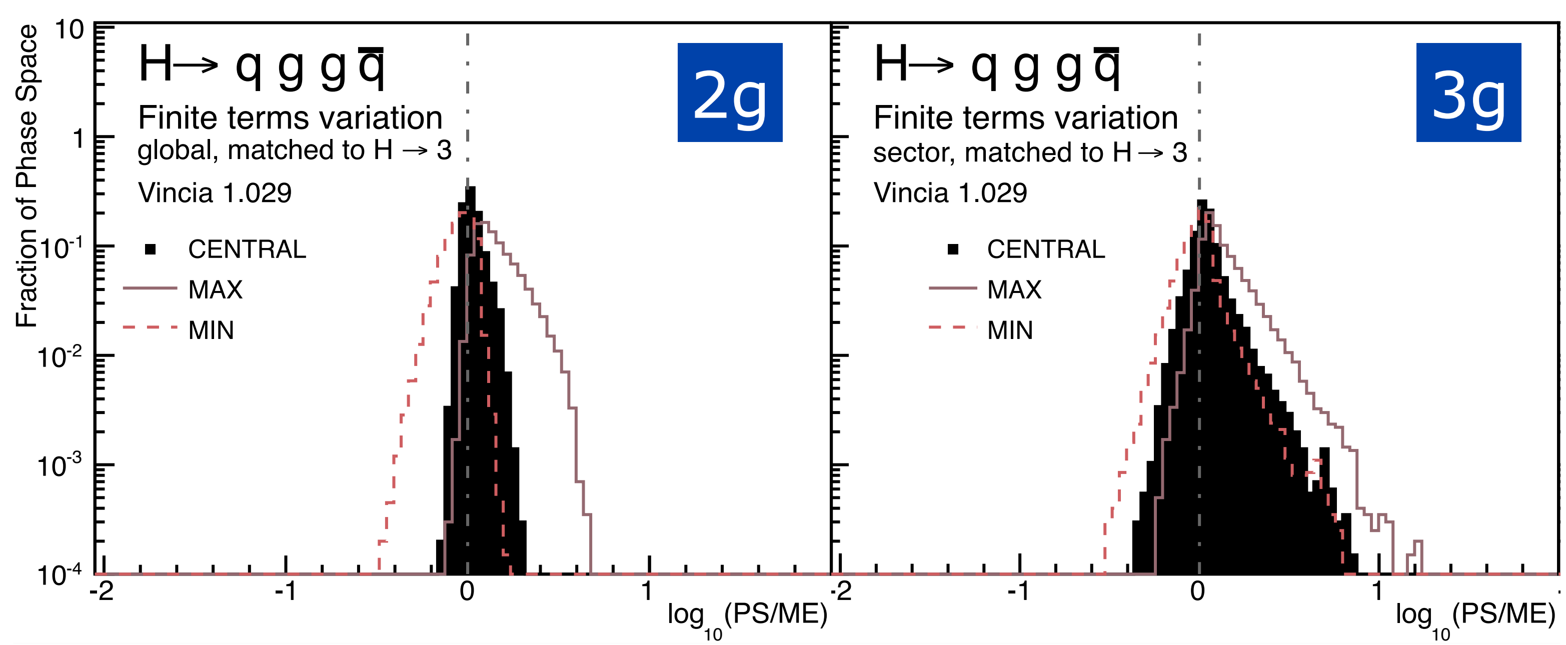


LO Shower Expansion / ME

Distribution of PS/ME ratio (summed over helicities)

Vincia shower already quite close to ME
→ small corrections

Note: precision not greatly improved by helicity dependence



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Larkoski, Lopez-Villarejo, Skands, PRD 87 (2013) 054033

