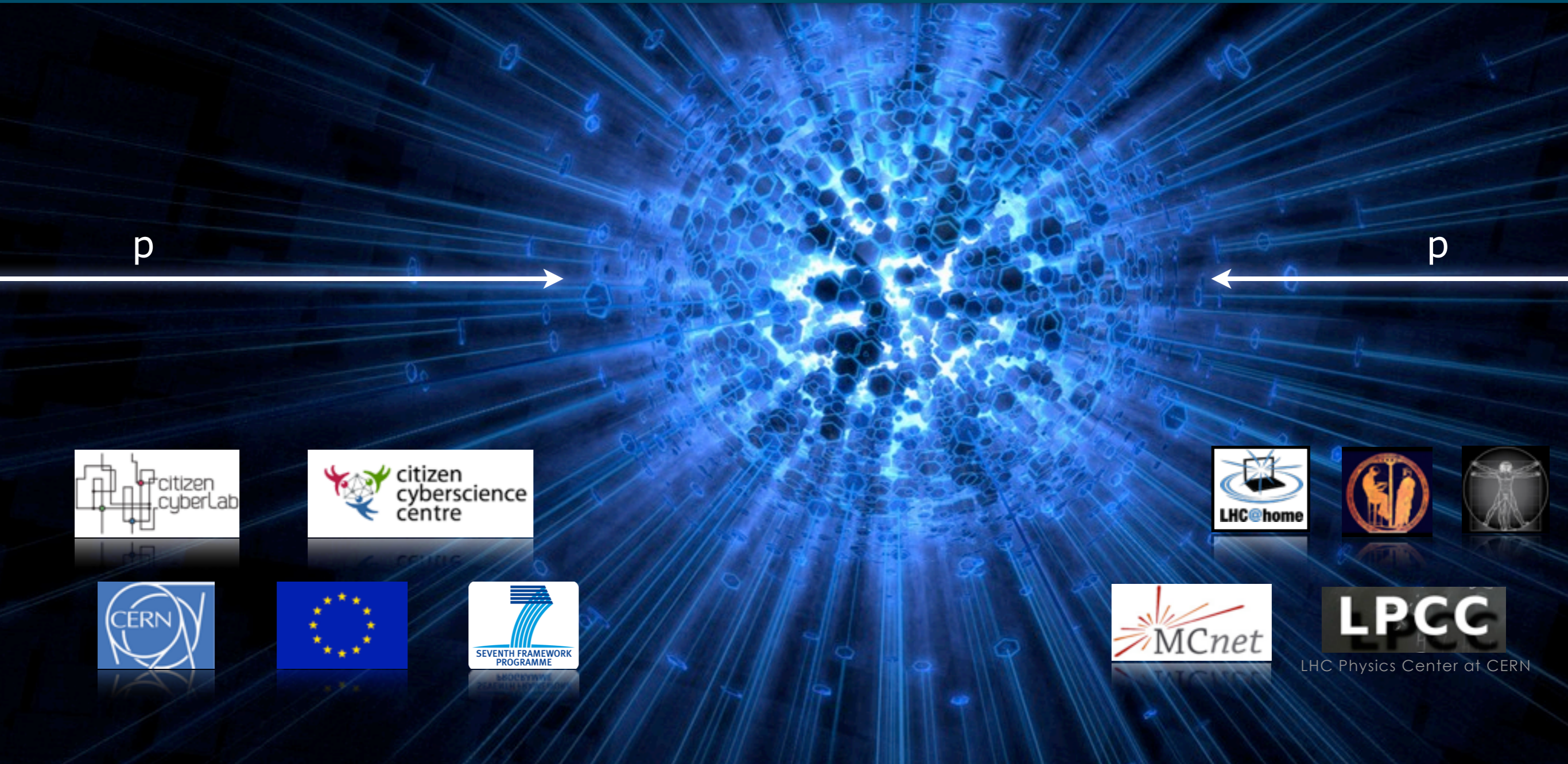
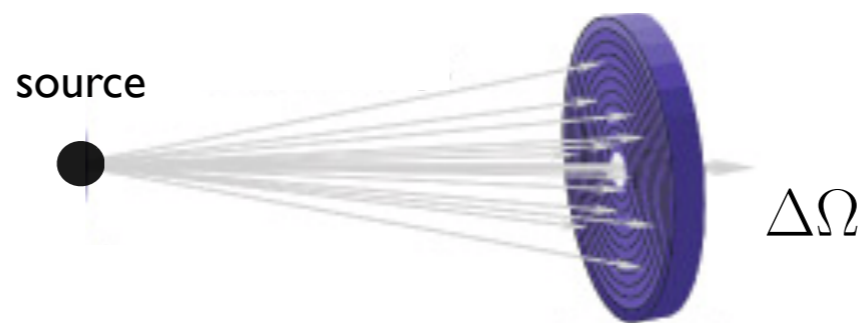


Event Generator Physics

Peter Skands (CERN Theoretical Physics Dept)



Scattering Experiments



LHC detector
Cosmic-Ray detector
Neutrino detector
X-ray telescope
...

→ Integrate differential cross sections over specific phase-space regions

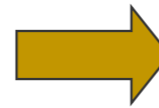
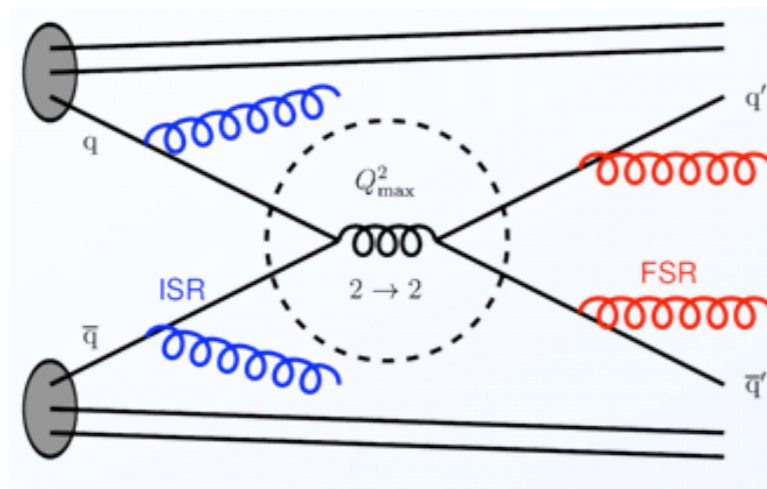
Predicted number of counts
= integral over solid angle

$$N_{\text{count}}(\Delta\Omega) \propto \int_{\Delta\Omega} d\Omega \frac{d\sigma}{d\Omega}$$

In particle physics:
Integrate over all quantum histories
(+ interferences)

↑
Lots of dimensions?
Complicated integrands?
→ Use Monte Carlo

General-Purpose Event Generators



Calculate Everything \approx solve QCD \rightarrow requires compromise!

Improve lowest-order perturbation theory,
by including the 'most significant' corrections
 \rightarrow complete events (can evaluate any observable you want)

The Workhorses

PYTHIA : Successor to JETSET (begun in 1978). Originated in hadronization studies: Lund String.
HERWIG : Successor to EARWIG (begun in 1984). Originated in coherence studies: angular ordering.
SHERPA : Begun in 2000. Originated in "matching" of matrix elements to showers: CKKW-L.
+ MORE SPECIALIZED: ALPGEN, MADGRAPH, HELAC, ARIADNE, VINCIA, WHIZARD, (a)MC@NLO, POWHEG, HEJ, PHOJET, EPOS, QGSJET, SIBYLL, DPMJET, LDCMC, DIPSY, HIJING, CASCADE, GOSAM, BLACKHAT, ...

Factorization

Why is Fixed Order QCD not enough?

: It requires all resolved scales $\gg \Lambda_{\text{QCD}}$ **AND** no large hierarchies

Trivially untrue for QCD

We're colliding, and observing, hadrons \rightarrow small scales

We want to consider high-scale processes \rightarrow large scale differences

$$\frac{d\sigma}{dX} = \sum_{a,b} \sum_f \int_{\hat{X}_f} f_a(x_a, Q_i^2) f_b(x_b, Q_i^2) \frac{d\hat{\sigma}_{ab \rightarrow f}(x_a, x_b, f, Q_i^2, Q_f^2)}{d\hat{X}_f} D(\hat{X}_f \rightarrow X, Q_i^2, Q_f^2)$$

PDFs: needed to compute inclusive cross sections

FFs: needed to compute (semi-)exclusive cross sections

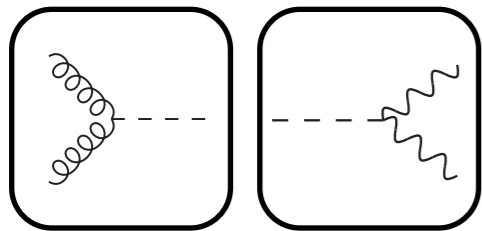
Resummed pQCD: All resolved scales $\gg \Lambda_{\text{QCD}}$ **AND** X Infrared Safe

*)pQCD = perturbative QCD

Divide and Conquer

Factorization → Split the problem into many (nested) pieces
+ Quantum mechanics → Probabilities → Random Numbers

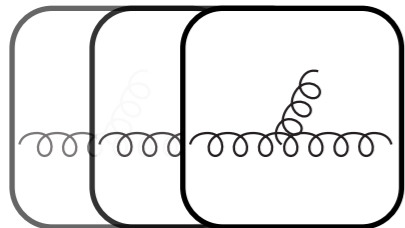
$$\mathcal{P}_{\text{event}} = \mathcal{P}_{\text{hard}} \otimes \mathcal{P}_{\text{dec}} \otimes \mathcal{P}_{\text{ISR}} \otimes \mathcal{P}_{\text{FSR}} \otimes \mathcal{P}_{\text{MPI}} \otimes \mathcal{P}_{\text{Had}} \otimes \dots$$



Hard Process & Decays:

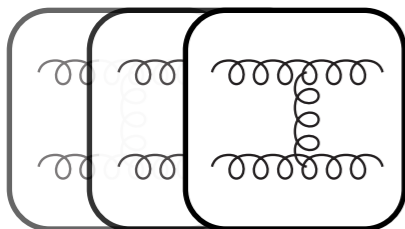
Use (N)LO matrix elements

→ Sets "hard" resolution scale for process: Q_{MAX}



Initial- & Final-State Radiation (ISR & FSR):

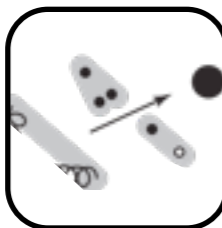
Altarelli-Parisi equations → differential evolution, dP/dQ^2 , as function of resolution scale; run from Q_{MAX} to ~ 1 GeV (More later)



MPI (Multi-Parton Interactions)

Additional (soft) parton-parton interactions: LO matrix elements

→ Additional (soft) "Underlying-Event" activity



Hadronization

Non-perturbative model of color-singlet parton systems → hadrons

(PYTHIA)



PYTHIA anno 1978 (then called JETSET)

LU TP 78-18
November, 1978

A Monte Carlo Program for Quark Jet
Generation

T. Sjöstrand, B. Söderberg

A Monte Carlo computer program is presented, that simulates the fragmentation of a fast parton into a jet of mesons. It uses an iterative scaling scheme and is compatible with the jet model of Field and Feynman.

Note:

Field-Feynman was an early fragmentation model
Now superseded by the String (in PYTHIA) and
Cluster (in HERWIG & SHERPA) models.

```
SUBROUTINE JETGEN(N)
COMMON /JET/ K(100,2), P(100,5)
COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
IFLSGN=(10-IFLBEG)/5
W=2.*EBEG
I=0
IPD=0
C 1 FLAVOUR AND PT FOR FIRST QUARK
IFL1=IABS(IFLBEG)
PT1=SIGMA*SQRT(-ALOG(RANF(0)))
PHI1=6.2832*RANF(0)
PX1=PT1*COS(PHI1)
PY1=PT1*SIN(PHI1)
100 I=I+1
C 2 FLAVOUR AND PT FOR NEXT ANTIQUARK
IFL2=1+INT(RANF(0)/PUD)
PT2=SIGMA*SQRT(-ALOG(RANF(0)))
PHI2=6.2832*RANF(0)
PX2=PT2*COS(PHI2)
PY2=PT2*SIN(PHI2)
C 3 MESON FORMED, SPIN ADDED AND FLAVOUR MIXED
K(I,1)=MESO(3*(IFL1-1)+IFL2,IFLSGN)
ISPIN=INT(PS1+RANF(0))
K(I,2)=1+9*ISPIN+K(I,1)
IF(K(I,1).LE.6) GOTO 110
TMIX=RANF(0)
KM=K(I,1)-6+3*ISPIN
K(I,2)=8+9*ISPIN+INT(TMIX+CMIX(KM,1))+INT(TMIX+CMIX(KM,2))
C 4 MESON MASS FROM TABLE, PT FROM CONSTITUENTS
110 P(I,5)=PMAS(K(I,2))
P(I,1)=PX1+PX2
P(I,2)=PY1+PY2
PMTS=P(I,1)**2+P(I,2)**2+P(I,5)**2
C 5 RANDOM CHOICE OF X=(E+PZ)MESON/(E+PZ)AVAILABLE GIVES E AND PZ
X=RANF(0)
IF(RANF(0).LT.CX2) X=1.-X**(1./3.)
P(I,3)=(X*W-PMTS/(X*W))/2.
P(I,4)=(X*W+PMTS/(X*W))/2.
C 6 IF UNSTABLE, DECAY CHAIN INTO STABLE PARTICLES
120 IPD=IPD+1
IF(K(IPD,2).GE.8) CALL DECAY(IPD,I)
IF(IPD.LT.1.AND.I.LE.96) GOTO 120
C 7 FLAVOUR AND PT OF QUARK FORMED IN PAIR WITH ANTIQUARK ABOVE
IFL1=IFL2
PX1=-PX2
PY1=-PY2
C 8 IF ENOUGH E+PZ LEFT, GO TO 2
W=(1.-X)*W
IF(W.GT.WFIN.AND.I.LE.95) GOTO 100
N=I
RETURN
END
```

(PYTHIA)



PYTHIA anno 2013 (now called PYTHIA 8)

~ 100,000 lines of C++

What a modern MC generator has inside:

LU TP 07-28 (CPC 178 (2008) 852)
October, 2007

A Brief Introduction to PYTHIA 8.1

T. Sjöstrand, S. Mrenna, P. Skands

The Pythia program is a standard tool for the generation of high-energy collisions, comprising a coherent set of physics models for the evolution from a few-body hard process to a complex multihadronic final state. It contains a library of hard processes and models for initial- and final-state parton showers, multiple parton-parton interactions, beam remnants, string fragmentation and particle decays. It also has a set of utilities and interfaces to external programs. [...]

- Hard Processes (internal, interfaced, or via Les Houches events)
- BSM (internal or via interfaces)
- PDFs (internal or via interfaces)
- Showers (internal or inherited)
- Multiple parton interactions
- Beam Remnants
- String Fragmentation
- Decays (internal or via interfaces)
- Examples and Tutorial
- Online HTML / PHP Manual
- Utilities and interfaces to external programs

(some) Physics

cf. equivalent-photon
approximation
Weiszäcker, Williams
~ 1934

Charges Stopped
or kicked

Radiation

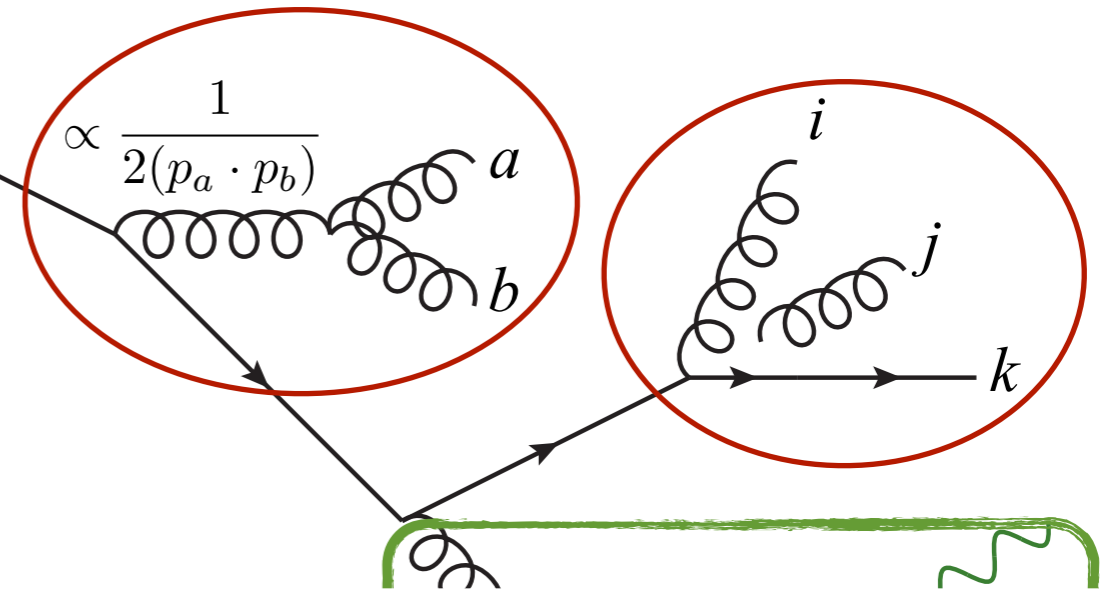
Radiation

a.k.a.
Bremsstrahlung
Synchrotron Radiation

The harder they stop, the harder the
fluctuations that continue to become radiation

Jets \approx Fractals

- **Most bremsstrahlung** is driven by divergent propagators \rightarrow simple structure
- **Amplitudes factorize in singular limits** (\rightarrow universal "conformal" or "fractal" structure)



Partons $ab \rightarrow$ "collinear": $P(z) =$ DGLAP splitting kernels, with $z =$ energy fraction $= E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\dots, a, b, \dots)|^2 \xrightarrow{a||b} g_s^2 C \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\dots, a + b, \dots)|^2$$

Gluon $j \rightarrow$ "soft": Coherence \rightarrow Parton j really emitted by (i,k) "colour antenna"