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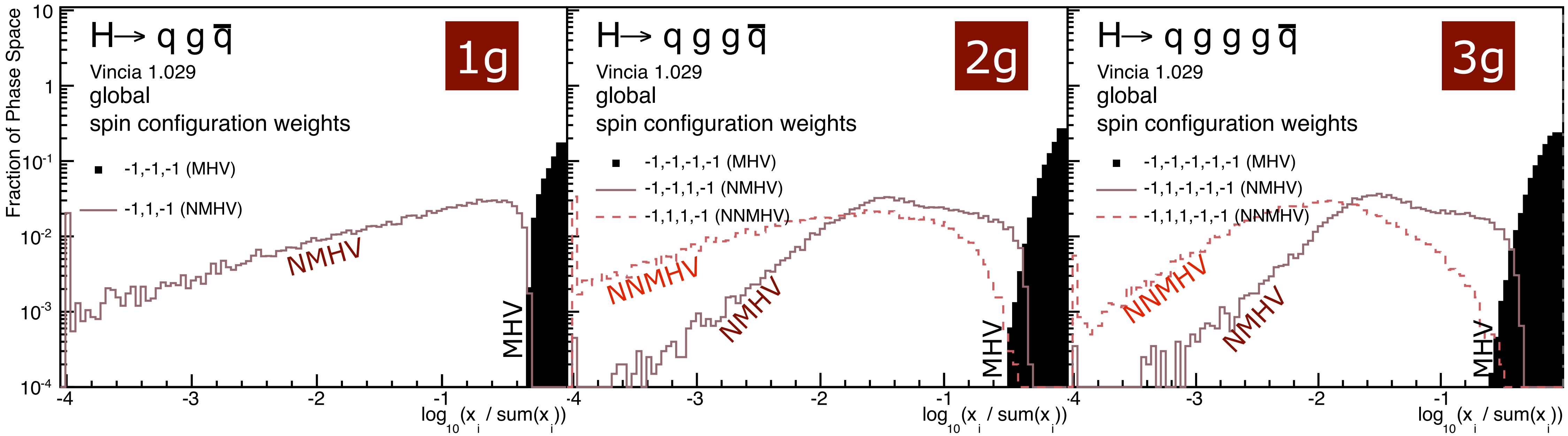