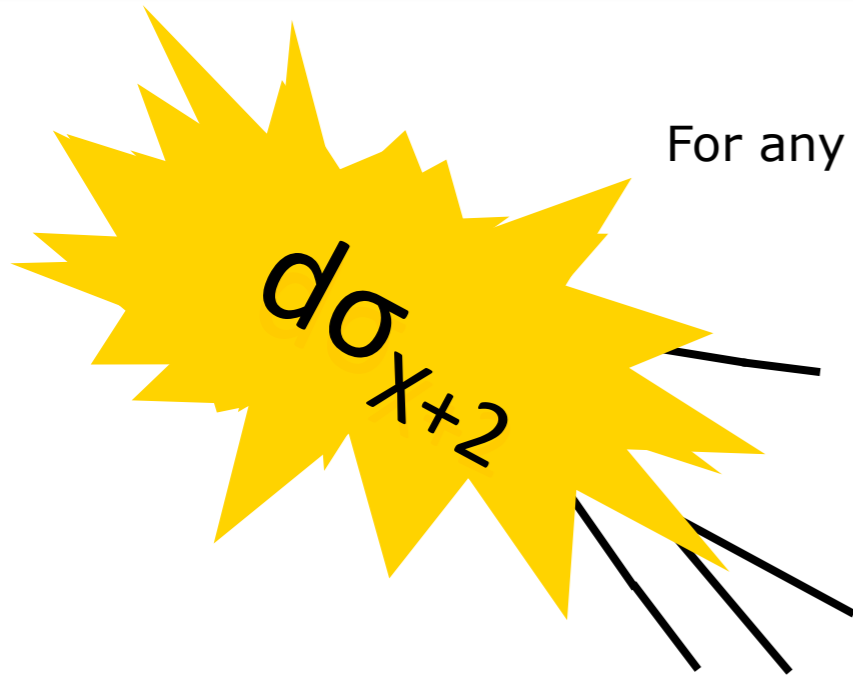
$$|\mathcal{M}_{F+1}(\dots, i, j, k, \dots)|^2 \xrightarrow{j_g \rightarrow 0} g_s^2 C \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\dots, i, k, \dots)|^2$$

+ scaling violation: $g_s^2 \rightarrow 4\pi\alpha_s(Q^2)$

See: PS, *Introduction to QCD*, TASI 2012, [arXiv:1207.2389](https://arxiv.org/abs/1207.2389)

Can apply this many times
 \rightarrow nested factorizations

Bremsstrahlung



For any basic process $d\sigma_X = \checkmark$ (calculated process by process)

$$d\sigma_{X+1} \sim N_C 2g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad \checkmark$$

$$d\sigma_{X+2} \sim N_C 2g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad \checkmark$$

$$d\sigma_{X+3} \sim N_C 2g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \dots$$

Factorization in Soft and Collinear Limits

$P(z)$: "DGLAP Splitting Functions"

$$|M(\dots, p_i, p_j \dots)|^2 \xrightarrow{i||j} g_s^2 C \frac{P(z)}{s_{ij}} |M(\dots, p_i + p_j, \dots)|^2$$

$$|M(\dots, p_i, p_j, p_k \dots)|^2 \xrightarrow{j_g \rightarrow 0} g_s^2 C \frac{2s_{ik}}{s_{ij}s_{jk}} |M(\dots, p_i, p_k, \dots)|^2$$

"Soft Eikonal" : generalizes to Dipole/Antenna Functions (more later)

Bremsstrahlung



For any basic process $d\sigma_X = \checkmark$ (calculated process by process)

$$d\sigma_{X+1} \sim N_C 2g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad \checkmark$$

$$d\sigma_{X+2} \sim N_C 2g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad \checkmark$$

$$d\sigma_{X+3} \sim N_C 2g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \dots$$

Singularities: mandated by gauge theory

Non-singular terms: process-dependent

$$\frac{|\mathcal{M}(Z^0 \rightarrow q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(Z^0 \rightarrow q_I \bar{q}_K)|^2} = g_s^2 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right]$$

SOFT COLLINEAR

$$\frac{|\mathcal{M}(H^0 \rightarrow q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(H^0 \rightarrow q_I \bar{q}_K)|^2} = g_s^2 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right]$$

SOFT COLLINEAR+F

Bremsstrahlung



For any basic process $d\sigma_X = \checkmark$ (calculated process by process)

$$d\sigma_{X+1} \sim N_C 2g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad \checkmark$$

$$d\sigma_{X+2} \sim N_C 2g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad \checkmark$$

$$d\sigma_{X+3} \sim N_C 2g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \dots$$

Iterated factorization

Gives us a universal approximation to ∞ -order tree-level cross sections.

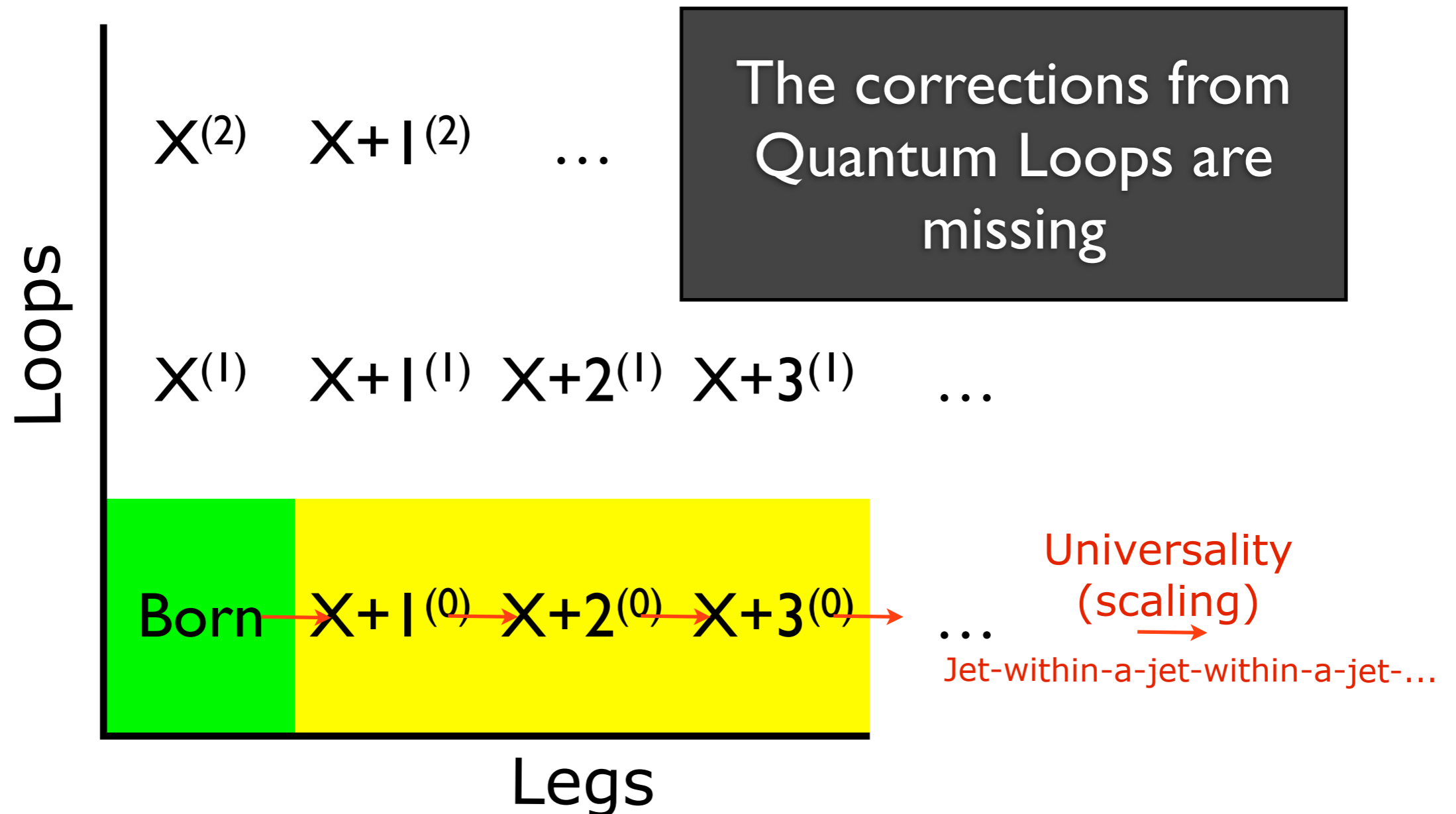
Exact in singular (strongly ordered) limit.

Finite terms (non-universal) \rightarrow Uncertainties for non-singular (hard) radiation

But something is not right ... Total σ would be infinite ...

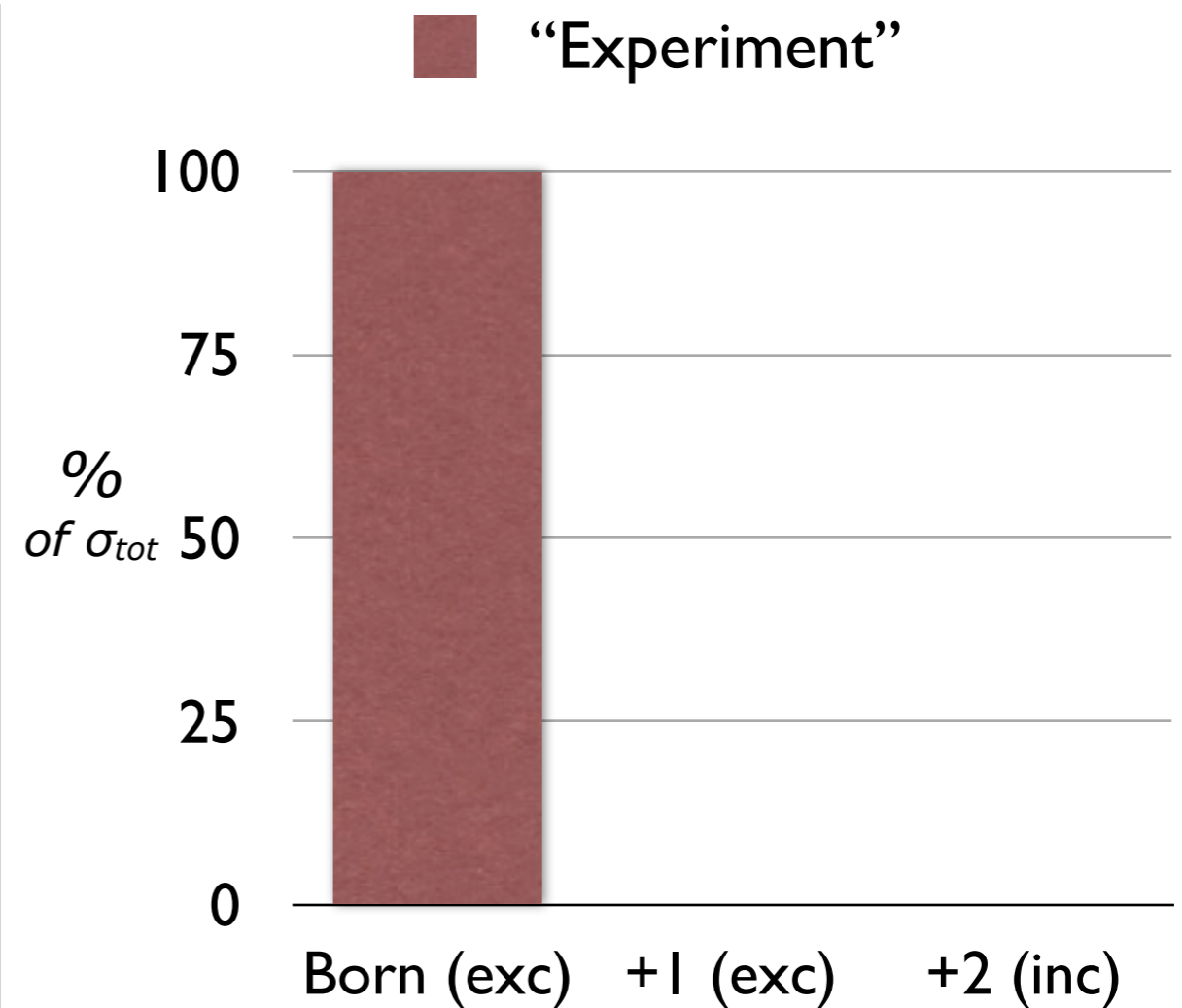
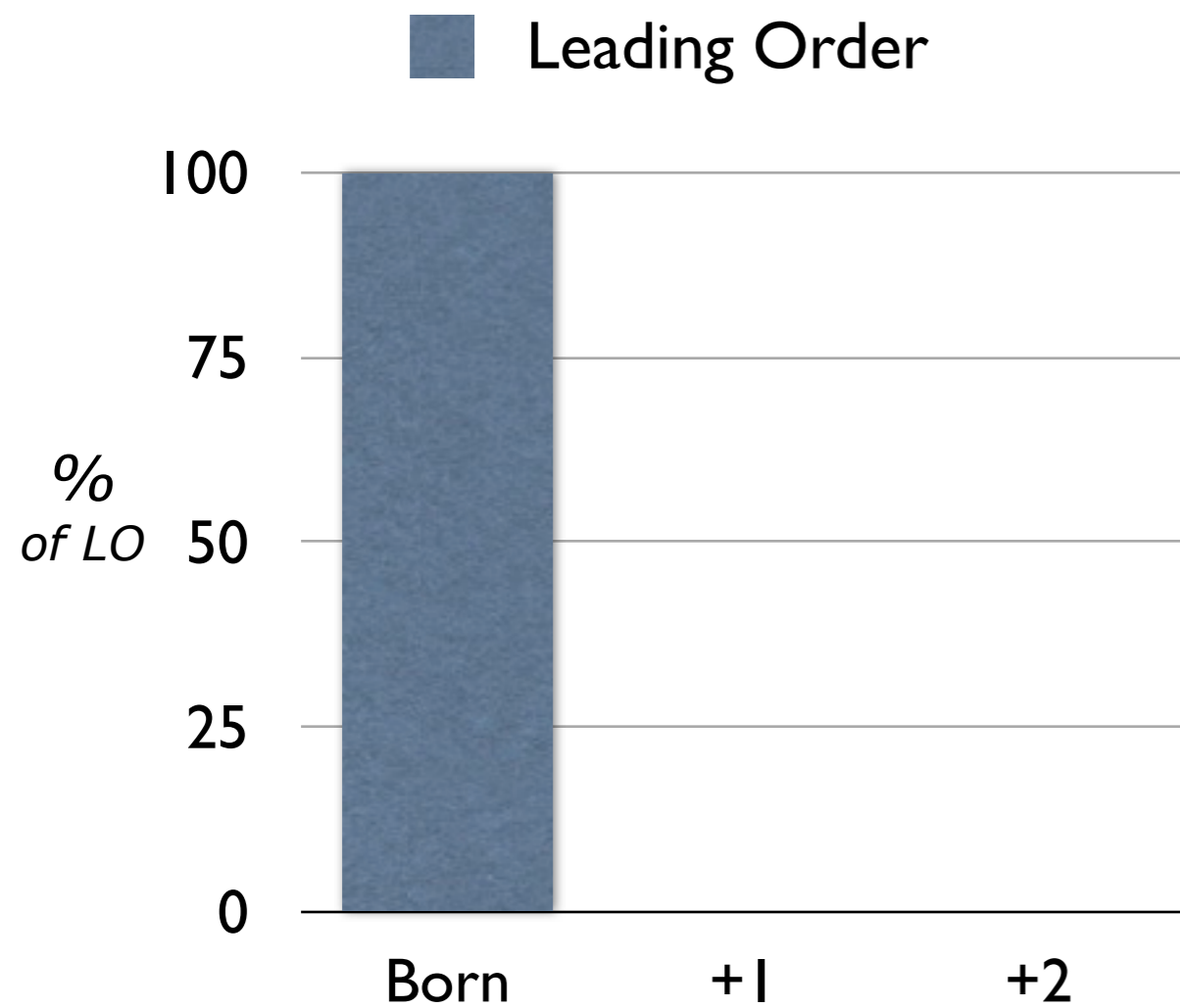
Loops and Legs

Coefficients of the Perturbative Series



Evolution

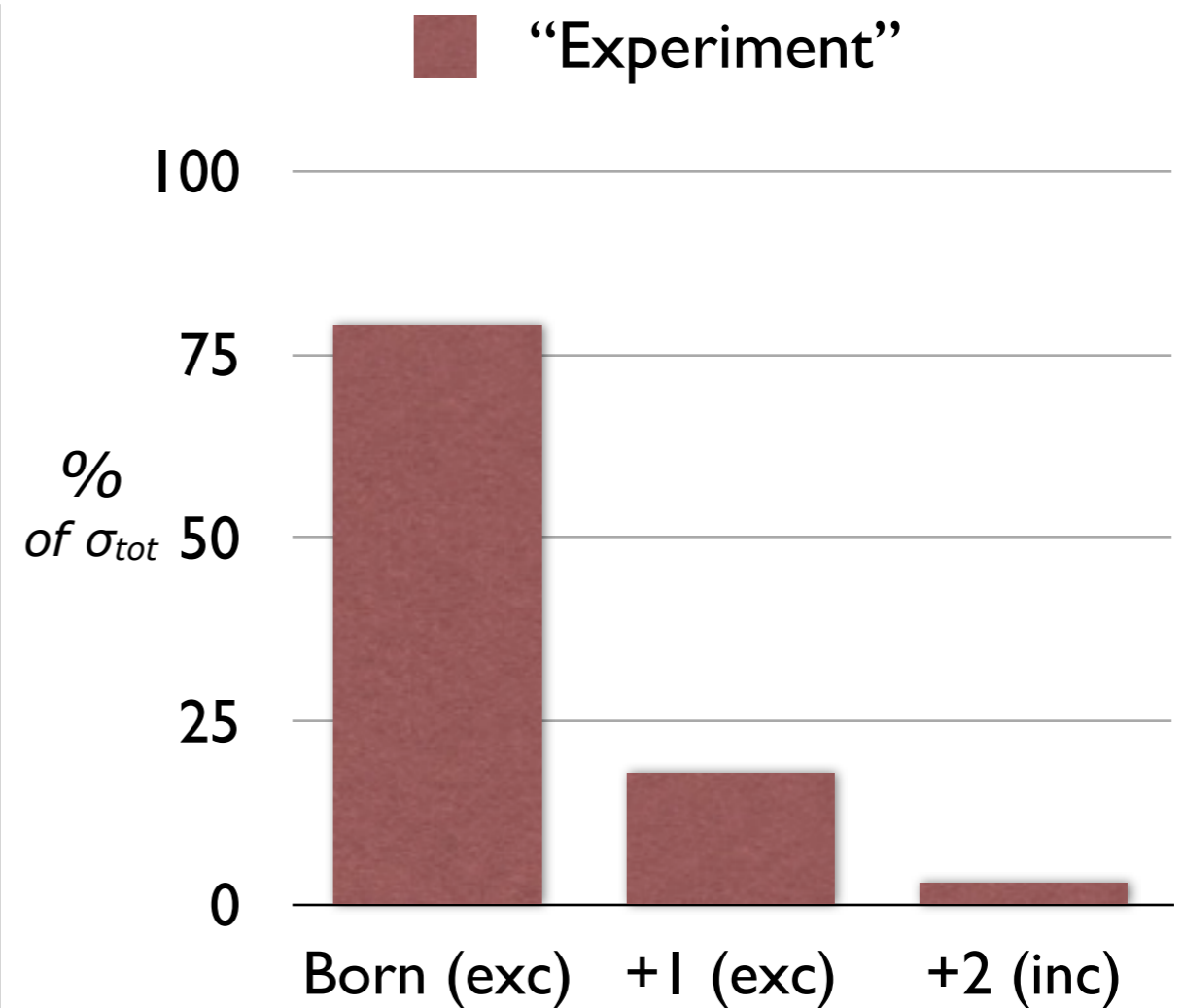
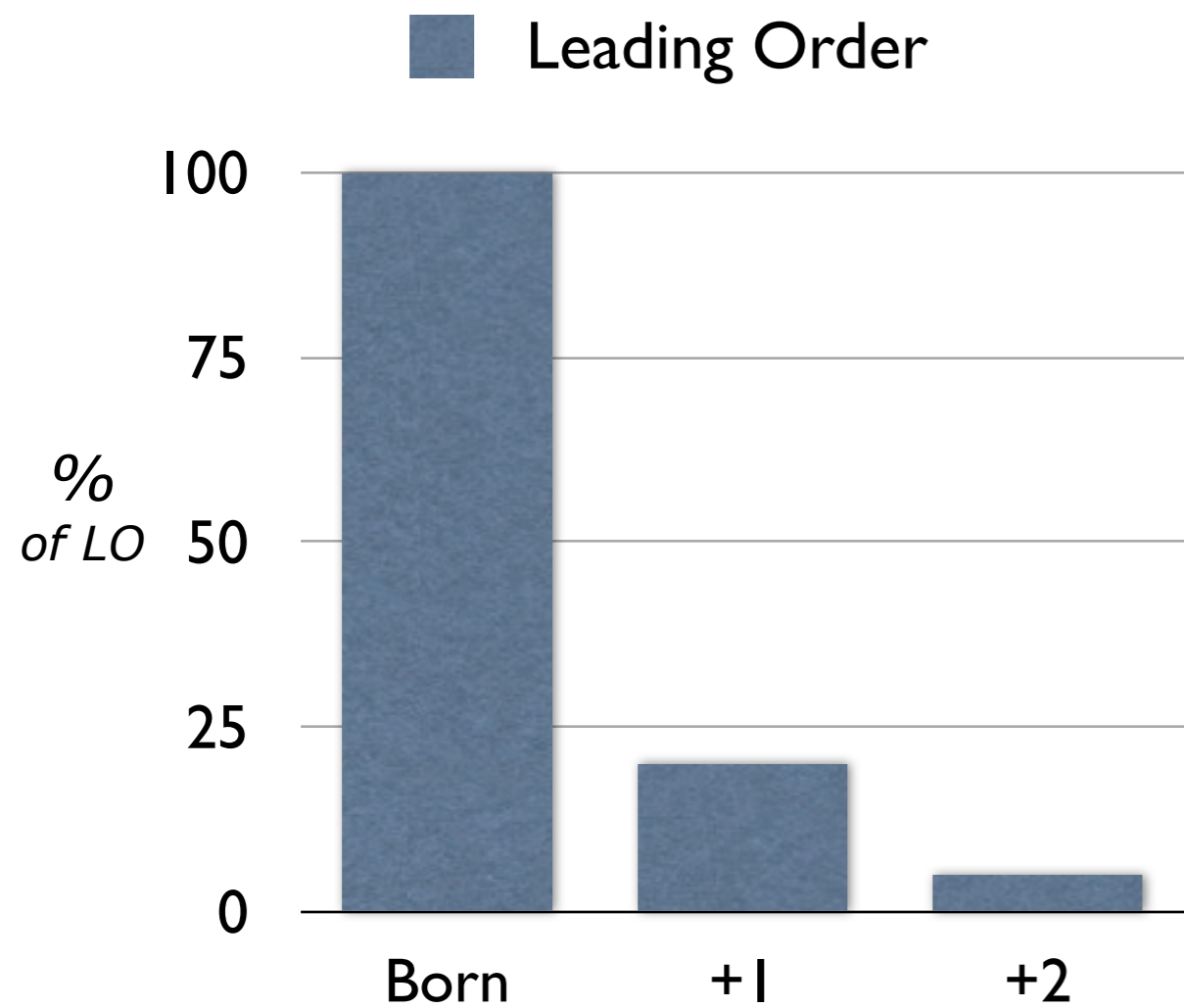
$$Q \sim Q_X$$



Exclusive = n and only n jets
Inclusive = n or more jets

Evolution

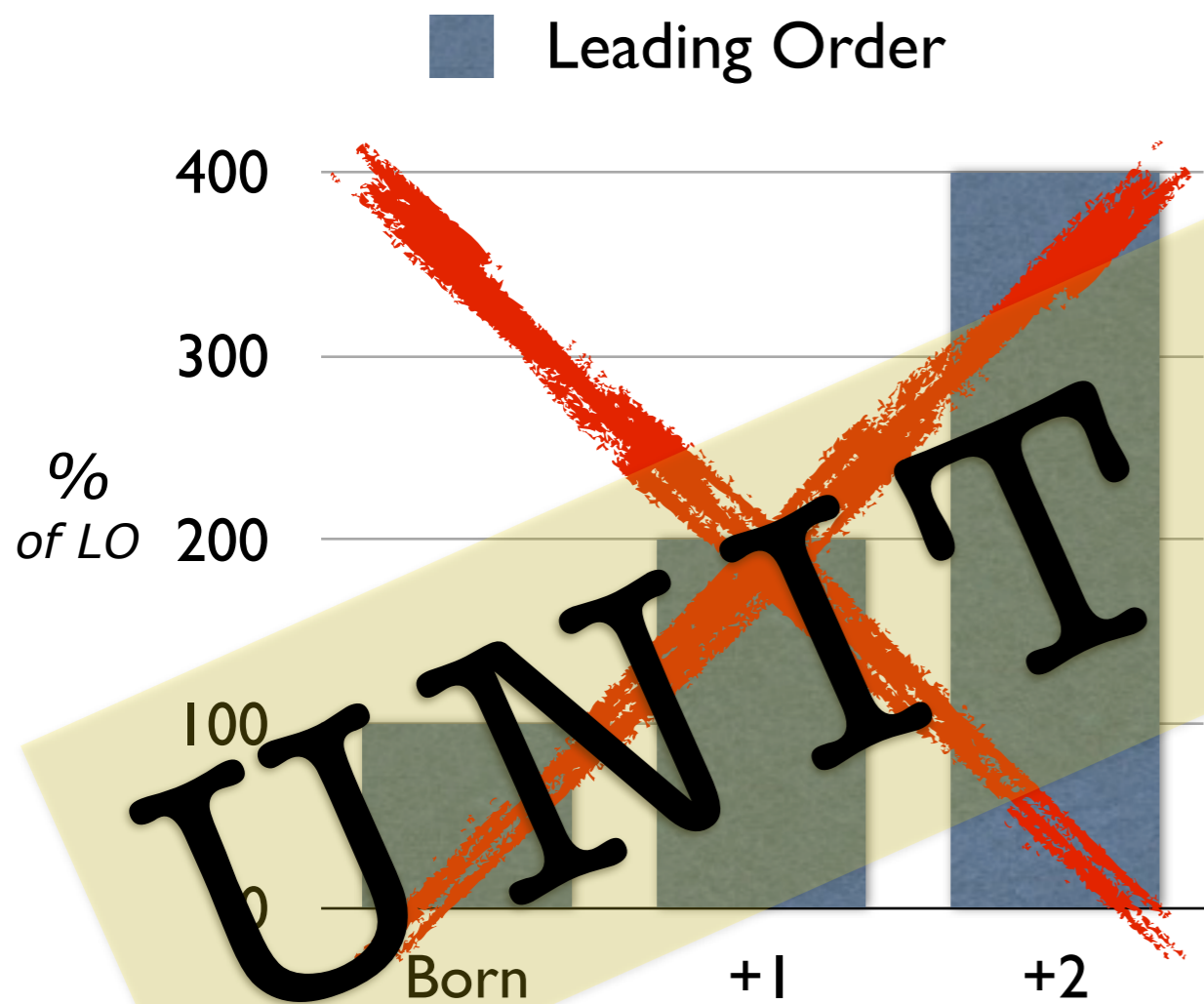
$$Q \sim \frac{Q_X}{\text{"A few"}}$$



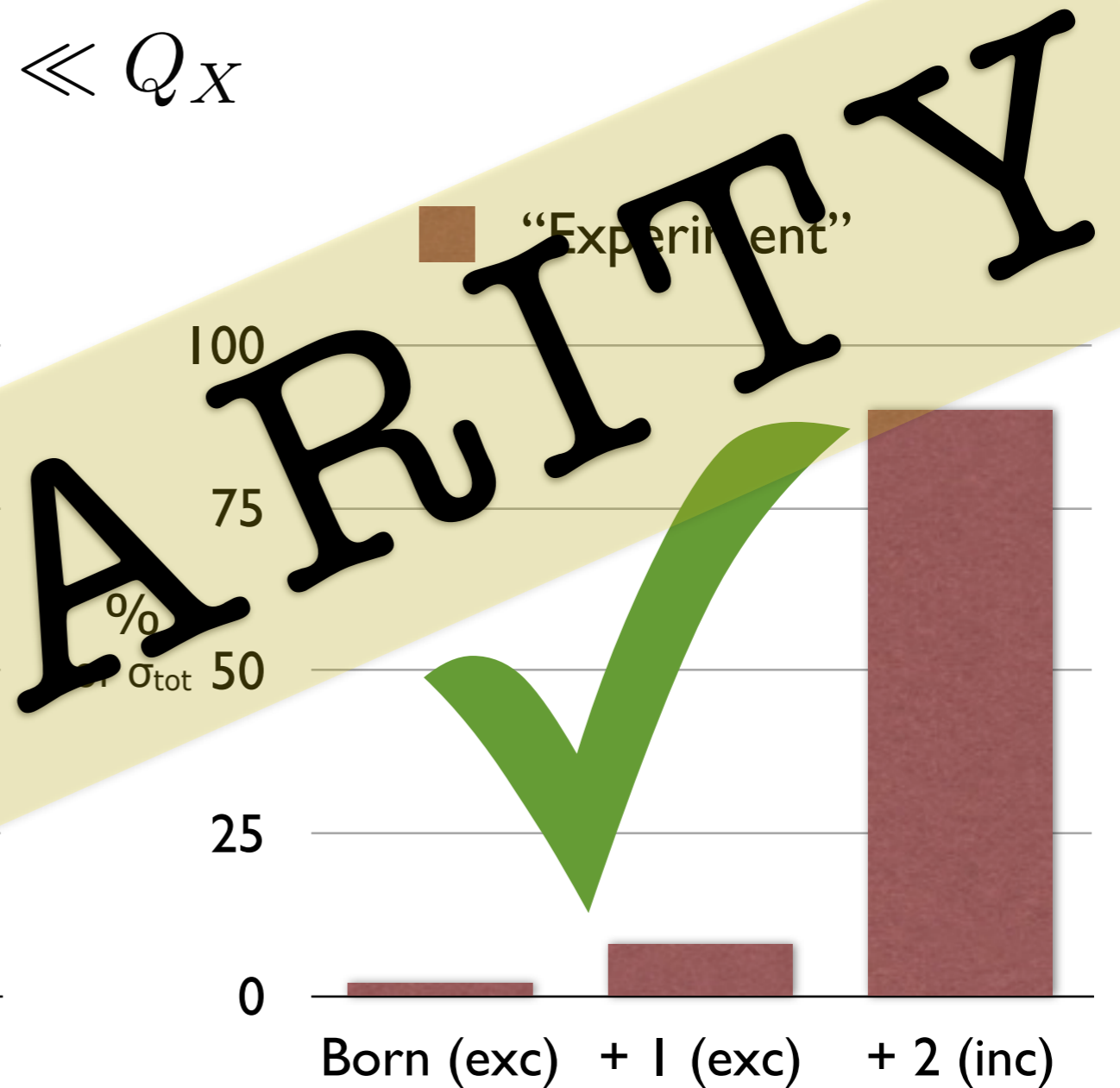
Exclusive = n and only n jets
Inclusive = n or more jets

Evolution

$$Q \ll Q_X$$



Cross Section Diverges



Cross Section Remains = Born (IR safe)
 Number of Partons Diverges (IR unsafe)

Unitarity → Evolution

Unitarity

Kinoshita-Lee-Nauenberg:
(sum over degenerate quantum states = finite)

$$\text{Loop} = - \text{Int}(\text{Tree}) + F$$

Parton Showers neglect F

→ *Leading-Logarithmic (LL) Approximation*

Imposed by Event *evolution*:

When (X) branches to (X+1):
Gain one (X+1). Lose one (X).

→ *evolution equation with kernel* $\frac{d\sigma_{X+1}}{d\sigma_X}$

Evolve in some measure of *resolution*
~ hardness, 1/time ... ~ fractal scale

→ **includes both real (tree) and virtual (loop) corrections**

- ▶ Interpretation: the structure evolves! (example: X = 2-jets)
 - Take a jet algorithm, with resolution measure “Q”, apply it to your events
 - At a very crude resolution, you find that everything is 2-jets

Evolution Equations

What we need is a differential equation

Boundary condition: a few partons defined at a high scale (Q_F)

Then evolves (or “runs”) that parton system down to a low scale (the hadronization cutoff ~ 1 GeV) \rightarrow It’s an evolution equation in Q_F

Close analogue: nuclear decay

Evolve an unstable nucleus. Check if it decays + follow chains of decays.

Decay constant

$$\frac{dP(t)}{dt} = c_N$$

Probability to remain undecayed in the time interval $[t_1, t_2]$

$$\Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} c_N dt\right) = \exp(-c_N \Delta t)$$

Decay probability per unit time

$$\frac{dP_{\text{res}}(t)}{dt} = \frac{-d\Delta}{dt} = c_N \Delta(t_1, t)$$

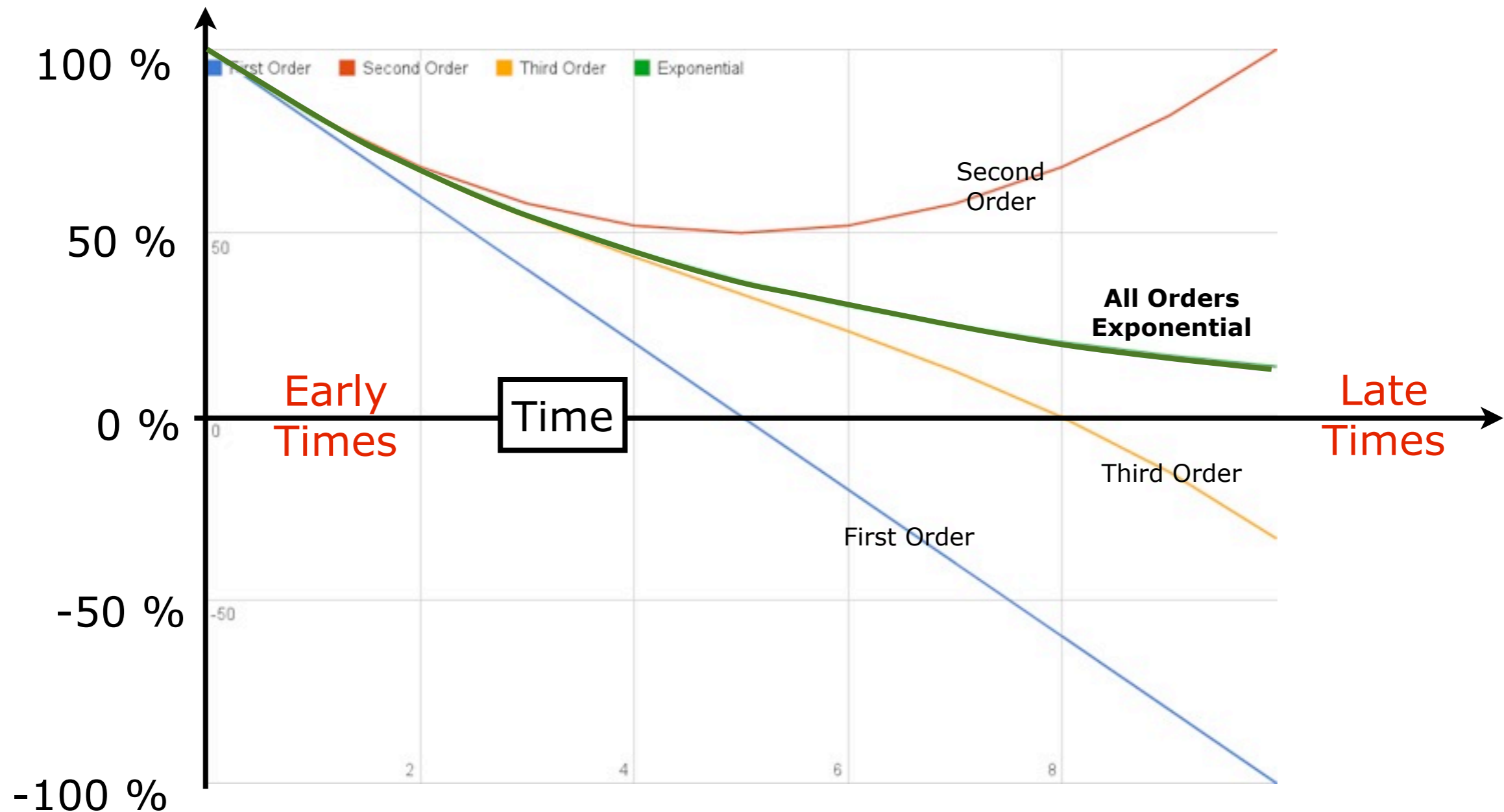
(requires that the nucleus did not already decay)

$$= 1 - c_N \Delta t + \mathcal{O}(c_N^2)$$

$\Delta(t_1, t_2)$: “Sudakov Factor”

Nuclear Decay

Nuclei remaining undecayed after time t = $\Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} dt \frac{d\mathcal{P}}{dt}\right)$



The Sudakov Factor

In nuclear decay, the Sudakov factor counts:

How many nuclei remain undecayed after a time t

Probability to remain undecayed in the time interval $[t_1, t_2]$

$$\Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} c_N dt\right) = \exp(-c_N \Delta t)$$

The Sudakov factor for a parton system counts:

The probability that the parton system doesn't evolve (branch) when we run the factorization scale ($\sim 1/\text{time}$) from a high to a low scale

Evolution probability per unit "time"

$$\frac{dP_{\text{res}}(t)}{dt} = \frac{-d\Delta}{dt} = c_N \Delta(t_1, t) \quad \begin{array}{l} \text{(replace } t \text{ by shower evolution scale)} \\ \text{(replace } c_N \text{ by proper shower evolution kernels)} \end{array}$$

What's the evolution kernel?