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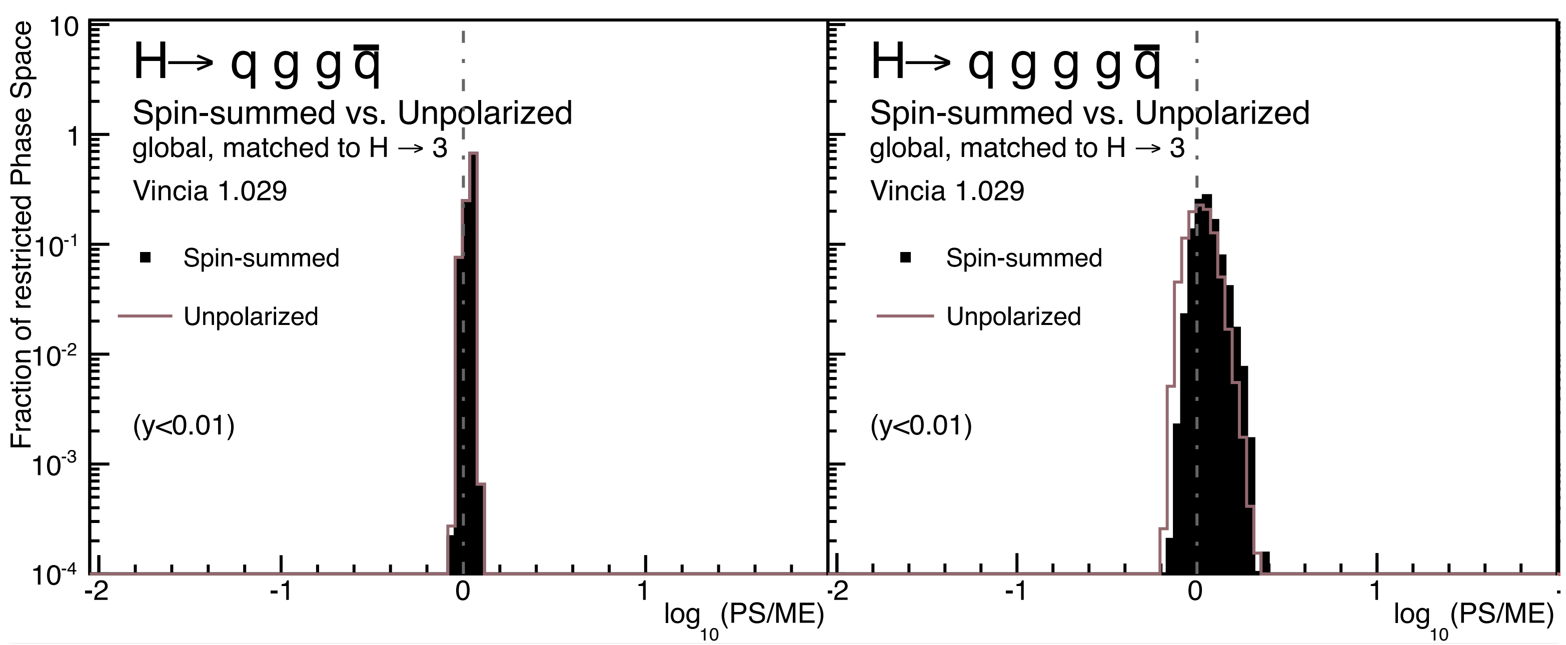
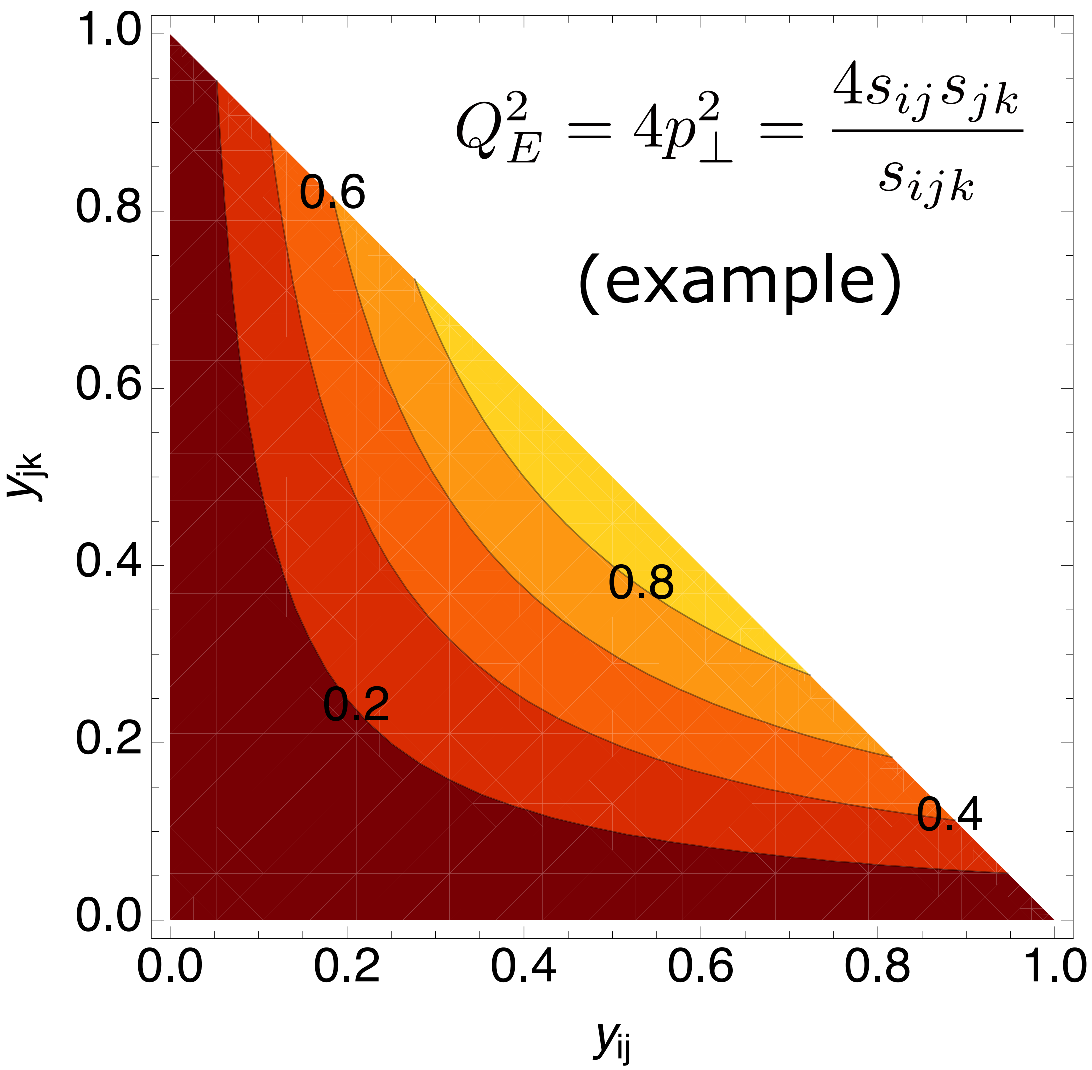