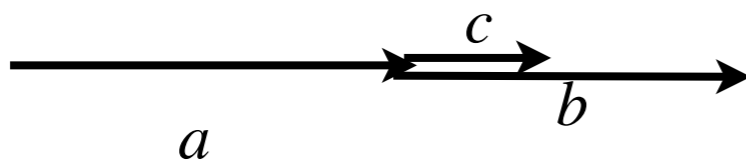
→ Yuri's lectures

DGLAP splitting functions

Can be derived (*in the collinear limit*) from requiring invariance of the physical result with respect to $Q_F \rightarrow RGE$

DGLAP
(E.g., PYTHIA)

$$d\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a \rightarrow bc}(z) dt dz .$$



$$p_b = z p_a$$

$$p_c = (1-z) p_a$$

$$P_{q \rightarrow qg}(z) = C_F \frac{1+z^2}{1-z} ,$$

$$P_{g \rightarrow gg}(z) = N_C \frac{(1-z(1-z))^2}{z(1-z)} ,$$

$$P_{g \rightarrow q\bar{q}}(z) = T_R (z^2 + (1-z)^2) ,$$

$$P_{q \rightarrow q\gamma}(z) = e_q^2 \frac{1+z^2}{1-z} ,$$

$$P_{l \rightarrow l\gamma}(z) = e_l^2 \frac{1+z^2}{1-z} ,$$

$$dt = \frac{dQ^2}{Q^2} = d \ln Q^2$$

... with Q^2 some measure of "hardness"
= event/jet resolution
measuring parton virtualities / formation time / ...

Note: there exist now also alternatives to AP kernels (with same collinear limits!): dipoles, antennae, ...

Coherence

QED: Chudakov effect (mid-fifties)



emulsion plate reduced ionization normal ionization

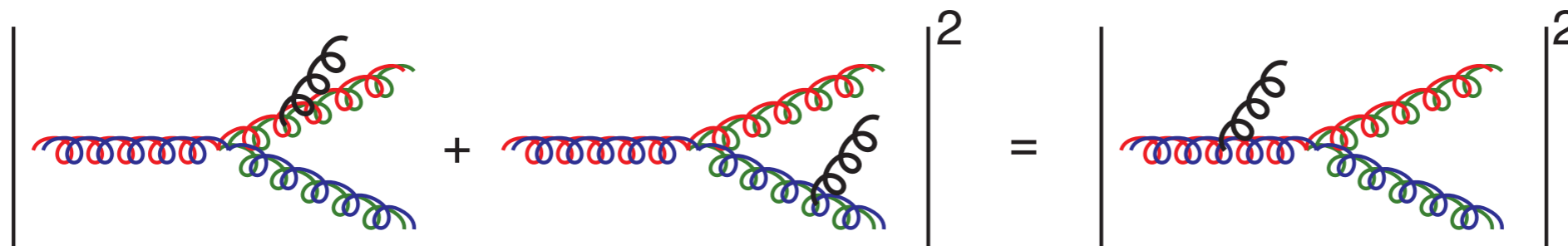
Approximations to Coherence:

Angular Ordering (HERWIG)

Angular Vetos (PYTHIA)

Coherent Dipoles/Antennae (ARIADNE, Catani-Seymour, VINCIA)

QCD: colour coherence for **soft** gluon emission



→ an example of an interference effect that can be treated probabilistically

More interference effects can be included by matching to full matrix elements

Coherence at Work

Example taken from: Ritzmann, Kosower, PS, PLB718 (2013) 1345

Example: quark-quark scattering in hadron collisions

Consider one specific phase-space point (eg scattering at 45°)

2 possible colour flows: a and b

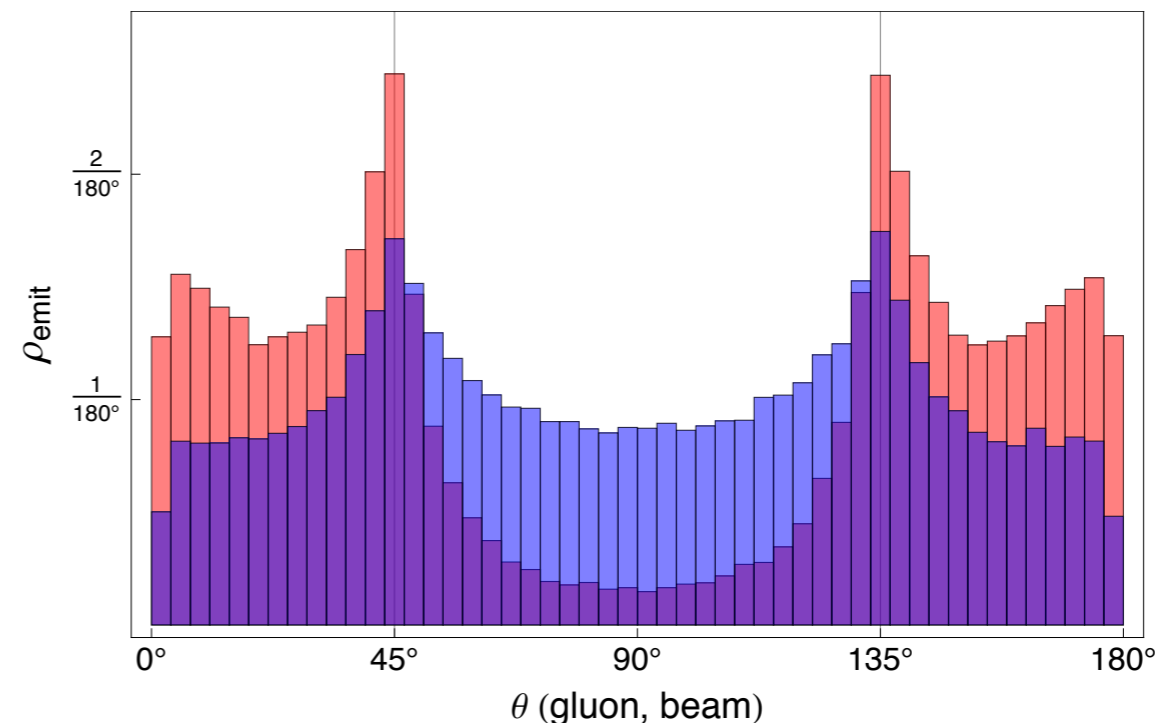
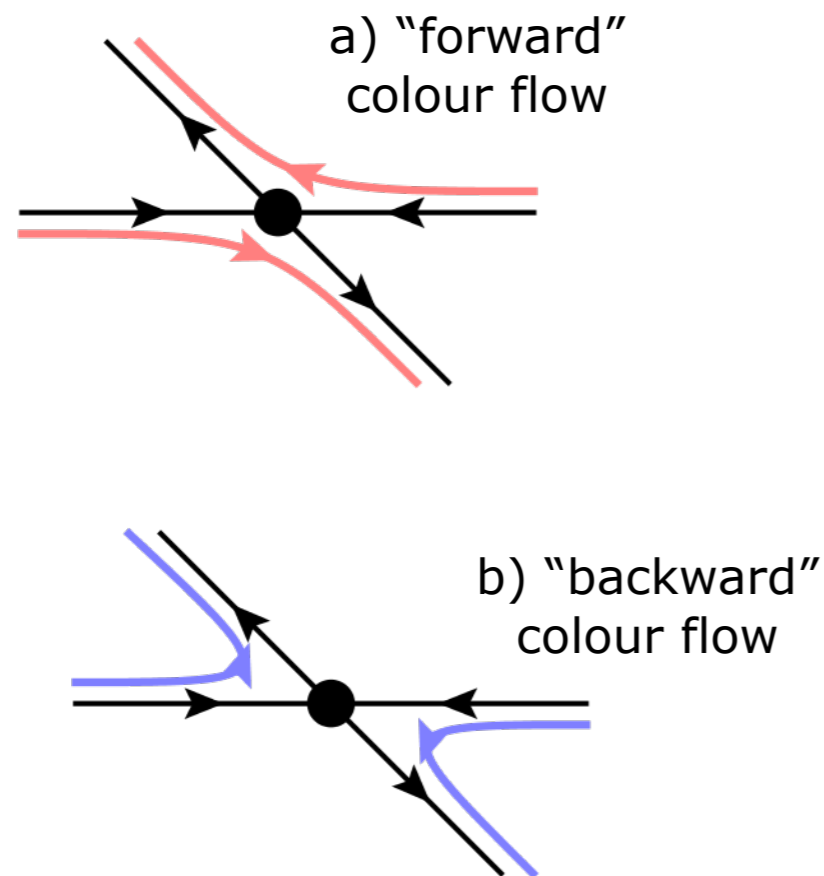


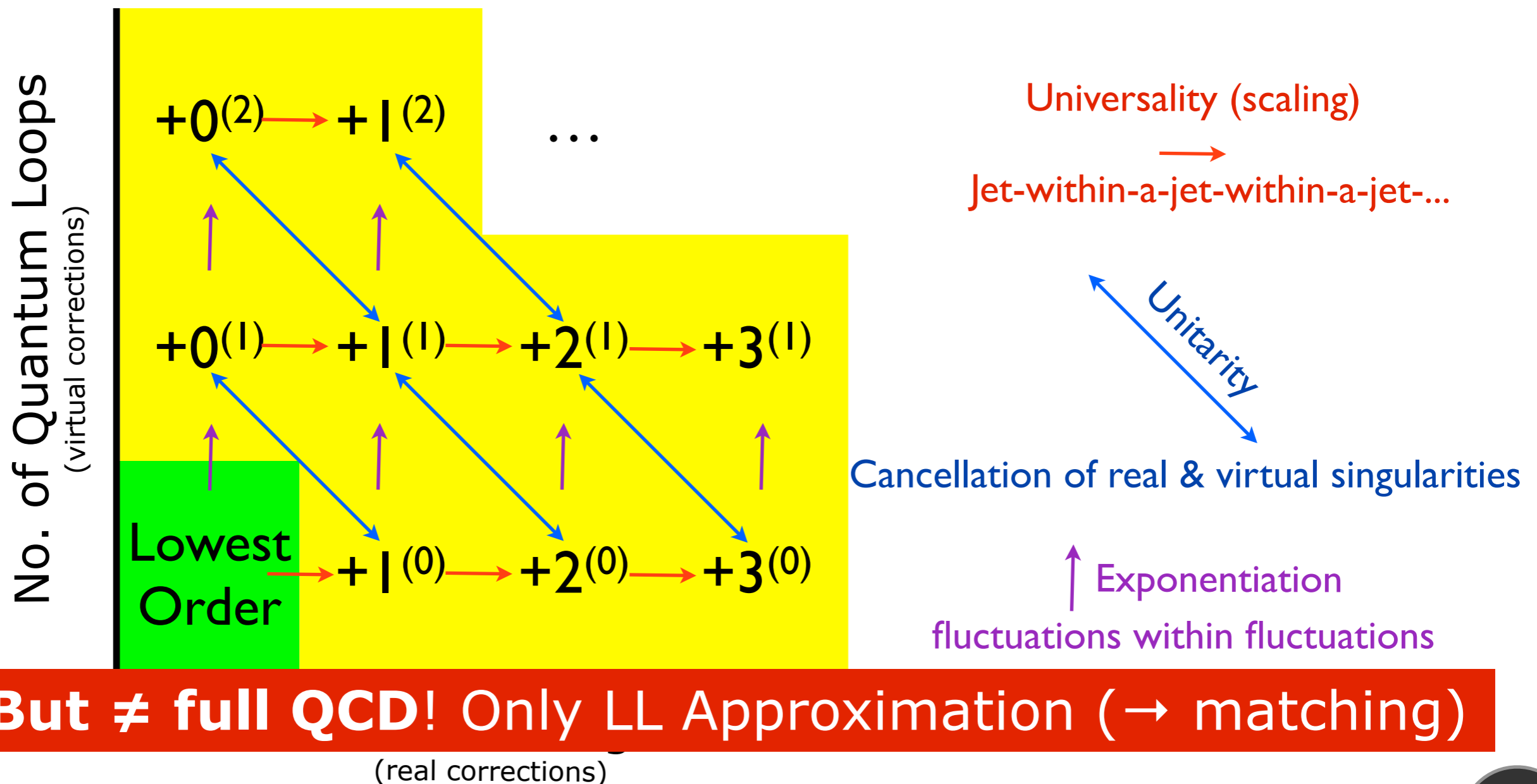
Figure 4: Angular distribution of the first gluon emission in $qq \rightarrow qq$ scattering at 45° , for the two different color flows. The light (red) histogram shows the emission density for the forward flow, and the dark (blue) histogram shows the emission density for the backward flow.

Another good recent example is the SM contribution to the Tevatron top-quark forward-backward asymmetry from coherent showers, see: PS, Webber, Winter, JHEP 1207 (2012) 151

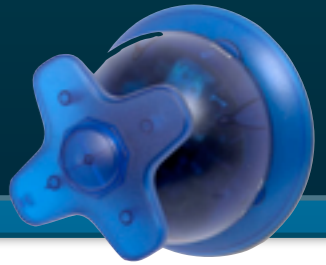
Bootstrapped Perturbation Theory

Start from an **arbitrary lowest-order** process (green = QFT amplitude squared)

Parton showers generate the bremsstrahlung terms of the rest of the perturbative series (approximate infinite-order resummation)



The Shower Operator



$$\text{Born} \quad \left. \frac{d\sigma_H}{d\mathcal{O}} \right|_{\text{Born}} = \int d\Phi_H |M_H^{(0)}|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))$$

H = Hard process
{p} : partons

But instead of evaluating \mathcal{O} directly on the Born final state,
first insert a showering operator

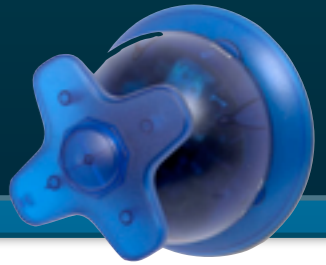
$$\text{Born} + \text{shower} \quad \left. \frac{d\sigma_H}{d\mathcal{O}} \right|_{\mathcal{S}} = \int d\Phi_H |M_H^{(0)}|^2 \mathcal{S}(\{p\}_H, \mathcal{O})$$

{p} : partons
S : showering operator

Unitarity: to first order, S does nothing

$$\mathcal{S}(\{p\}_H, \mathcal{O}) = \delta(\mathcal{O} - \mathcal{O}(\{p\}_H)) + \mathcal{O}(\alpha_s)$$

The Shower Operator



To ALL Orders

(Markov Chain)

$$S(\{p\}_X, \mathcal{O}) = \Delta(t_{\text{start}}, t_{\text{had}}) \delta(\mathcal{O} - \mathcal{O}(\{p\}_X))$$

“Nothing Happens” → “Evaluate Observable”

$$- \int_{t_{\text{start}}}^{t_{\text{had}}} dt \frac{d\Delta(t_{\text{start}}, t)}{dt} S(\{p\}_{X+1}, \mathcal{O})$$

“Something Happens” → “Continue Shower”

All-orders Probability that nothing happens

$$\Delta(t_1, t_2) = \exp \left(- \int_{t_1}^{t_2} dt \frac{d\mathcal{P}}{dt} \right)$$

(Exponentiation)
Analogous to nuclear decay
 $N(t) \approx N(0) \exp(-ct)$

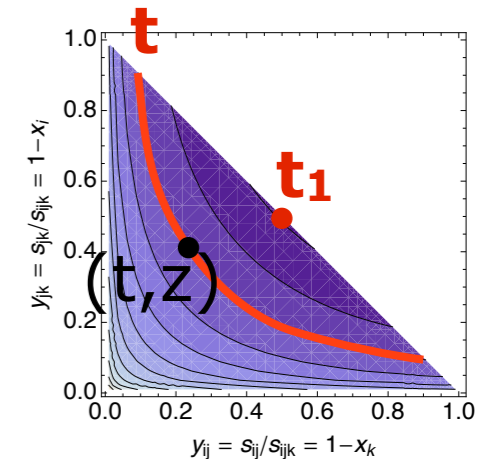
A Shower Algorithm

Note: on this slide, I use results from the theory of Random numbers, interesting in itself but would need more time to give details

1. Generate Random Number, $R \in [0,1]$

Solve equation $R = \Delta(t_1, t)$ for t (with starting scale t_1)

Analytically for simple splitting kernels,
else numerically (or by trial+veto)
→ t scale for next branching



2. Generate another Random Number, $R_z \in [0,1]$

To find second (linearly independent) phase-space invariant

Solve equation $R_z = \frac{I_z(z, t)}{I_z(z_{\max}(t), t)}$ for z (at scale t)

With the "primitive function" $I_z(z, t) = \int_{z_{\min}(t)}^z dz \frac{d\Delta(t')}{dt'} \Big|_{t'=t}$

3. Generate a third Random Number, $R_\varphi \in [0,1]$

Solve equation $R_\varphi = \varphi/2\pi$ for φ → Can now do 3D branching

Perturbative Ambiguities

The final states generated by a shower algorithm will depend on

1. The choice of perturbative evolution variable(s) $t^{[i]}$. ← Ordering & Evolution-scale choices
2. The choice of phase-space mapping $d\Phi_{n+1}^{[i]}/d\Phi_n$. ← Recoils, kinematics
3. The choice of radiation functions a_i , as a function of the phase-space variables.
4. The choice of renormalization scale function μ_R . ← Non-singular terms, Reparametrizations, Subleading Colour
5. Choices of starting and ending scales. ← Phase-space limits / suppressions for hard radiation and choice of hadronization scale

→ gives us additional handles for uncertainty estimates, beyond just μ_R
(+ ambiguities can be reduced by including more pQCD → matching!)

Jack of All Orders, Master of None?