Sudakov Integrals



2→3:

$$a_3^0 = \frac{1}{s}\left(\frac{2y_{ik}}{y_{ij}y_{jk}} + \frac{y_{ij}}{y_{jk}} + \frac{y_{jk}}{y_{ij}}\right)$$

$$g_s^2 C_A \int_{Q_3^2}^{s} a_3^0 \, \mathrm{d}\Phi_{\mathrm{ant}} = \frac{\alpha_s C_A}{2\pi}\left(\sum_{i=1}^{5} K_i I_i(s, Q_3^2)\right)$$

$$K_1 = 1\,, \quad K_2 = -2 \quad K_3 = 2, \quad K_4 = -\delta_{Ig} - \delta_{Kg}, \quad K_5 = 1.$$

$$I_1 = \left[-\mathrm{Li}_2\left(\frac{1}{2}\left(1+\sqrt{1-y_3^2}\right)\right) + \mathrm{Li}_2\left(\frac{1}{2}\left(1-\sqrt{1-y_3^2}\right)\right) - \frac{1}{2}\ln\left(\frac{4}{y_3^2}\right)\ln\left(\frac{1-\sqrt{1-y_3^2}}{1+\sqrt{1-y_3^2}}\right)\right]$$

$$I_2 = \left[-2\sqrt{1-y_3^2} + \ln\left(\frac{1+\sqrt{1-y_3^2}}{1-\sqrt{1-y_3^2}}\right)\right]$$

$$I_3 = \left[-\frac{1}{2}\sqrt{1-y_3^2} + \frac{1}{4}\ln\left(\frac{1+\sqrt{1-y_3^2}}{1-\sqrt{1-y_3^2}}\right)\right]$$

$$I_4 = \left[-\frac{13\sqrt{1-y_3^2}}{36} + \frac{1}{36}y_3^2\sqrt{1-y_3^2} + \frac{1}{3}\ln\left[1+\sqrt{1-y_3^2}\right] - \frac{\ln\left(y_3^2\right)}{6}\right]$$

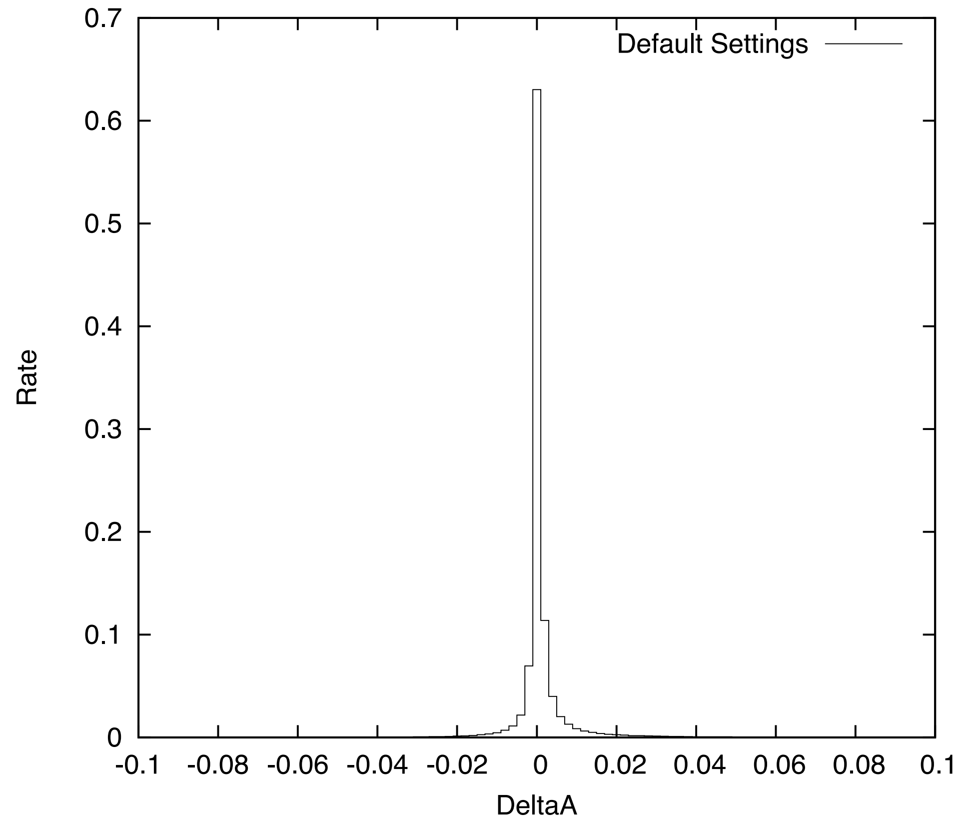
$$I_5 = \frac{1}{24}\left[2\left(3C_{00} - (C_{01}+C_{10})(-1+y_3^2)\sqrt{1-y_3^2} - 3\,C_{00}\,y_3^2\ln\left(\frac{1+\sqrt{1-y_3^2}}{1-\sqrt{1-y_3^2}}\right)\right)\right].$$