Nice to have all-orders solution

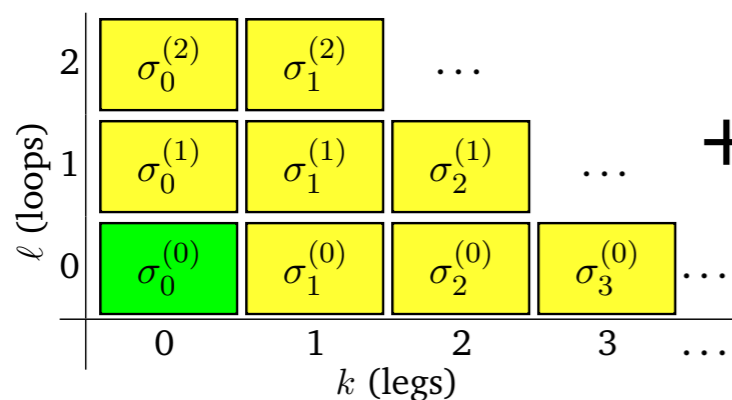
But it is only exact in the singular (soft & collinear) limits

→ gets the bulk of bremsstrahlung corrections right, but fails equally spectacularly: for hard wide-angle radiation: **visible, extra jets**

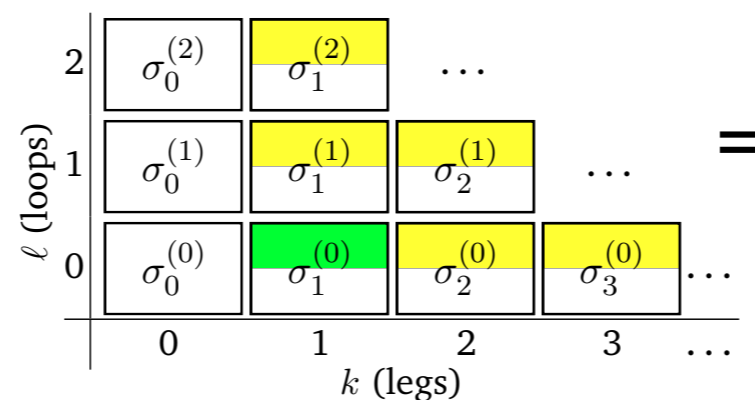
... which is exactly where fixed-order calculations work!

So combine them!

F @ LO×LL



F+1 @ LO×LL



F & F+1 @ LO×LL



See: PS, *Introduction to QCD*, TASI 2012, [arXiv:1207.2389](https://arxiv.org/abs/1207.2389)

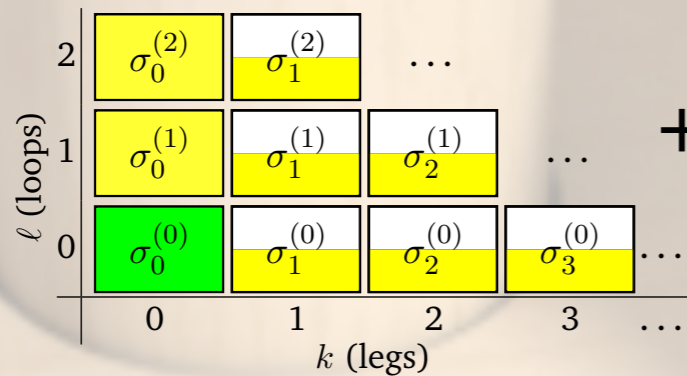
Matching 1: Slicing

Examples: MLM, CKKW, CKKW-L

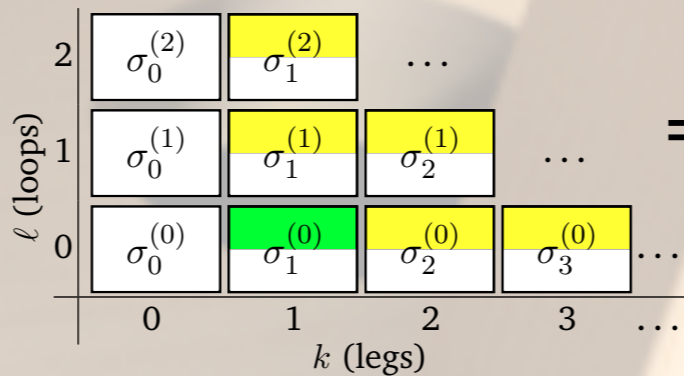
First emission: "the HERWIG correction"

Use the fact that the angular-ordered HERWIG parton shower has a "dead zone" for hard wide-angle radiation (Seymour, 1995)

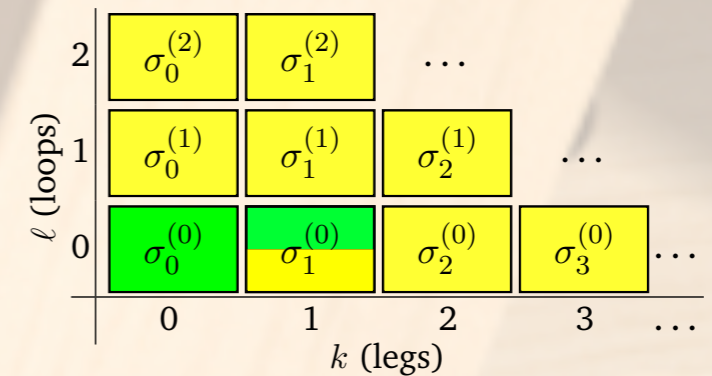
F @ LO×LL-Soft (HERWIG Shower)



F+1 @ LO×LL (HERWIG Corrections)

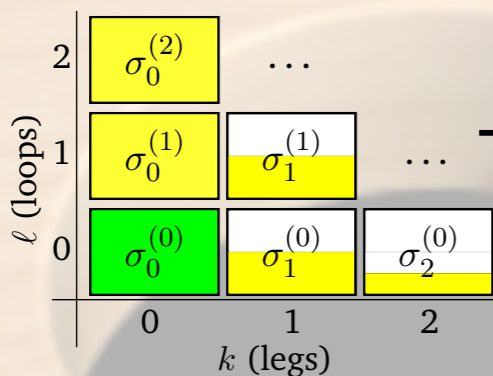


F @ LO_1×LL (HERWIG Matched)

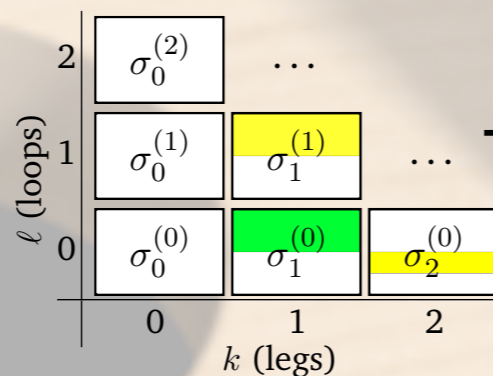


Many emissions: the MLM & CKKW-L prescriptions

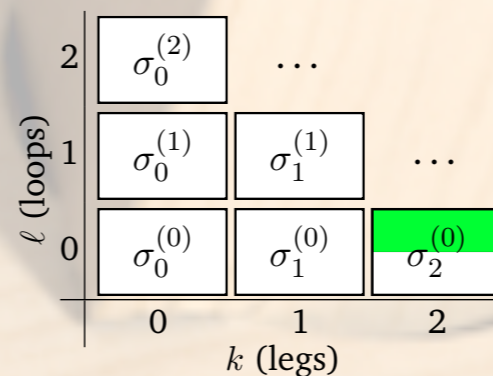
F @ LO×LL-Soft (excl)



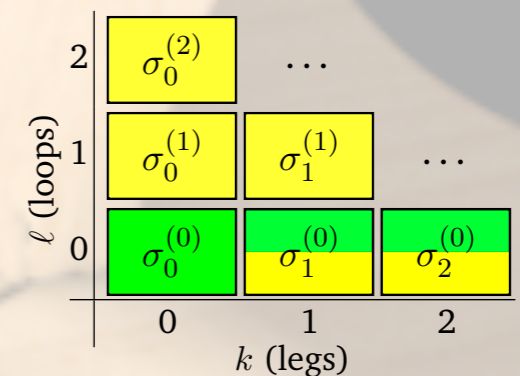
F+1 @ LO×LL-Soft (excl)



F+2 @ LO×LL (incl)



F @ LO_2×LL (MLM & (L)-CKKW)



(CKKW & Lönnblad, 2001)

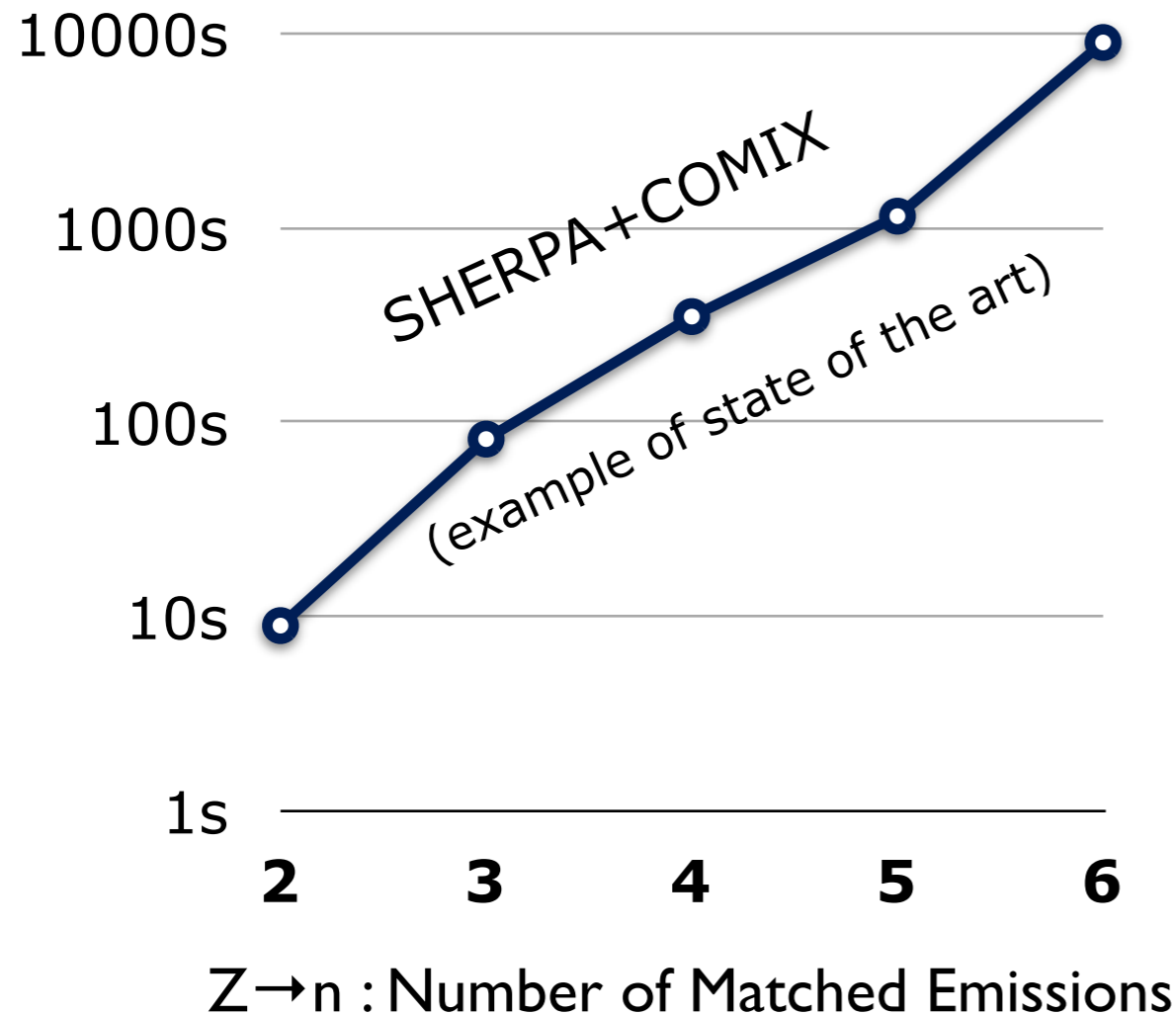
(Mangano, 2002)

(+many more recent; see Alwall et al., EPJC53(2008)473)

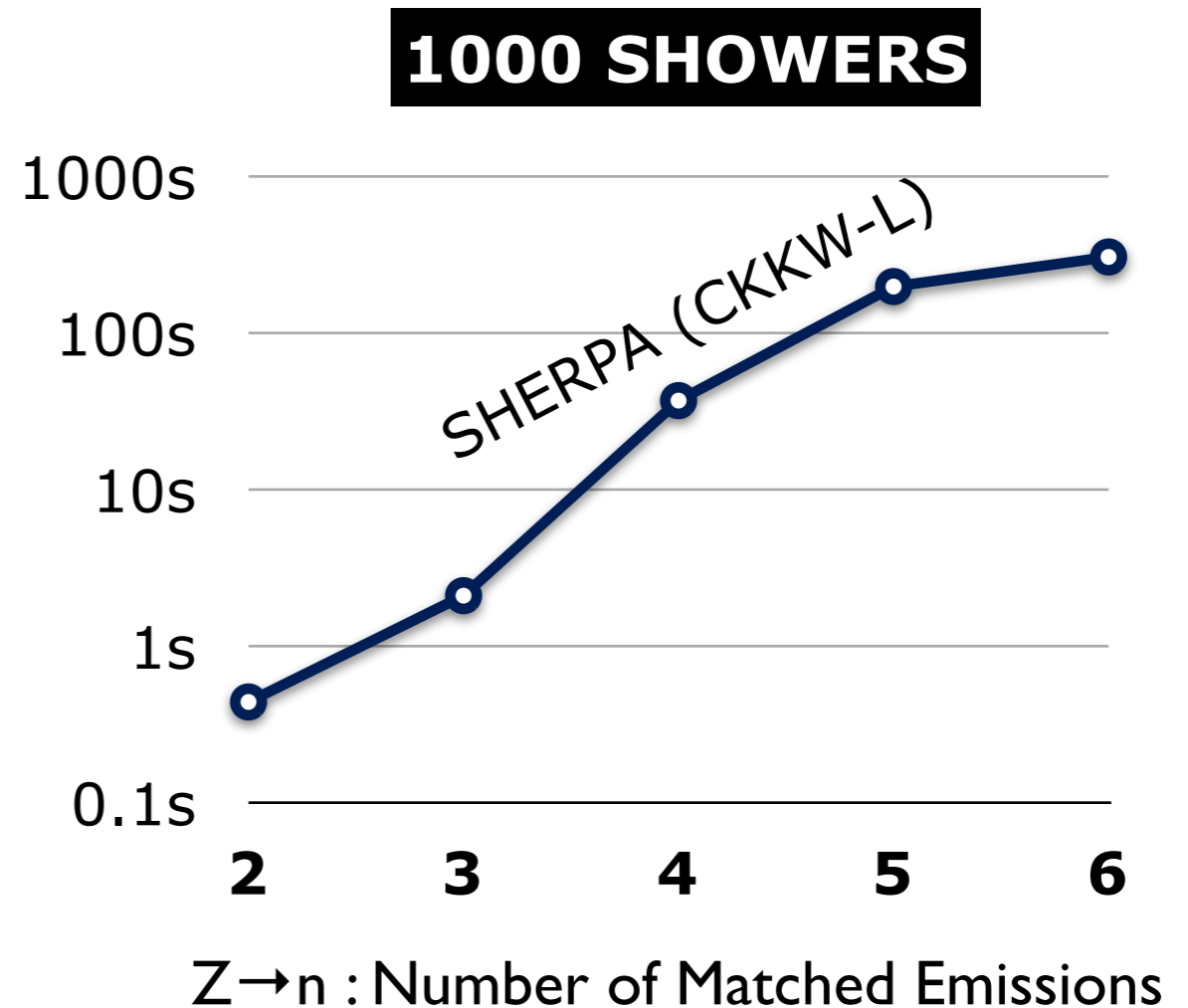
Image Credits: istockphoto

Slicing: The Cost

1. Initialization time
(to pre-compute cross sections and warm up phase-space grids)



2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)

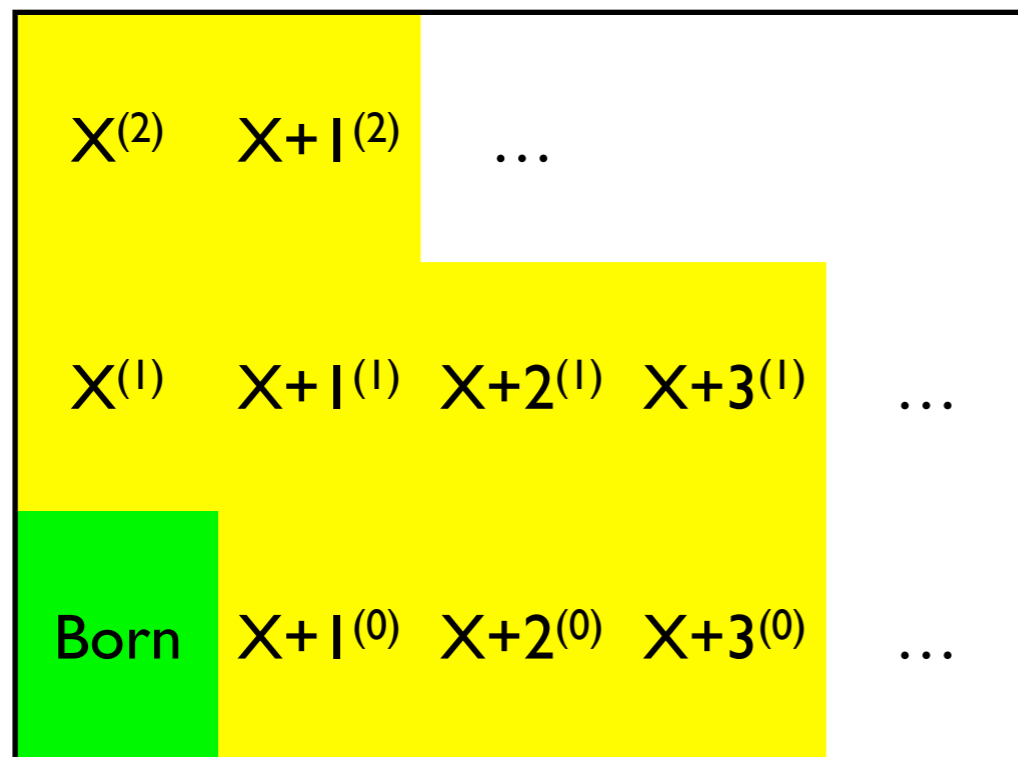


Z → udsb ; Hadronization OFF ; ISR OFF ; udsc MASSLESS ; b MASSIVE ; $E_{CM} = 91.2$ GeV ; $Q_{match} = 5$ GeV
 SHERPA 1.4.0 (+COMIX) ; PYTHIA 8.1.65 ; VINCIA 1.0.29 (+MADGRAPH 4.4.26) ;
 gcc/gfortran v 4.7.1 -O2 ; single 3.06 GHz core (4GB RAM)