3→4: C_A piece (for strong ordering)

$$-g_s^2\sum_{j=1}^{2} C_A \int_0^{s_j}(1-O_{E_j})\, d_3^0 \mathrm{d}\Phi_{\mathrm{ant}} = -\frac{\alpha_s C_A}{2\pi}\left(\sum_{i=1}^{5} K_i I_i(s_{qg}, Q_3^2)\right) - \frac{\alpha_s C_A}{2\pi}\left(\sum_{i=1}^{5} K_i I_i(s_{g\bar{q}}, Q_3^2)\right)$$

The δA Terms - Speed

Hartgring, Laenen, Skands, arXiv:1303.4974



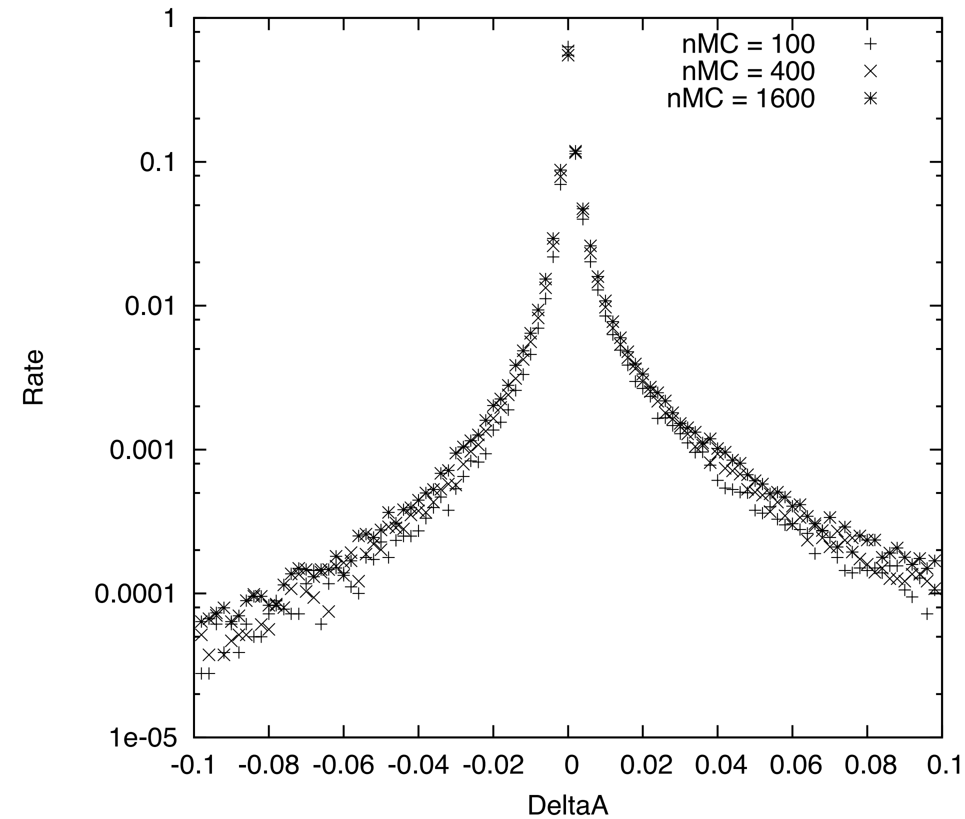


Figure 14: Distribution of the size of the δA terms (normalized so the LO result is unity) in actual VINCIA runs. *Left:* linear scale, default settings. *Right:* logarithmic scale, with variations on the minimum number of MC points used for the integrations (default is 100).

Speed:

	LO level $Z \to$	NLO level $Z \to$	Time / Event [milliseconds]	Speed relative to PYTHIA $\frac{1}{\text{Time}}$ / PYTHIA 8
PYTHIA 8	$2, 3$	2	0.4	1
VINCIA (NLO off)	$2, 3, 4, 5$	2	2.2	$\sim 1/5$
VINCIA (NLO on)	$2, 3, 4, 5$	$2, 3$	3.0	$\sim 1/7$ ← OK

Loop Corrections

Giele, Kosower, Skands, Phys.Rev. D78 (2008) 014026
Hartgring, Laenen, Skands, arXiv:1303.4974

Pedagogical Example: $Z^0 \to q\bar{q}$ First Order (~POWHEG)

Fixed Order: Exclusive 2-jet rate (2 and only 2 jets), at Q = Q_{had}

$$= \underbrace{|M_0^0|^2}_{\text{Born}} \left(1 + \underbrace{\frac{2\,\mathrm{Re}[M_0^0 M_0^{1*}]}{|M_0^0|^2}}_{\text{Virtual}} + \underbrace{\int_0^{Q^2_{\mathrm{had}}} \mathrm{d}\Phi_{\mathrm{ant}}\; g_s^2\, \mathcal{C}\; A_{g/q\bar{q}}}_{\text{Unresolved Real}} \right)$$

$$= \frac{|M_1^0|^2}{|M_0^0|^2}$$

Loop Corrections

Giele, Kosower, Skands, Phys.Rev. D78 (2008) 014026
Hartgring, Laenen, Skands, arXiv:1303.4974

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$$= \frac{|M_1^0|^2}{|M_0^0|^2}$$

LO Vincia: Exclusive 2-jet rate (2 and only 2 jets), at Q = Q_{had}

$$\underbrace{|M_0^0|^2}_{\text{Born}}\; \underbrace{\Delta(s, Q^2_{\mathrm{had}})}_{\text{Sudakov}} = |M_0^0|^2 \left(\underbrace{1 - \int_{Q^2_{\mathrm{had}}}^{s} \mathrm{d}\Phi_{\mathrm{ant}}\; g_s^2\, \mathcal{C}\; A_{g/q\bar{q}} + \mathcal{O}(\alpha_s^2)}_{\text{Approximate Virtual + Unresolved Real}} \right)$$

Loop Corrections

Giele, Kosower, Skands, Phys.Rev. D78 (2008) 014026
Hartgring, Laenen, Skands, arXiv:1303.4974

Pedagogical Example: $Z^0 \to q\bar{q}$ First Order (~POWHEG)

Fixed Order: Exclusive 2-jet rate (2 and only 2 jets), at Q = Q_{had}

$$= \underbrace{|M_0^0|^2}_{\text{Born}} \left(1 + \underbrace{\frac{2\,\mathrm{Re}[M_0^0 M_0^{1*}]}{|M_0^0|^2}}_{\text{Virtual}} + \underbrace{\int_0^{Q^2_{\mathrm{had}}} \mathrm{d}\Phi_{\mathrm{ant}}\; g_s^2\; \mathcal{C}\; A_{g/q\bar{q}}}_{\text{Unresolved Real}} \right) \qquad = \frac{|M_1^0|^2}{|M_0^0|^2}$$