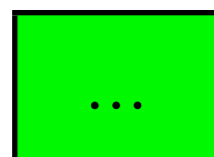
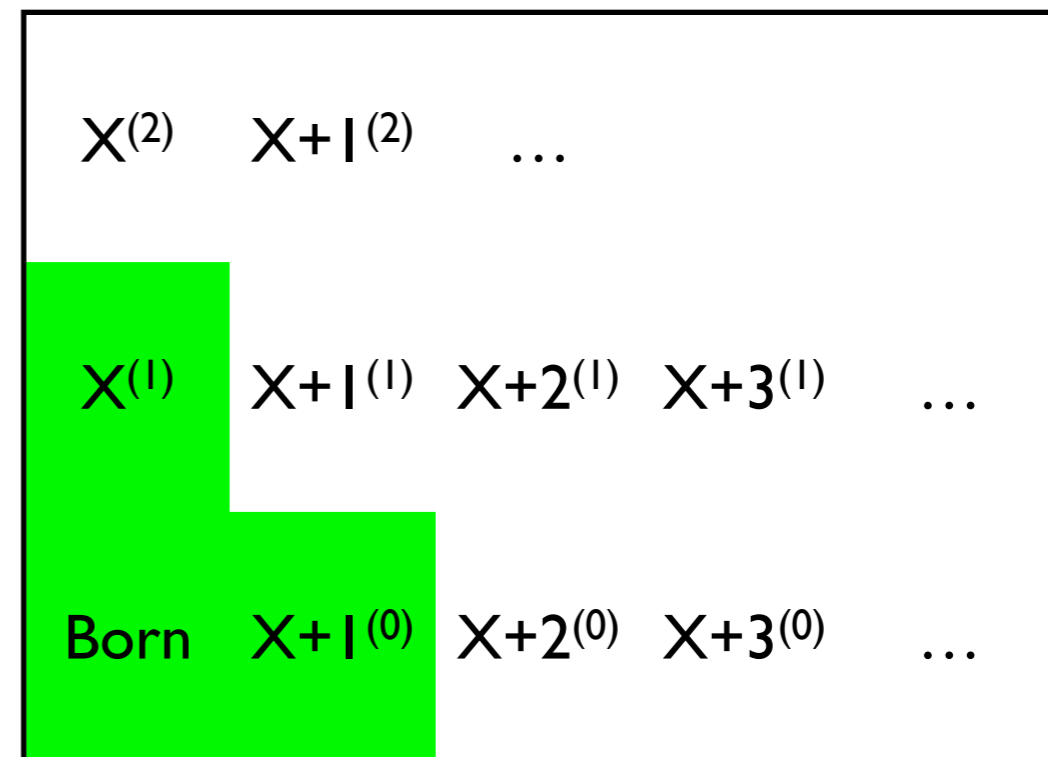
Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

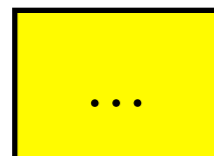
LO × Shower



NLO



Fixed-Order Matrix Element

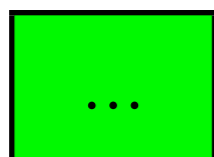
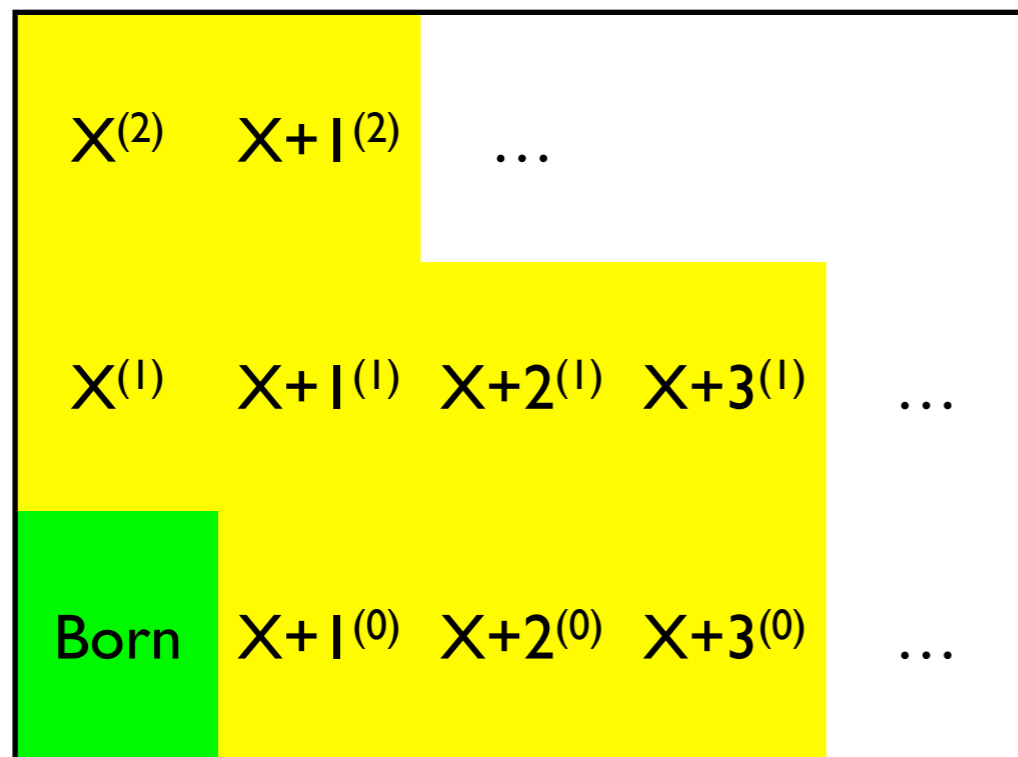


Shower Approximation

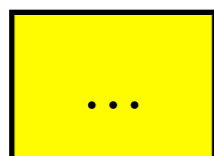
Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

LO \times Shower

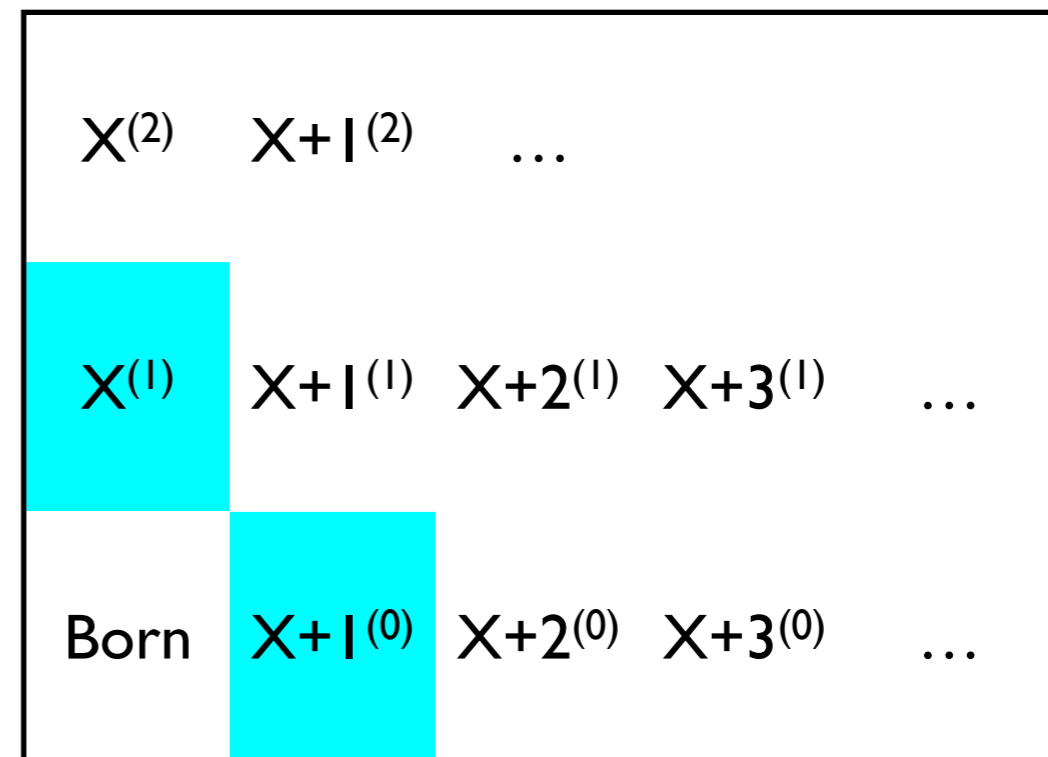


Fixed-Order Matrix Element



Shower Approximation

NLO - Shower_{NLO}



Expand shower approximation to NLO analytically, then subtract:

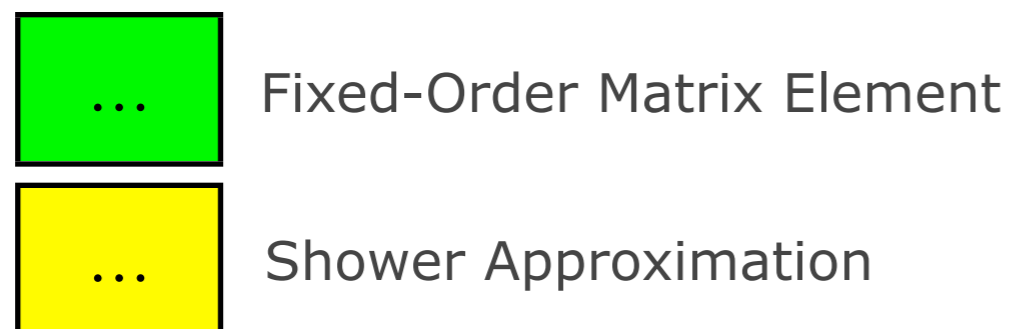
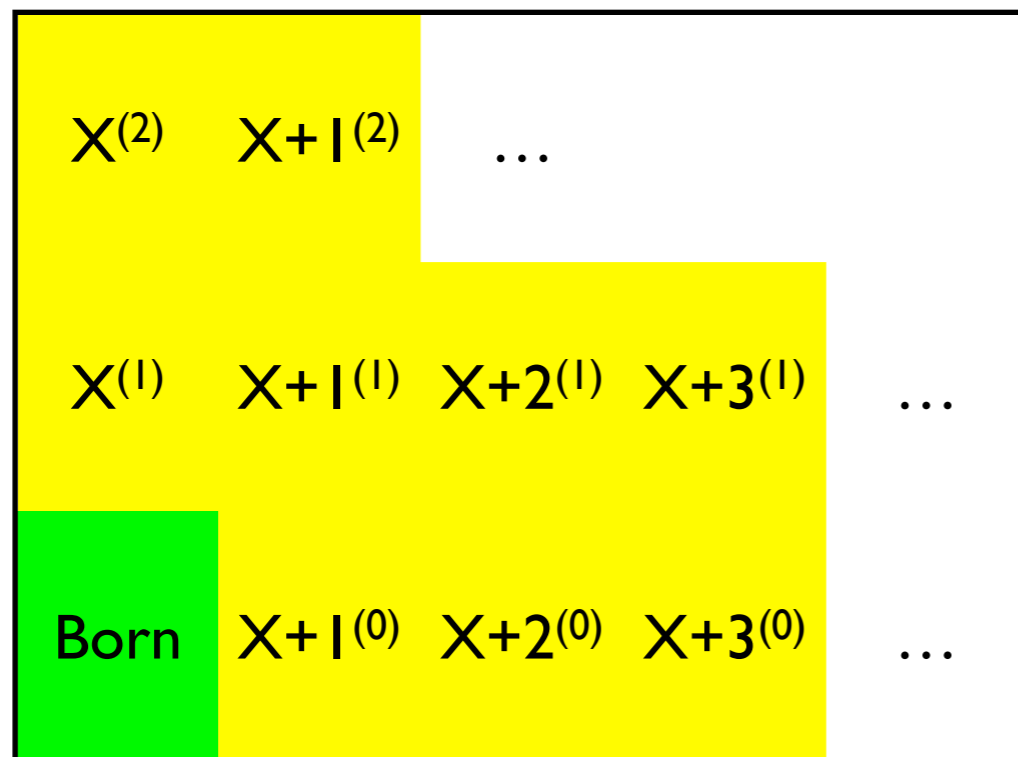


Fixed-Order ME minus Shower Approximation (NOTE: can be < 0 !)

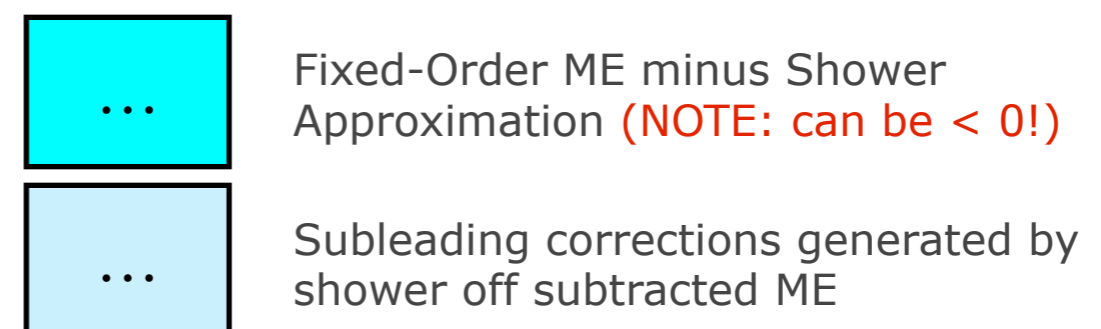
Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

LO × Shower



(NLO - Shower_{NLO}) × Shower



Matching 2: Subtraction

Examples: MC@NLO, aMC@NLO

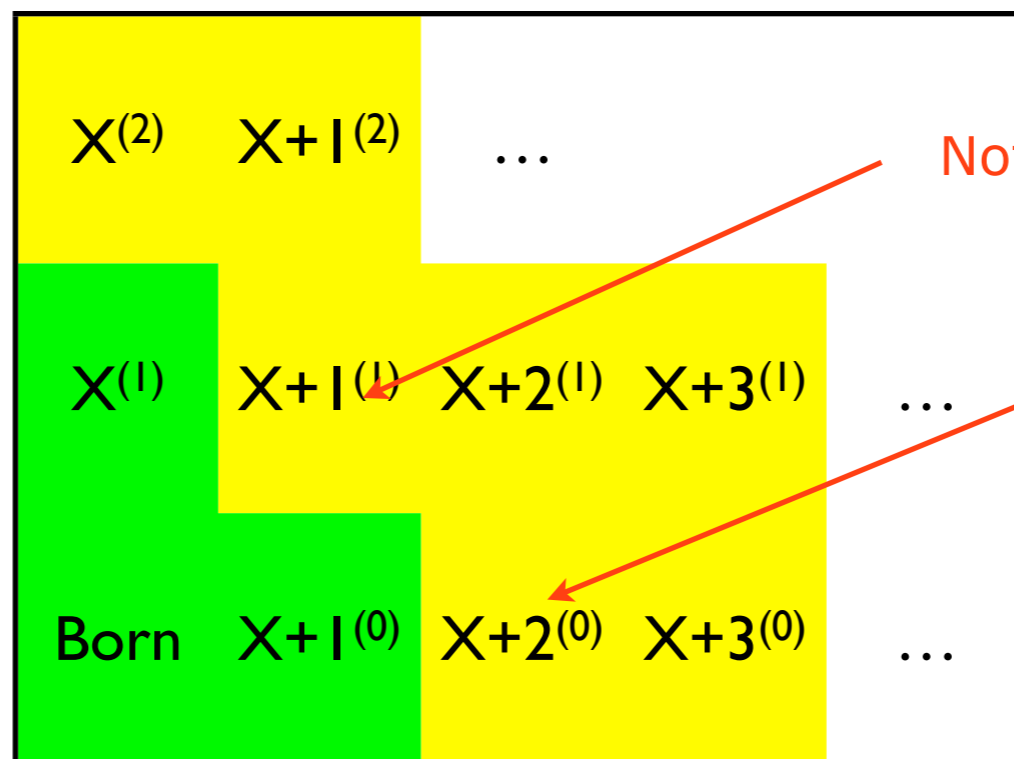
Combine \rightarrow MC@NLO Frixione, Webber, JHEP 0206 (2002) 029

Consistent NLO + parton shower (though correction events can have $w < 0$)

Recently, has been almost fully automated in aMC@NLO

Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli, JHEP 1202 (2012) 048

NLO: for X inclusive
LO for X+1
LL: for everything else



Note 1: NOT NLO for X+1

Note 2: Multijet tree-level matching still superior for X+2

NB: $w < 0$ are a problem because they kill efficiency:

Extreme example: 1000 positive-weight - 999 negative-weight events \rightarrow statistical precision of 1 event, for 2000 generated (for comparison, normal MC@NLO has $\sim 10\%$ neg-weights)

Matching 3: ME Corrections

Standard Paradigm:

Have ME for $X, X+1, \dots, X+n$;

Want to combine and add showers → “The Soft Stuff”

Double counting, IR divergences, multiscale logs

Works pretty well at low multiplicities

Still, only corrected for “hard” scales; Soft still pure LL.

At high multiplicities:

Efficiency problems: slowdown from need to compute and generate phase space from $d\sigma_{X+n}$, and from unweighting (efficiency also reduced by negative weights, if present)

Scale hierarchies: smaller single-scale phase-space region

Powers of alphaS pile up

Better Starting Point: a QCD fractal?



(shameless VINCIA promo)



(plug-in to PYTHIA 8 for ME-improved final-state showers, uses helicity matrix elements from MadGraph)

Interleaved Paradigm:

Have shower; want to improve it using ME for $X, X+1, \dots, X+n$.

Interpret all-orders shower structure as a trial distribution

Quasi-scale-invariant: intrinsically multi-scale (resums logs)

Unitary: automatically unweighted (& IR divergences \rightarrow multiplicities)

More precise expressions imprinted via veto algorithm: ME corrections at LO, NLO, ... \rightarrow soft *and* hard corrections

No additional phase-space generator or σ_{X+n} calculations \rightarrow **fast**

Automated Theory Uncertainties

For each event: vector of output weights (central value = 1)

+ Uncertainty variations. Faster than N separate samples; only one sample to analyse, pass through detector simulations, etc.