LO Vincia: Exclusive 2-jet rate (2 and only 2 jets), at Q = Q_{had}

$$\underbrace{|M_0^0|^2}_{\text{Born}}\; \underbrace{\Delta(s, Q^2_{\mathrm{had}})}_{\text{Sudakov}} = |M_0^0|^2 \left(1 - \underbrace{\int_{Q^2_{\mathrm{had}}}^{s} \mathrm{d}\Phi_{\mathrm{ant}}\; g_s^2\; \mathcal{C}\; A_{g/q\bar{q}}}_{\text{Approximate Virtual + Unresolved Real}} + \mathcal{O}(\alpha_s^2) \right)$$

NLO Correction: Subtract and correct by difference

$$\left.\begin{aligned} \frac{2\,\mathrm{Re}[M_0^0 M_0^{1*}]}{|M_0^0|^2} &= \frac{\alpha_s}{2\pi}\, 2C_F\; \left(2I_{q\bar{q}}(\epsilon, \mu^2/m_Z^2) - 4\right) \\ \int_0^s \mathrm{d}\Phi_{\mathrm{ant}}\; 2C_F\; g_s^2\; A_{g/q\bar{q}} &= \frac{\alpha_s}{2\pi}\, 2C_F\; \left(-2I_{q\bar{q}}(\epsilon, \mu^2/m_Z^2) + \frac{19}{4}\right) \end{aligned}\right\} \quad |M_0^0|^2 \to \left(1 + \frac{\alpha_s}{\pi}\right) |M_0^0|^2$$

IR Singularity Operator

IR Singularity Operators

Gehrmann, Gehrmann-de Ridder, Glover, JHEP 0509 (2005) 056

$q\bar{q} \to qg\bar{q}$ antenna function

$$X^0_{ijk} = S_{ijk,IK} \frac{|\mathcal{M}^0_{ijk}|^2}{|\mathcal{M}^0_{IK}|^2}$$

$$A^0_3(1_q, 3_g, 2_{\bar{q}}) = \frac{1}{s_{123}} \left(\frac{s_{13}}{s_{23}} + \frac{s_{23}}{s_{13}} + 2\frac{s_{12}s_{123}}{s_{13}s_{23}} \right)$$

Integrated antenna

$$\mathcal{P}oles\left(\mathcal{A}^0_3(s_{123})\right) = -2\mathbf{I}^{(1)}_{q\bar{q}}\left(\epsilon, s_{123}\right)$$

$$\mathcal{F}inite\left(\mathcal{A}^0_3(s_{123})\right) = \frac{19}{4} .$$

$$\mathcal{X}^0_{ijk}(s_{ijk}) = \left(8\pi^2 \left(4\pi\right)^{-\epsilon} e^{\epsilon\gamma}\right) \int \mathrm{d}\Phi_{X_{ijk}}\, X^0_{ijk}.$$

Singularity Operators

$$\mathbf{I}^{(1)}_{q\bar{q}}\left(\epsilon, \mu^2/s_{q\bar{q}}\right) = -\frac{e^{\epsilon\gamma}}{2\Gamma\left(1-\epsilon\right)} \left[\frac{1}{\epsilon^2} + \frac{3}{2\epsilon}\right] \mathrm{Re}\left(-\frac{\mu^2}{s_{q\bar{q}}}\right)^{\epsilon}$$

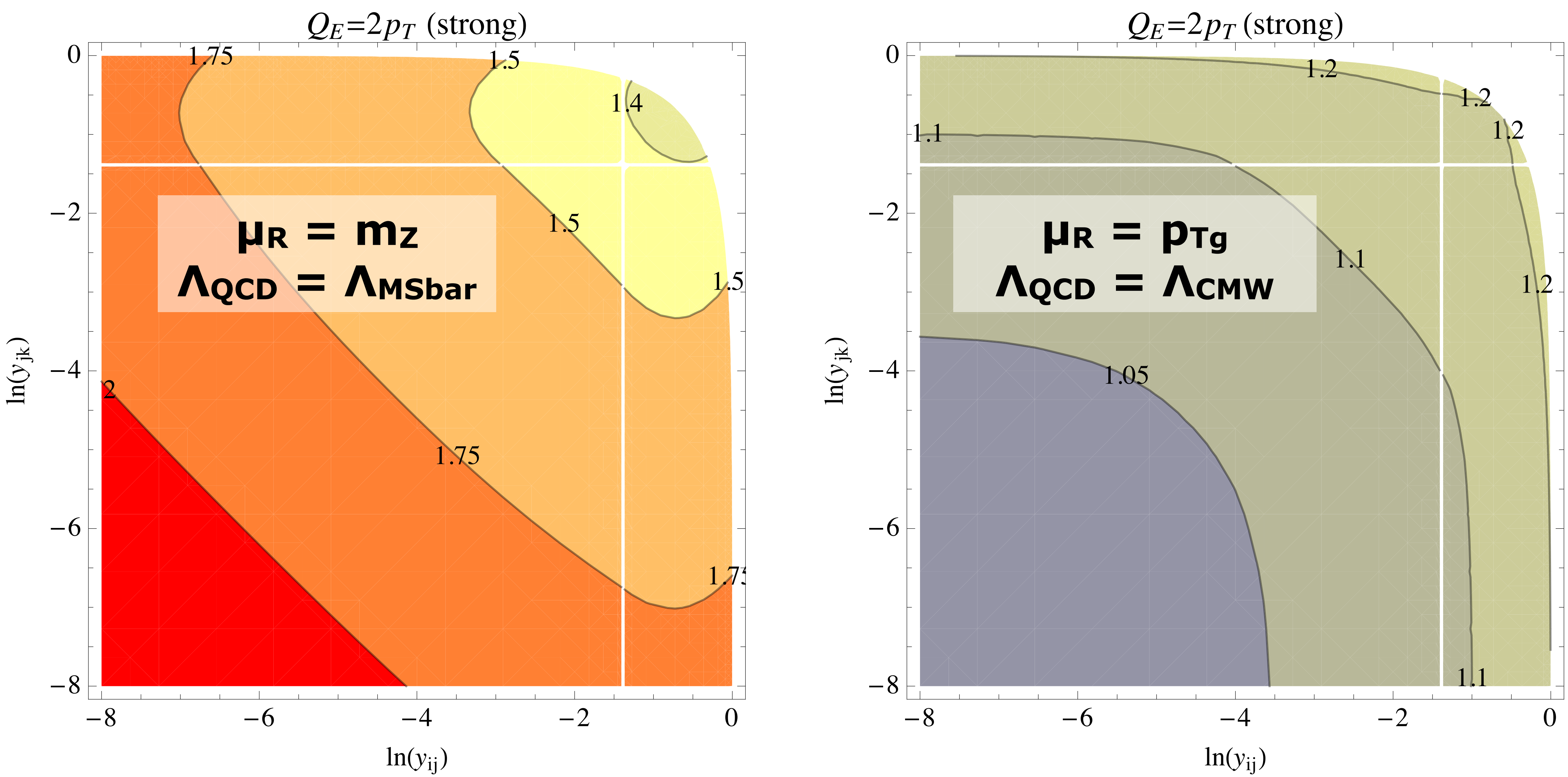
$$\mathbf{I}^{(1)}_{qg}\left(\epsilon, \mu^2/s_{qg}\right) = -\frac{e^{\epsilon\gamma}}{2\Gamma\left(1-\epsilon\right)} \left[\frac{1}{\epsilon^2} + \frac{5}{3\epsilon}\right] \mathrm{Re}\left(-\frac{\mu^2}{s_{qg}}\right)^{\epsilon}$$ for qg→qgg

$$\mathbf{I}^{(1)}_{qg,F}\left(\epsilon, \mu^2/s_{qg}\right) = \frac{e^{\epsilon\gamma}}{2\Gamma\left(1-\epsilon\right)} \frac{1}{6\epsilon} \mathrm{Re}\left(-\frac{\mu^2}{s_{qg}}\right)^{\epsilon}$$ for qg→qq’q’

Choice of μ_R

Renormalization: 1) Choose $\mu_R \sim p_{Tjet}$ (absorbs universal β-dependent terms)

2) Translate from MSbar to CMW scheme ($\Lambda_{CMW} \sim 1.6\ \Lambda_{MSbar}$ for coherent showers)



Markov Evolution in: Transverse Momentum, $\alpha_S(M_Z) = 0.12$