LO: Giele, Kosower, Skands, [PRD84\(2011\)054003](#)

NLO: Hartgring, Laenen, Skands, [arXiv:1303.4974](#)

Matching 3: ME Corrections

Examples: PYTHIA, POWHEG, VINCIA

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \stackrel{LL}{\sim} \sum_{i \in \text{ant}} a_i |M_F|^2$$

Correct to Matrix Element

$$a_i \rightarrow \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

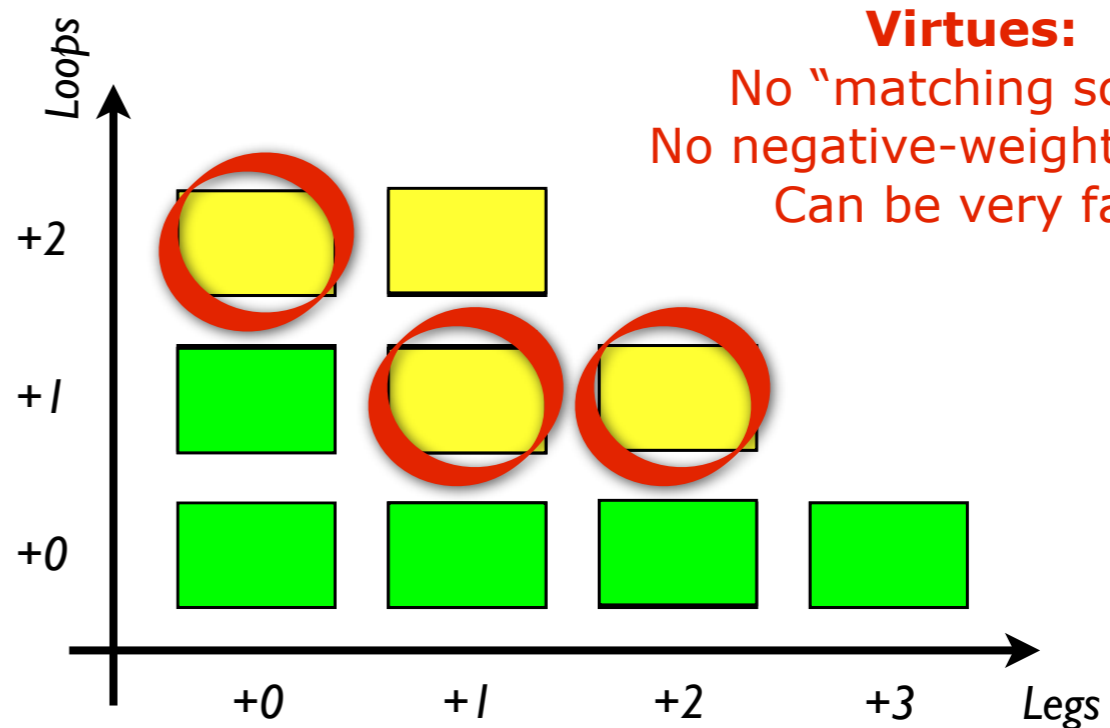
Unitarity of Shower

$$\text{Virtual} = - \int \text{Real}$$

Correct to Matrix Element

$$|M_F|^2 \rightarrow |M_F|^2 + 2\text{Re}[M_F^1 M_F^0] + \int \text{Real}$$

Repeat



Virtues:
 No "matching scale"
 No negative-weight events
 Can be very fast

First Order

PYTHIA: LO₁ corrections to most SM and BSM decay processes, and for pp → Z/W/H (Sjöstrand 1987)

POWHEG (& POWHEG BOX): LO₁ + NLO₀ corrections for generic processes (Frixione, Nason, Oleari, 2007)

Multileg NLO:

VINCIA: LO_{1,2,3,4} + NLO_{0,1} (shower plugin to PYTHIA 8; formalism for pp soon to appear) (see previous slide)

MiNLO-merged POWHEG: LO_{1,2} + NLO_{0,1} for pp → Z/W/H

UNLOPS: for generic processes (in PYTHIA 8, based on POWHEG input) (Lönnblad & Prestel, 2013)

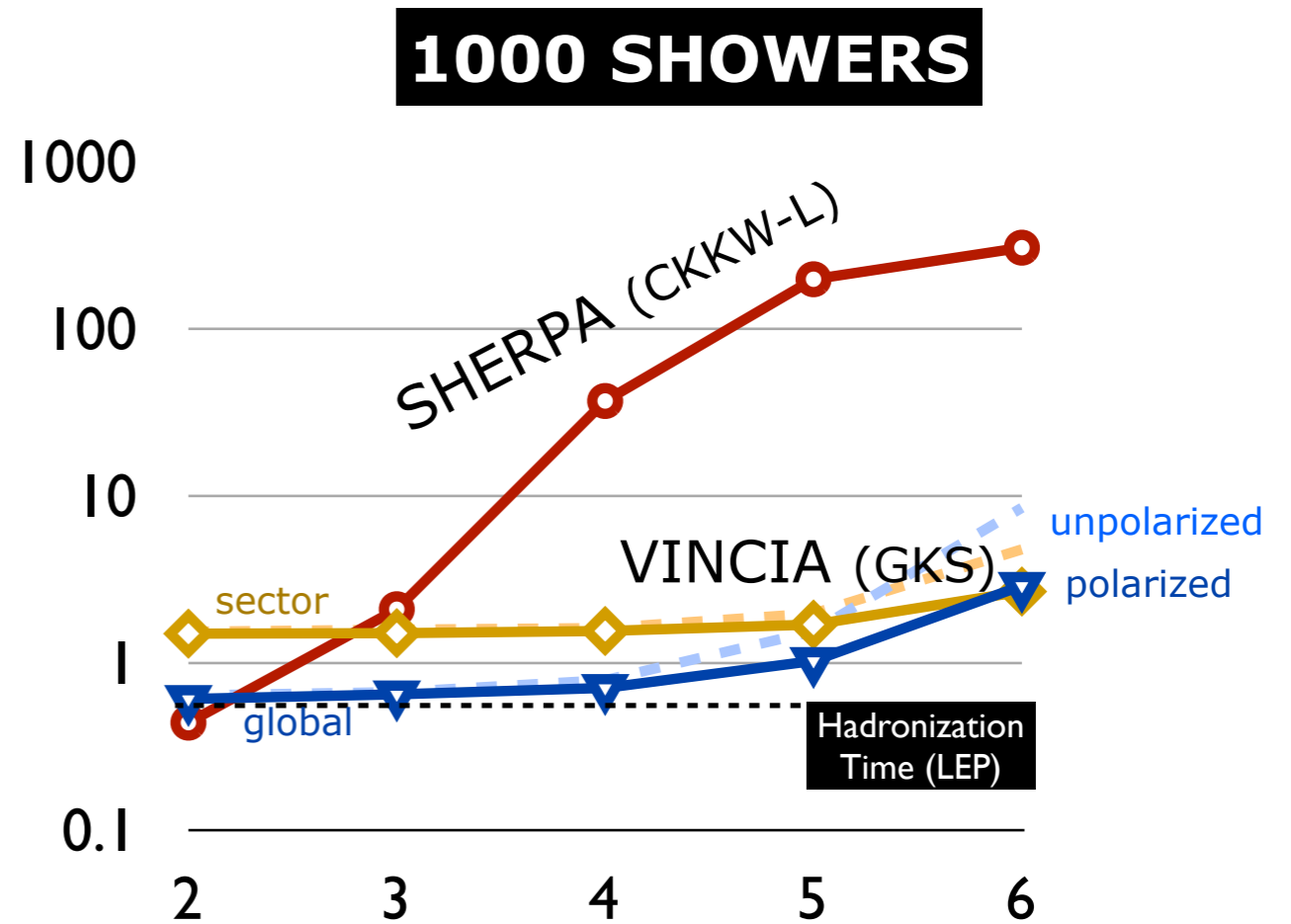
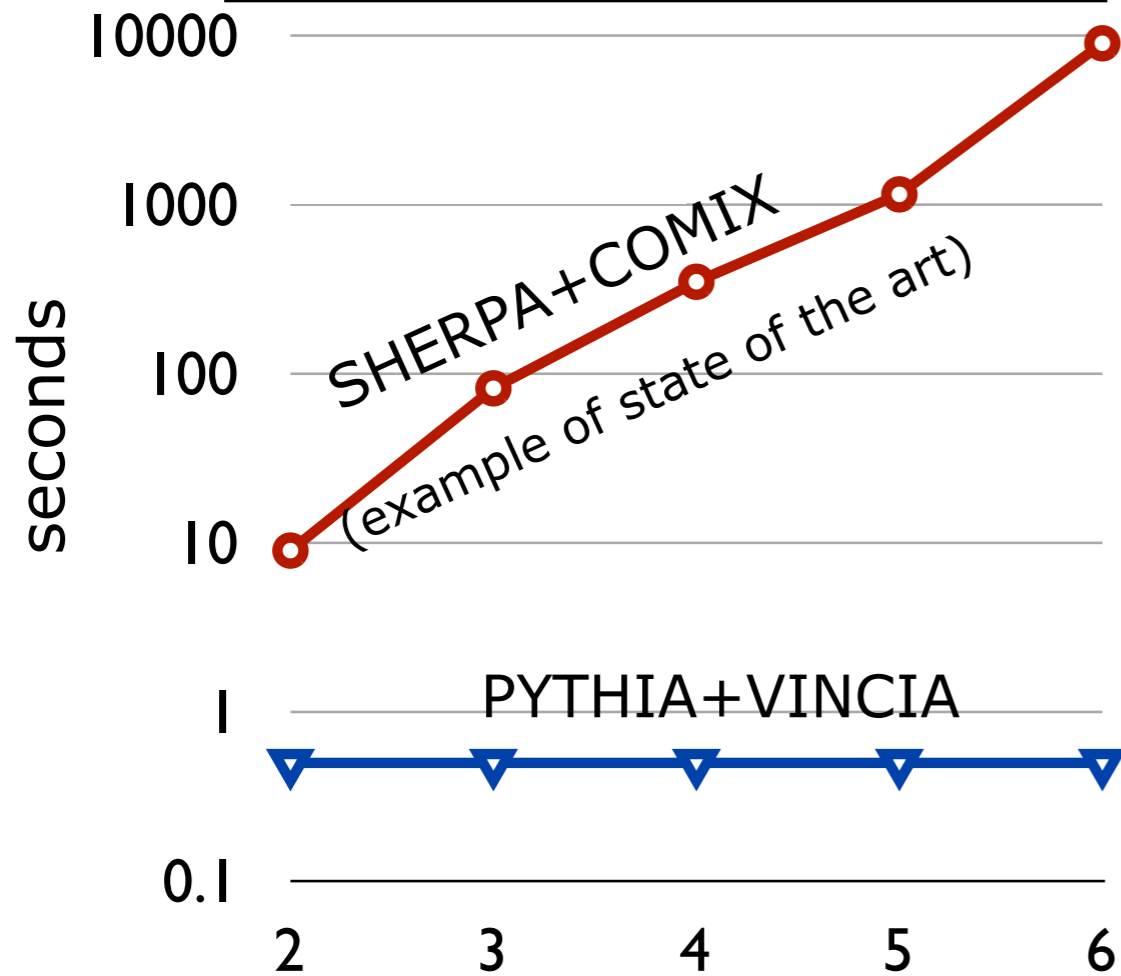


Speed

Larkoski, Lopez-Villarejo, Skands, [PRD 87 \(2013\) 054033](#)

1. Initialization time
(to pre-compute cross sections and warm up phase-space grids)

2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)



Z → n : Number of Matched Legs

Z → n : Number of Matched Legs

Z → uds c b ; Hadronization OFF ; ISR OFF ; u d s c MASSLESS ; b MASSIVE ; E_{CM} = 91.2 GeV ; Q_{match} = 5 GeV
SHERPA 1.4.0 (+COMIX) ; PYTHIA 8.1.65 ; VINCIA 1.0.29 + MADGRAPH 4.4.26 ;
gcc/gfortran v 4.7.1 -O2 ; single 3.06 GHz core (4GB RAM)

The Tyranny of Carlo



J. D. Bjorken

“Another change that I find disturbing is the rising tyranny of Carlo. No, I don’t mean that fellow who runs CERN, but the other one, with first name Monte.

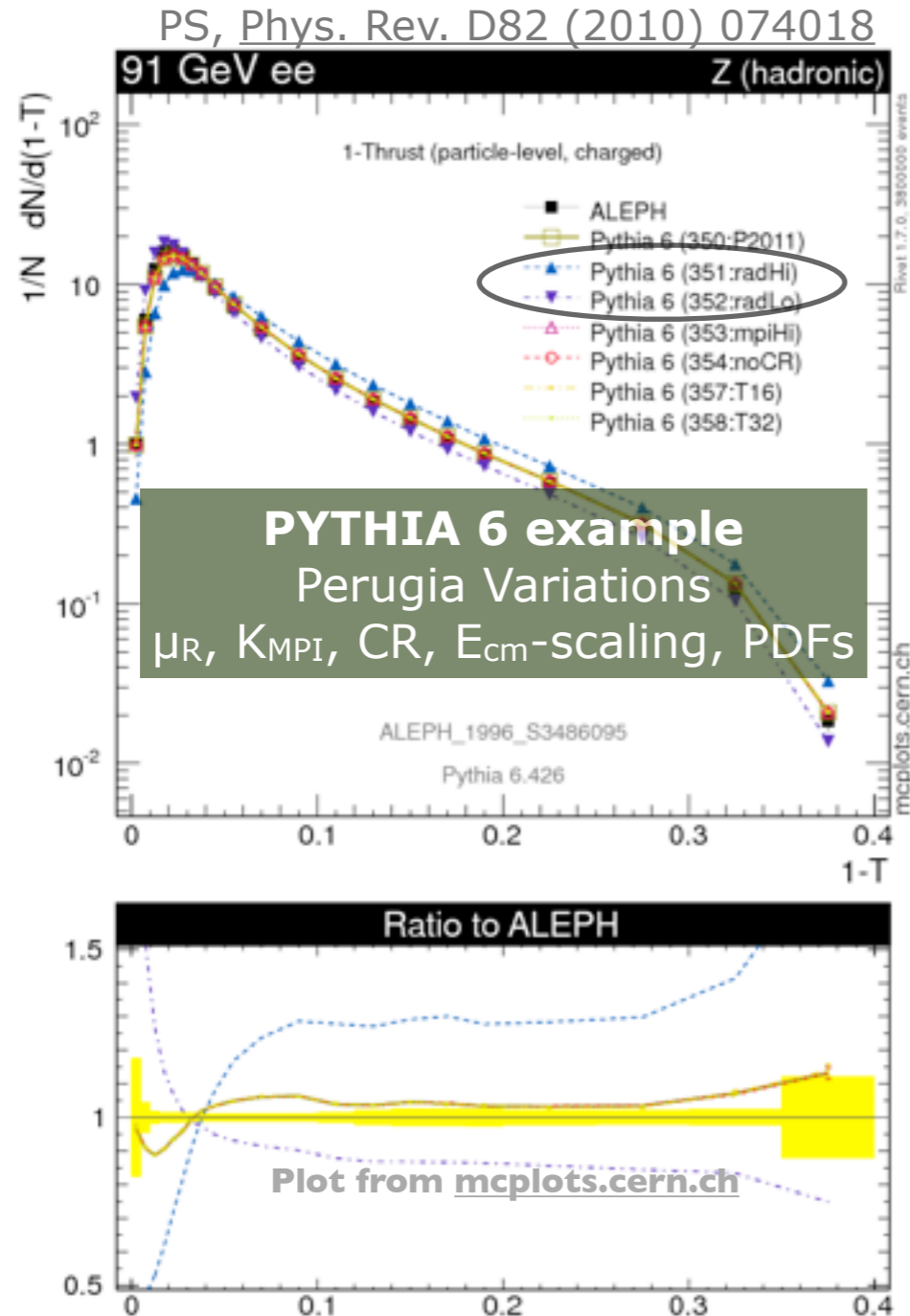
The simultaneous increase in detector complexity and in computation power has made simulation techniques an essential feature of contemporary experimentation. The Monte Carlo simulation has become the major means of visualization of not only detector performance but also of physics phenomena. So far so good.

But it often happens that the physics simulations provided by the the MC generators carry the authority of data itself. They look like data and feel like data, and if one is not careful they are accepted as if they were data. All Monte Carlo codes come with a GIGO (garbage in, garbage out) warning label. But the GIGO warning label is just as easy for a physicist to ignore as that little message on a packet of cigarettes is for a chain smoker to ignore. I see nowadays experimental papers that claim **agreement with QCD** (translation: someone’s simulation labeled QCD) and/or **disagreement with an alternative piece of physics** (translation: an unrealistic simulation), without much evidence of the inputs into those simulations.”

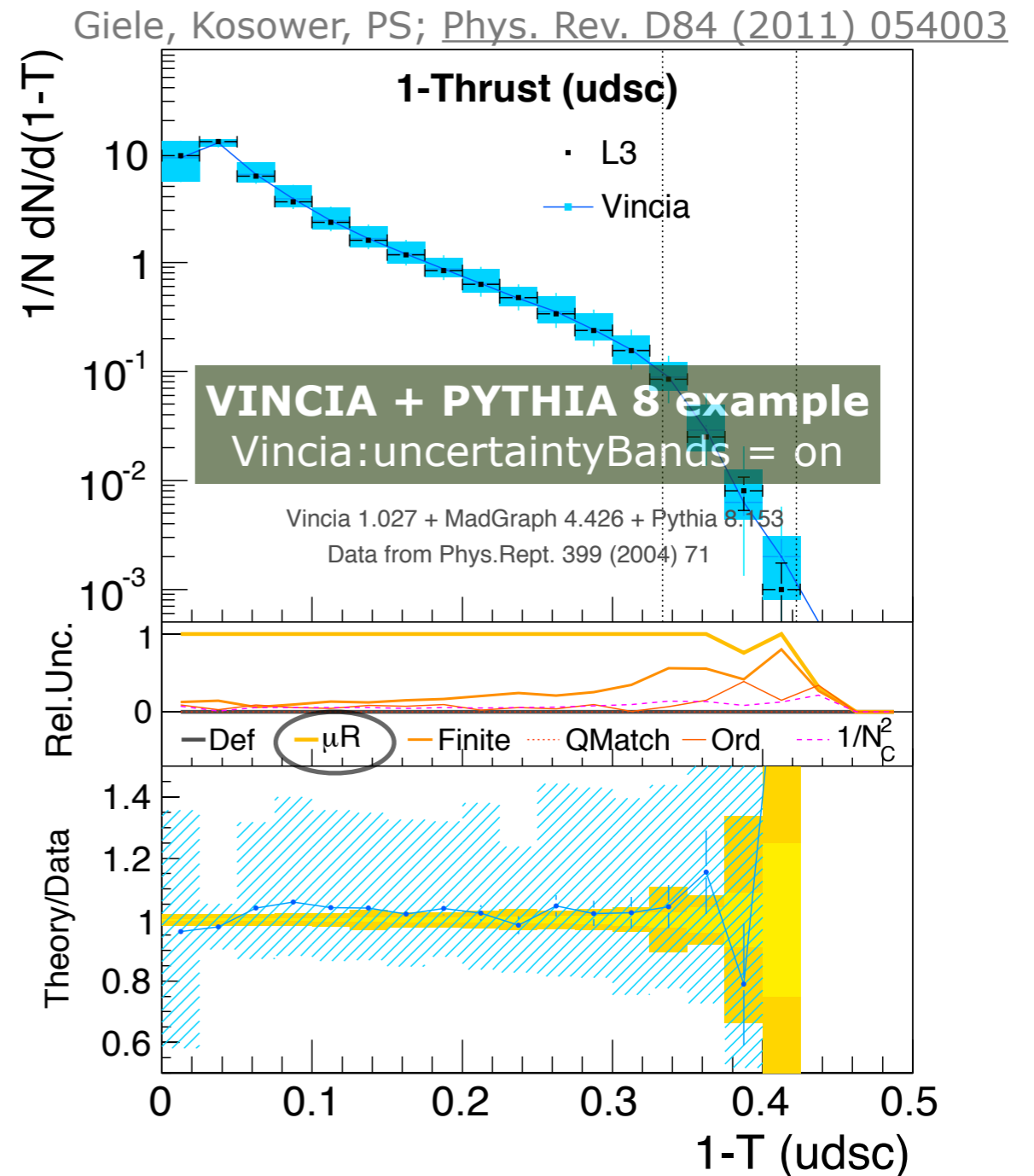
Account for parameters + pertinent cross-checks and validations
Do serious effort to estimate uncertainties, by salient MC variations

Uncertainty Estimates

a) Authors provide specific "tune variations"
 Run once for each variation → envelope

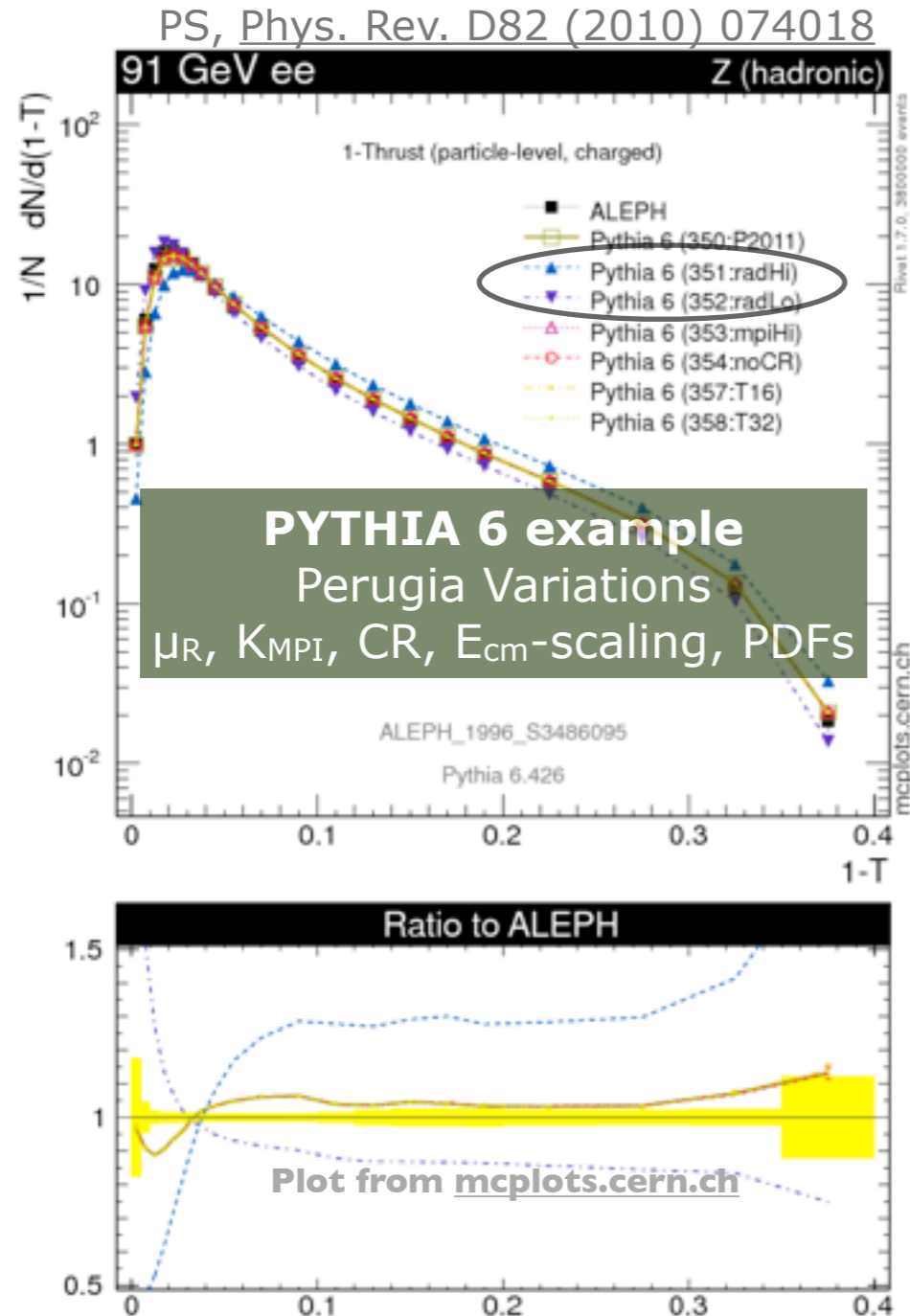


b) **One** shower run
 + unitarity-based uncertainties → envelope

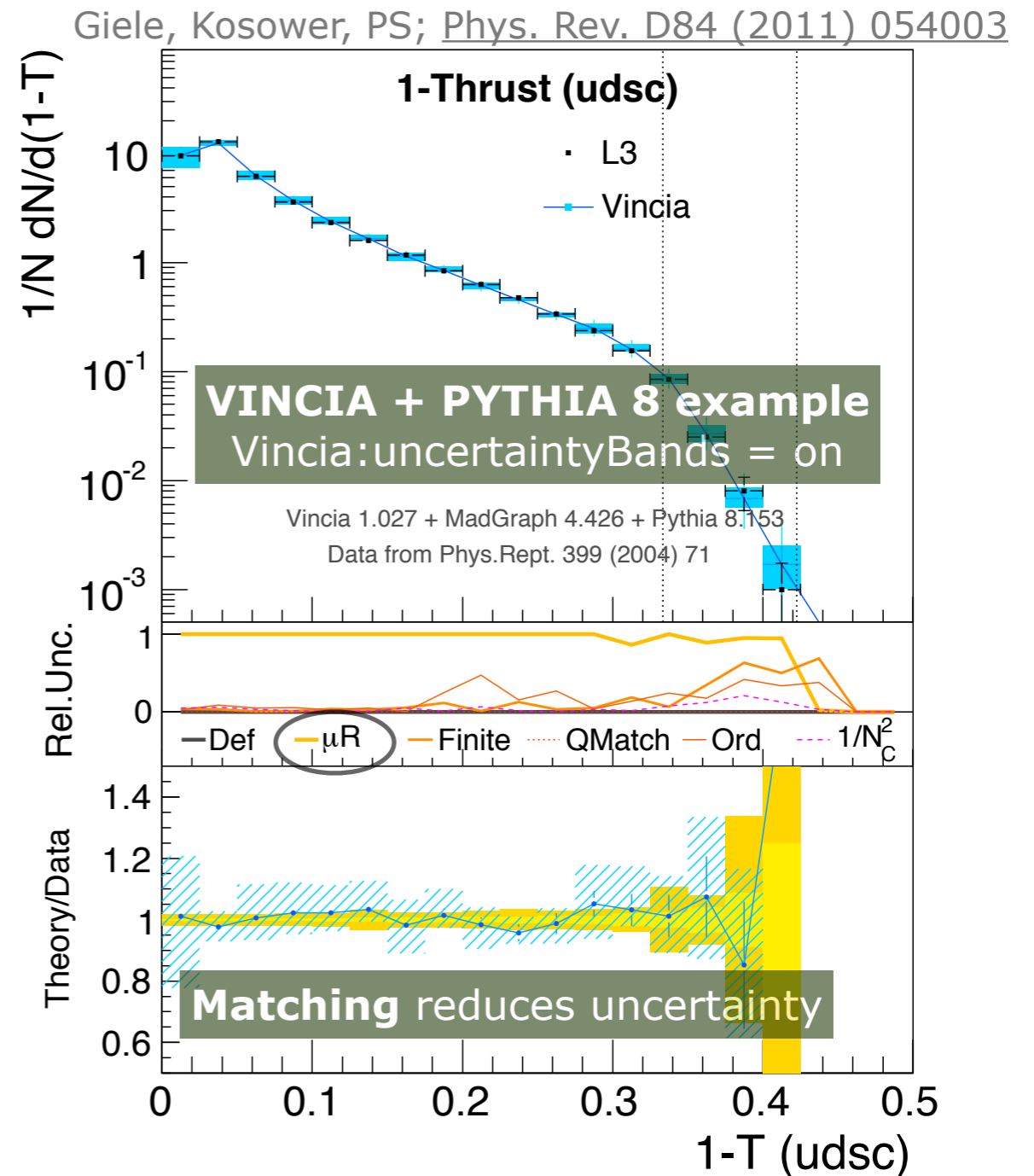


Uncertainty Estimates

a) Authors provide specific "tune variations"
 Run once for each variation → envelope



b) **One** shower run
 + unitarity-based uncertainties → envelope



Summary: Parton Showers

Aim: generate events in as much detail as mother nature

→ Make stochastic choices \sim as in Nature (Q.M.) → Random numbers

Factor complete event probability into separate universal pieces, treated independently and/or sequentially (Markov-Chain MC)

Improve lowest-order perturbation theory by including 'most significant' corrections

Resonance decays (e.g., $t \rightarrow bW^+$, $W \rightarrow qq'$, $H^0 \rightarrow \gamma^0 \gamma^0$, $Z^0 \rightarrow \mu^+ \mu^-$, ...)

Bremsstrahlung (FSR and ISR, exact in collinear and soft* limits)

Hard radiation (matching, discussed tomorrow)

Hadronization (strings/clusters, discussed tomorrow)

Additional Soft Physics: multiple parton-parton interactions, Bose-Einstein correlations, colour reconnections, hadron decays, ...

Coherence*

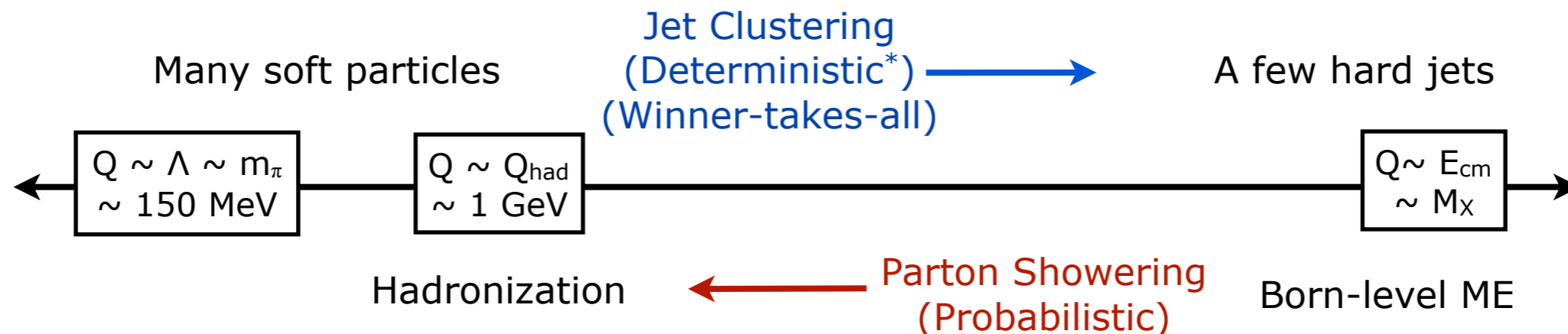
Soft radiation → Angular ordering or Coherent Dipoles/Antennae

See also: **1)** MCnet Review (long): [Phys.Rept. 504 \(2011\) 145-233](#) and/or **2)** PDG Review on Monte Carlo Event Generators, and/or **3)** PS, TASI Lectures (short): [arXiv:1207.2389](#)

Jets vs Showers

Jet clustering algorithms

Map event from low resolution scale (i.e., with many partons/hadrons, most of which are soft) to a higher resolution scale (with fewer, hard, IR-safe, jets)



Parton shower algorithms

Map a few hard partons to many softer ones

Probabilistic → closer to nature. Not uniquely invertible by any jet algorithm*

(* See “Qjets” for a probabilistic jet algorithm, [arXiv:1201.1914](https://arxiv.org/abs/1201.1914))

(* See “Sector Showers” for a deterministic shower, [arXiv:1109.3608](https://arxiv.org/abs/1109.3608))

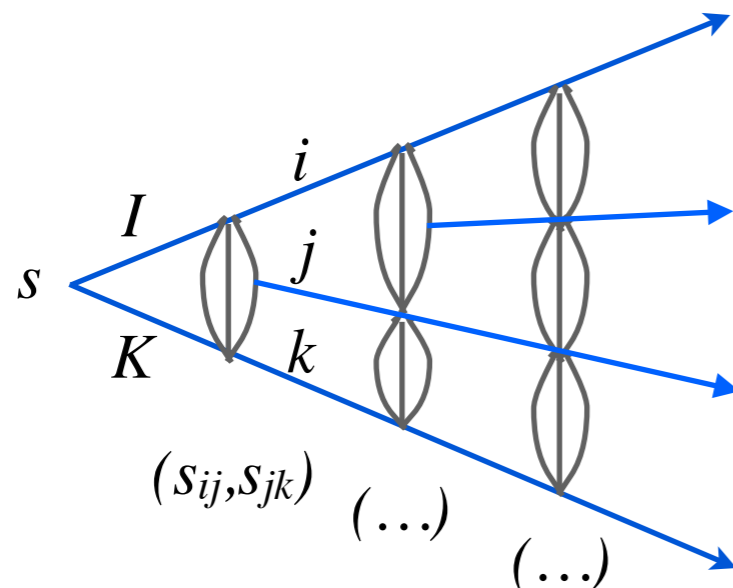
Antennae

Observation: the evolution kernel is responsible for generating real radiation.

- Choose it to be the ratio of the real-emission matrix element to the Born-level matrix element
- AP in coll limit, but also includes the Eikonal for soft radiation.

Dipole-Antennae
(E.g., ARIADNE, VINCIA)

$$d\mathcal{P}_{IK \rightarrow ijk} = \frac{ds_{ij}ds_{jk}}{16\pi^2s} a(s_{ij}, s_{jk})$$



2→3 instead of 1→2
(→ all partons on shell)

$$a_{q\bar{q} \rightarrow qg\bar{q}} = \frac{2C_F}{s_{ij}s_{jk}} (2s_{ik}s + s_{ij}^2 + s_{jk}^2)$$

$$a_{qg \rightarrow qgg} = \frac{C_A}{s_{ij}s_{jk}} (2s_{ik}s + s_{ij}^2 + s_{jk}^2 - s_{ij}^3)$$

$$a_{gg \rightarrow ggg} = \frac{C_A}{s_{ij}s_{jk}} (2s_{ik}s + s_{ij}^2 + s_{jk}^2 - s_{ij}^3 - s_{jk}^3)$$

$$a_{qg \rightarrow q\bar{q}'q'} = \frac{T_R}{s_{jk}} (s - 2s_{ij} + 2s_{ij}^2)$$

$$a_{gg \rightarrow g\bar{q}'q'} = a_{qg \rightarrow q\bar{q}'q'}$$

... + non-singular terms

The "CKKW" Prescription

Catani, Krauss, Kuhn, Webber, JHEP11(2001)063
Lönblad, JHEP05(2002)046

Start from a set of fixed-order MEs

Separate Phase-Space Integrations

$$\sigma_F^{\text{inc}}$$

$$\sigma_{F+1}^{\text{inc}}(Q_{\text{cut}})$$

$$\sigma_{F+2}^{\text{inc}}(Q_{\text{cut}})$$

Wish to add showers while eliminating Double Counting:
Transform inclusive cross sections, for "X or more", to exclusive ones, for "X and only X"

Jet Algorithm (CKKW) → Recluster back to F → "fake" brems history
Or use statistical showers (Lönblad), now done in all implementations
Reweight each internal line by shower Sudakov factor & each vertex by $\alpha_s(\mu_{\text{PS}})$

$$\sigma_{F+1}^{\text{exc}}(Q_{F+1})$$

$$\sigma_{F+2}^{\text{exc}}(Q_{F+2})$$

Reweight each external line by shower Sudakov factor

$$\sigma_F^{\text{exc}}(Q_{\text{cut}})$$

$$\sigma_{F+1}^{\text{exc}}(Q_{\text{cut}})$$

Now add a genuine parton shower → remaining evolution down to confinement scale

Start from Q_{cut}

Start from Q_{F+2}

Automatic Uncertainty Estimates

One shower run (VINCIA + PYTHIA)
+ unitarity-based uncertainties → envelope
 Giele, Kosower, PS; [Phys. Rev. D84 \(2011\) 054003](#)

```
*----- PYTHIA Event and Cross Section Statistics -----*
```

Subprocess	Code	Number of events			sigma +- delta	
		Tried	Selected	Accepted	(estimated)	(mb)
f fbar -> gamma*/Z0	221	10511	10000	10000	4.143e-05	0.000e+00
sum		10511	10000	10000	4.143e-05	0.000e+00

```
*----- End PYTHIA Event and Cross Section Statistics -----*
```

```
*----- VINCIA Statistics -----*
```

Number of nonunity-weight events = none
 Number of negative-weight events = none

This run	weight(i) i =	IsUnw	Avg Wt <w>	Avg Dev <w-1>	rms(dev)	kUnwt 1/<w>	Expected Max wt	effUnw <w>/MaxWt
User settings	0	yes	1.000	0.000	-	1.000	-	-
Var : VINCIA defaults	1	yes	1.000	0.000	-	1.000	1.000	1.000
Var : AlphaS-Hi	2	no	0.996	-3.89e-03	-	1.004	22.414	4.44e-02
Var : AlphaS-Lo	3	no	1.020	1.99e-02	-	0.981	43.099	2.37e-02
Var : Antennae-Hi	4	no	1.000	2.61e-04	-	1.000	5.417	0.185
Var : Antennae-Lo	5	no	0.996	-4.33e-03	-	1.004	10.753	9.26e-02
Var : NLO-Hi	6	yes	1.000	0.000	-	1.000	1.000	1.000
Var : NLO-Lo	7	yes	1.000	0.000	-	1.000	1.000	1.000
Var : Ordering-Stronger	8	no	1.004	4.48e-03	-	0.996	14.225	7.06e-02
Var : Ordering-mDaughter	9	no	1.033	3.25e-02	-	0.968	55.954	1.85e-02
Var : Subleading-Color-Hi	10	no	1.001	7.37e-04	-	0.999	1.505	0.665
Var : Subleading-Color-Lo	11	no	1.006	6.44e-03	-	0.994	5.283	0.191

```
*----- End VINCIA Statistics -----*
```